Companion to the Public School Arithmetic

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COMPANION

TO THE

PUBLIC SCHOOL ARITHMETIC.

FOR THE USE OF TEACHERS AND PRIVATE STUDENTS.

BY

WM. SCOTT, B.A.,

PRINCIPAL OF THE TORONTO NORMAL SCHOOL,

AND

CHARLES A. BARNES, M.A.,

PUBLIC SCHOOL INSPECTOR.

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PREFACE.

This is a work intended for the use of teachers and private students.

In preparing the Public School Arithmetic, it was deemed advisable to omit the answers to the purely mechanical examples. This book gives the answers to all the examples so that the teacher may be saved the trouble of working them.

The time of the teacher is so fully occupied by the ordinary school duties, that it is not always possible to find an opportunity for solving all the more difficult problems, hence a work such as this becomes a great convenience.

To supply the younger teachers with the experience gained from years of teaching this subject, Suggestions for Teaching Arithmetic have been added. In the preparation of these the authors desire to acknowledge the valuable aid given to them by W. H. Elliott, B.A., Vice-Principal of the Toronto Normal School; A. McIntosh, Head Master, R. W. Murray, First Assistant, T. M. Porter, Second Assistant, of the Provincial Model School, Toronto; Miss M. T. Scott, Principal of the Presbyterian Ladies' College, and W. Wilson, Principal of the Model School, Toronto Junction.

A collection of seven hundred and twenty-one problems, covering all the fields of the public school course, together with the answer to each, and where there might be difficulty in the solution, a hint as to the mode of procedure are given.

To facilitate certain mechanical calculations, a table of squares, cubes, square roots and cube roots of all integral numbers up to 1000 has been inserted.

Indications of errors, omissions or obscurities will be thankfully received.

TORONTO, July 24th, 1901.

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3. Sixteen; sixty-one; seventy-one; eighty-one; eighteen; ninety-two.

4. Twenty-nine; ninety-three; thirty-nine; forty-seven; seventy-four; forty-nine.

5. Ninety-four; eighty-six; sixty-eight; seventy-six; sixty-seven; thirty-two.

6. Twenty-three; thirty-four; forty-three; forty-five; fifty-four; fifty-six.

7. Sixty-five; seventy-eight; eighty-seven; ninety-one; nineteen; twenty-four.

8. Forty-two; twenty five; fifty-two; twenty-six; sixty-two; eighty-two.

9. Twenty-eight; ninety-five; fifty-nine; sixty-nine; ninety-six; ninety-seven.

10. Seventy-nine; forty-eight; eighty-four; sixty-four; forty-six; one hundred.

EXERCISE II. PAGE 8.

1. Five hundred; seven hundred; eight hundred; two hundred; six hundred; four hundred.

2. Eight hundred and thirty; four hundred and seventy; six hundred and nine; seven hundred and fourteen; eight hundred and six; nine hundred and ninety.

3. Six hundred and seven; seven hundred and six; six hundred and seventy; seven hundred and sixty; nine hundred and one; one hundred and nine.

4. Eight hundred and forty-seven; seven hundred and eighty-four; four hundred and seventy-eight; seven hundred and forty-eight; eight hundred and seventy-four; four hundred and eighty-seven.

5. Two hundred and five; four hundred and six; five hundred and seven; seven hundred and five; five hundred and seventy; seven hundred and fifty.

6. Four thousand, seven hundred and five; five thousand, eight hundred and forty-three; nine thousand, one hundred and sixty-eight; two thousand and forty-nine; four thousand, nine hundred and five; four thousand, two hundred and thirty-one.

7. Eight thousand and seven; eight thousand and seventy; eight thousand, seven hundred; seven thousand and eighty; seven thousand and eight; nine thousand and forty-five.

8. Five thousand and sixty-eight; nine thousand and eight; six thousand, two hundred and two; one thousand, five hundred and eight; three thousand, seven hundred and forty-six; five thousand, seven hundred and eighty-six.

9. Two thousand, one hundred and thirty-four; one thousand, two hundred and thirty-four; one thousand, two hundred and forty-three; four thousand, three hundred and twenty-one; four thousand, one hundred and thirty-two; eight thousand and nine.

10. Nine thousand and one; one thousand and sixty-nine; nine thousand, six hundred and one; nine thousand, one hundred and six; nine thousand, six hundred and ten; nine thousand and eighty.

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1. Forty thousand, eight hundred and seventy-six; fifty-four thousand and eighty-nine; thirty thousand and seventy; ninety-one thousand, one hundred and eleven.

2. Fifty-four thousand and eighty-nine; forty thousand and fifty-eight; thirty-nine thousand and seven; forty thousand, one hundred and six.

3. Eighty-one thousand and ninety-five; fifty-nine thousand, one hundred; ninety-five thousand, one hundred and one; thirty-seven thousand, six hundred and ninety-eight.

4. Eight hundred and nineteen thousand and seventy-five; eight hundred and ninty-one thousand, four hundred and ninety-two; two hundred and ninety-four thousand, seven hundred and sixteen; four hundred and ten thousand, one hundred and one.

5. Two hundred and ten thousand and ten; nine hundred and eighteen thousand and five; two hundred and thirty-one thousand and forty-six; two hundred and seventeen thousand and six.

6. One million, four hundred and seventy-six thousand, nine hundred and thirty-one; three million, seven hundred and ninety-six thousand, eight hundred and forty-two; nine hundred and ten thousand and ten; two million, one hundred and three thousand and one.

7. Three million, ninety-one thousand and six; four million, seven thousand and seven; one hundred and ninety thousand, one hundred; three million, nineteen thousand and forty-seven.

8. Seventeen million, ninety-one thousand, eight hundred and seven; ninety-three million, seven hundred and four thousand, nine hundred and five; nine hundred and three thousand, seven hundred and five; twenty-one million, ten thousand, one hundred and one.

9. Forty million, ninety-seven thousand and six; thirty million, nine hundred and ten thousand, four hundred and five; nine million, three hundred and seventeen thousand, nine hundred and sixty; seventy million, six hundred and five.

10. Two billion, three hundred and nine million, four hundred thousand, one hundred and six; ninety-one billion, four hundred and ninety-one million, five hundred and ninety-one thousand, six hundred and ninety-one; one billion, thirteen million, two hundred and ten thousand and twelve; seven billion, sixty million, seven hundred thousand and one.

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1. 15; 21; 34; 78; 87; 19; 91.

2. 100; 107; 316; 440; 909; 919; 991.

3. 1000; 1402; 8006; 8060; 2496.

4. 1011; 10604; 29470; 80990.

5. 804016; 291704.

6. 1101001; 5004030.

7. 954000806.

8. 7707707.

9. 146147047.

10. 536347972.

11. 6000095148; 700000000001.

12. 99000037004.

13. 864538217953.

14. 40004400414.

15. 49000000058798.

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5. CIX; CIV; CV; CCXIX; CCCXXIX.

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7. CDIV; CDXLIV; CDXCIX; DLXXXIX; DXCIV.

8. DCCXCVI; DCCLXXXIX; DCCLXXVII; DCCLXVI; DCXCIX.

9. DCCCLXXV; DCCCLVII; DLXXVIII; DCCCXCIV; CMLXXXIV.

10. CMLXXIII; CMLXV; CMXCIX; CMLXXXVII; CMXLIV.

11. MCM; MDCCCLXXXIX; MDCCCXCIX; MDCCCXLIX; MCMIV.

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2. 111; 109; 144; 229; 504; 249; 709.

3. 404; 730; 540; 444; 529; 624.

4. 833; 484; 333; 434.

5. 505; 430; 297; 244; 323; 555.

6. 1111; 1500; 1014; 990; 999; 1354.

7. 1290; 1555; 1444; 1009; 1222; 1490.

8. 2222; 947; 1906; 1400; 1909; 1555.

9. 1744; 1449; 1666; 2009; 1119.

10. 1599; 1900; 1901; 1899.

EXERCISE VII. PAGE 10.

1. 10; 100; 1000.

2. CMXCIX.

3. The value of the letter to the left is to be subtracted from that to the right; IV; IX; XL; XC.

4. I can be written to the left of V and X.

5. X can be placed to the left of I, V, L and C.

6. 1, I; 2, II; 3, III; 4, IV; 5, V; 6, VI; 7, VII; 8, VIII; 9, IX.

7. V, L, and D cannot be repeated. If repeated, they would give one of the next higher order.

8. The earth is ninety-two million, eight hundred and ninety thousand miles distant from the sun. The length of the equator is forty-three million, eight hundred and twenty-seven thousand and thirty-three yards. The cheese factories of Ontario produced one hundred and thirty-seven million, three hundred and sixty-two thousand, nine hundred and sixteen pounds of cheese in eighteen hundred and ninety-seven. The milk used in making the cheese was one billion, four hundred and fifty-five million, nine hundred and thirty-seven thousand, one hundred and forty-eight pounds. In eighteen hundred and ninety-eight, the ninety-eight creameries making returns to the Bureau of Industries produced nine million, eight thousand, nine hundred and ninety-two pounds of butter, valued at one million, six hundred and thirty-two thousand, two hundred and thirty-four dollars.

9. V; X; XV; XXV; XXXV.

10. 100; 110; 120; 130; 200; 210; 220; 230; 300; 310; 320; 330.

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1. 76; 67; 77; 89; 69.

2. 988; 896; 1098; 999; 998.

3. 8999; 11636; 9987; 9998; 9968.

4. 18576; 20687; 20987; 19585; 19967.

5. 204996; 225469; 196678; 187898; 214829.

6. 7497. *7.* 8789. *8.* 7898. *9.* 35888. *10.* 39476.

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1. 49 ct. *2.* $47. *3.* 209 bu. *4.* 87 mi. *5.* 124 mi. *6.* 945769. *7.* 969 yd. *8.* 1367 mi. *9.* 786. *10.* 1668 pages.

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1. 1433; 1330; 1020; 1314; 1312; 13312.
2. 15242; 14151; 14422; 14322; 133133; 132243.
3. 141722; 162222; 152651; 157526; 173245; 160513.
4. 184254; 198598; 178467; 206676: 189305; 172498.
5. 200396; 246121; 226250; 285326; 221632; 238796.
6. 249180; 192931; 320105; 217479; 276014; 258123.
7. 21164; 268399; 209846; 2634792; 25869; 242007.
8. 268655; 294395; 269181; 303376; 249409; 303737.
9. 277929; 304154; 289039; 289446: 308648; 285296.
10. 358489; 302884; 231950; 266875; 368887; 301027.

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1. 3432890; 4584711; 3328081; 5437312; 4801749.
2. 3380656; 4164757; 4212613; 4644035; 5870413.
3. 3885336; 4150872; 3515001; 3929106; 3462627.
4. 1521900; 4312101; 5473906; 1109747; 14126839.
5. 40393623; 47219804; 465850334; 30717060; 57286686.
6. 52888932; 366547583; 377268904; 530116458; 436075579.
7. 30970654; 6772830; 4564359; 11556690; 39154681.
8. 65331127; 11813141; 12048191: 22755959; 23313249.
9. 186811552; 77223388; 29185155; 49579202; 467645729.
10. 62637126; 661580628; 723283090: 550941556; 746063693.

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1. 62075. *2.* 10601. *3.* 5186837. *4.* 3309278. *5.* 5199140. *6.* 13786. *7.* 2683. *8.* 10288. *9.* 11947. *10.* 13254.

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1. 87432779. *2.* 66751805. *3.* 68318319. *4.* 2384473. *5.* 2000000. *6.* 5263704. *7.* 21044. *8.* 53497. *9.* 457709135. *10.* 3315109.

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1. 303. *2.* 354 A. *3.* 272 marbles. *4.* 158 mi. *5.* $7353. *6.* 1273 animals. *7.* 885 trees. *8.* MCXVII. *9.* 46391. *10.* 226.

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1. 40 marbles. *2.* $45. *3.* 870 pages. *4.* 300 mi. *5.* 1000 ct. *6.* $1900. *7.* $270. *8.* 98 ct. *9.* 208 mi. *10.* $7856.

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1. 9054. *2*. $9000. *3*. 814 baskets. *4*. 106 ft. *5*. 170 ct. *6*. $129. *7*. 144 ct. *8*. 500 ct. *9*. 160 A. *10*. 90 sheep; $389.

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1. $9991. *2*. $350. *3*. 197795. *4*. $52948276. *5*. 3617 sheep. *6*. 510 mi. *7*. $100. *8*. $120. *9*. 195 ct. *10*. 3666.

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1. 76 mi. *2*. 1159620. *3*. $45000. *4*. 1171460 oranges. *5*. $3811. *6*. 78. *7*. $31839. *8*. $12085. *9*. $33820. *10*. $33020.

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1. 1990977. *2*. 72720, 147720. *3*. $18565. *4*. 154 yd. *5*. 31136. *6*. 131 ct. *7*. 365 days. *8*. 12413. *9*. 1832. *10*. $140900.

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2. 541; 526; 333; 563; 351; 446.
3. 622; 432; 513; 311; 242; 517.
4. 4635; 2745; 5245; 4711; 1111; 3531.
5. 20532; 76464; 51236; 36335; 20142; 18155.
6. 14234; 15524; 26152; 26324; 52433; 834251.
7. 24730; 16330; 16230; 36310; 44002; 86443.
8. 6232; 24414; 3445; 3117; 2543; 46444.
9. 5215; 105542; 7724; 3232; 845462.
10. 125553; 505072; 224; 245000; 67302.

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1. 111 cows. *2*. 40 chickens. *3*. $23. *4*. 113 pages. *5*. 272 gal. *6*. $150. *7*. 254 bbl. *8*. 1232 sheep. *9*. 433 A. *10*. $1200.

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4. 2215; 1414; 2343; 3212; 2962; 3925.
5. 1345; 1254; 1265; 3312; 1524; 1018.
6. 18354; 25233; 33222; 35343; 17427; 27728.
7. 105054; 107014; 302073; 304022; 177244; 312816.
8. 226173; 114542; 234121; 213653; 132590; 679924.
9. 457022; 318234; 515255; 625223; 725675; 876899.
10. 491978; 261636; 294928; 680929; 281939; 577184.

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1. 545; 788. *2.* 622; 112.
3. 1353; 5396. *4.* 29597; 28655.
5. 22610; 6779. *6.* 387712; 476993.
7. 601; 479779. *8.* 322229; 199600.
9. 457710; 191188. *10.* 185118; 409086.

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1. 3658; 4256; 5328. *2.* 5366; 4656; 4296.
3. 5257; 7766; 4454. *4.* 3257; 775; 3252.
5. 6389; 2766; 2682. *6.* 3157; 1444; 4474.
7. 4387; 822; 5275. *8.* 4127; 2077; 6255.
9. 4549; 3885; 3786. *10.* 5667; 1382; 2456.

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1. 32423; 7775; 277553. *2.* 20884; 60622; 443674.
3. 35524; 16653; 236387. *4.* 23691; 15343; 378778.
5. 47171; 25665; 315829. *6.* 55857; 38755; 350646.
7. 59094; 46558; 351705. *8.* 42875; 37810; 382316.
9. 9795; 30542; 345467. *10.* 6272; 30878; 434717.

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1. 4989050; 4997999. *2.* 1700261; 6088013.
3. 8974088; 75489688. *4.* 99019119; 6363637.
5. 6821430; 7276195. *6.* 798499; 4099996.
7. 7733440; 9797978. *8.* 8526021; 18351936.
9. 999989; 4058809. *10.* 3800909; 3770438.

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1. 74. *2.* 286032. *3.* 6319267. *4.* 32299. *5.* 676. *6.* 2714. *7.* 526084. *8.* 2578. *9.* 6667315. *10.* 141998.

EXERCISE XXVIII. PAGE 26.

1. 89 sheep. *2.* $257. *3.* $468. *4.* 24168 bu. *5.* $1779. *6.* $2168. *7.* $49. *8.* 1432 mi. *9.* 162482. *10.* 158677 cows.

EXERCISE XXIX. PAGE 26.

1. $807. *2.* 39 marbles. *3.* 13999 men. *4.* $3816. *5.* 351636 min. *6.* 439 pages. *7.* $21506957. *8.* 6300836 ft. *9.* 117993. *10.* 83849884 lb.

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1. 27 ct. *2.* 288. *3.* $11993. *4.* $35. *5.* 1642. *6.* 89 yr. *7.* 983 yr. *8.* 3688 ft. *9.* 1325 votes. *10.* 3969356.

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1. 45. *2.* $198. *3.* 215 bu. *4.* $3250. *5.* $32. *6.* $107950. *7.* $7178. *8.* 5 mi., 137 yd. *9.* 225 mi. *10.* 2210.

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1. $245. *2.* 1950. *3.* 1487 A. *4.* 45 yr., 38 yr. *5.* Loss, $2410. *6.* Jas. 31, Jno. 26. *7.* 45 A. *8.* 70 da. *9.* 18 marbles. *10.* 21171, 14896, 18691.

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1. 6746. *2.* 12968. *3.* 1606. *4.* 2658225. *5.* 6529. *6.* 925358. *7.* 2534. *8.* 73699. *9.* 2155. *10.* 8453. *11.* 23718. *12.* 74128.

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1. 6080408; 8604864; 846084; 680486; 6408604.
2. 1948468; 9926482; 1759432; 19694912; 14169134.
3. 1536128; 1156934; 1416192; 1197492; 1935730.
4. 1726856; 1536994; 1401216; 734992; 1569934.
5. 10369362; 10363935; 13630368; 19603068; 10636392.
6. 26305425; 28942092; 26308368; 28454895; 23698458.
7. 13616536; 2536972; 16549616; 6713372; 27133748.
8. 21749956; 27156052; 3914596; 1383948; 16559056.
9. 1152280; 27160305; 17376320; 12682360; 6172835.
10. 22985225; 48412345; 39465430; 35404520; 38424815.
11. 28948225; 44753470; 27394730; 24488245; 43238245.
12. 22446384; 204426384; 22311858; 251344206; 25180824.

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1. 26254716; 41354790; 30484236; 22136562; 47081712.
2. 47345790; 28257882; 50944776; 41367474; 53267976.
3. 52198636; 53519851; 59197488; 63483546; 53794335.
4. 1719746; 26197948; 37944438; 29775494; 49045199.
5. 28615342; 28609378; 21636118; 34283578; 69135801.
6. 29136432; 25968520; 36836504; 18765080; 34821384.
7. 38352688; 71101248; 73876536; 63172344; 53606472.
8. 61479736; 20505024; 12289688; 26452200; 40130584.
9. 11111085; 50789376; 18661185; 38111103; 51880563.
10. 27583101; 58310775; 72813627; 49219155; 70776783.
11. 15846201; 47805255; 28395837; 13806819; 46219419.
12. 76897450; 70000680; 95060780; 56789430; 76847960.

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1. 757283566; 1135925349; 1514567132; 1893208915; 2271850698; 2650492481.

2. 957200648; 1435800972; 1914401296; 2393001620; 2871601944; 3350202268.

3. 757283668; 1135925502; 1514567336; 1893209170; 2271851004; 2650492838.

4. 936743668; 1405115502; 1873487336; 2341859170; 2810231004; 3278602838.

5. 3567201388; 4459001735; 5350802082; 6242602429; 7134402776; 8026203123.

6. 2431560780; 3039450975; 3647341170; 4255231365; 4863121560; 5471011755.

7. 4339170395; 5207004474; 6074838553; 6942672632; 7810506711; 8678340790.

8. 3391709345; 4070051214; 4748393083; 5426734952; 6105076821; 6783418690.

9. 2391890325; 2870268390; 3348646455; 3827024520; 4305402585; 4783780650.

10. 2091739230; 2510087076; 2928434922; 3346782768; 3765130614; 4183478460.

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1. 73185; 136612; 87822; 170765.

2. 94990; 122130; 169625; 284970.

3. 226752; 255096; 297612; 347214.

4. 201264; 431280; 469616; 517536.

5. 60936; 63475; 91404; 159957.

6. 205024; 288315; 345978; 461304.

7. 368880; 384250; 461100; 537950.

8. 677950; 774800; 871650; 968500.

9. 346005; 415206; 484407; 622809.

10. 146250; 195000; 243750; 292500.

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1. 1649466; 2434926; 3220386; 4005846; 5576766.

2. 4153795; 4834745; 5515695; 6196645; 1293805.

3. 1026571; 1342439; 1500373; 2290043; 2921779.

4. 5051124; 5633946; 6152010; 4144512; 3820722.

5. 11207664; 16524120; 21840576; 27731784.

6. 55361880; 87094880; 33881080; 62294320.

7. 29272750; 23147990; 62058230; 87908320.

8. 23515182; 31123035; 54407676; 54100288.

9. 395846112; 691940584; 494708876; 892826560.

10. 442875810; 616023720; 617415864; 783690063.

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1. 1200 mi. *2.* $195625. *3.* 607488 oz. *4.* 1632000 mi. *5.* 63168 ct. *6.* 702 trees. *7.* 2920 mi. *8.* 95424 hills. *9.* $130455. *10.* 75087 lemons.

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1. 1530000 ct. *2.* 32076 yd. *3.* 2368 nails. *4.* 62640 mi. *5.* 171216 in. *6.* 383600 ct. *7.* 482040 ct. *8.* $204000. *9.* 525600 min. *10.* 5285484 papers.

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1. 43050. *2.* $2241190. *3.* $5. *4.* 553 ct. *5.* $516. *6.* Gain, $32. *7.* 94720 ct. *8.* 2472 ct. *9.* 1037 mi. *10.* Gain, $83.

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1. 172484. *2.* 233189. *3.* 1062347. *4.* 67419143. *5.* 15403465. *6.* 2016978981. *7.* 1123193115. *8.* 63096033. *9.* 50700. *10.* 14254.

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1. 55986805. *2.* 1516. *3.* 62985. *4.* 9989001. *5.* 7051253. *6.* $30613. *7.* 57415 ct. *8.* 15725910 A. *9.* 703 stooks, 8436 sheaves. *10.* 99693084.

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1. 12; 23; 48; 42.
2. 234; 236; 418; 473.
3. 321; 233; 229; 292.
4. 121; 142; 184; 156.
5. 117; 127; 41; 149.
6. 103; 61; 81; 160.
7. 107; 121; 82; 135.
8. 121; 105; 116; 91.
9. 91; 102; 81; 40.
10. 571; 821; 749; 301.

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1. 214321; 104231; 121340; 212413; 231430.
2. 78998; 67897; 175896; 268979; 157985.
3. 132023; 52315; 154572; 288319; 248561.
4. 133692; 196181; 203373; 168627; 140683.
5. 36952; 100207; 148360; 71437; 185530.
6. 114126; 102057; 140292; 155246; 116667.
7. 102116; 135101; 83538; 45923; 90149.
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1. 2089317; 1392878; 1044658-2; 835726-4; 696439; 596947-5; 522329-2.

2. 3351703; 2234468-2; 1675851-2; 1340681-1; 1117234-2; 957629-3; 837925-6.

3. 2033933-1; 1355955-2; 1016966-3; 813573-2; 677977-5; 581123-6; 508483-3.

4. 3391933-1; 2261289; 1695966-3; 1356773-2; 1130644-3; 969123-6; 847983-3.

5. 1389028-2; 1041771-2; 833417-1; 694514-2; 595298; 520885-6; 463009-5.

6. 1359534-1; 1019650-3; 815720-3; 679767-1; 582657-4; 509825-3; 453178-1.

7. 1023547-1; 767660-2; 614128-2; 511773-4; 438663-1; 383830-2; 341182-4.

8. 767603; 614082-2; 511735-2; 438630-2; 383801-4; 341156-8; 307041-2.

9. 1003169-2; 802535-3; 668779-4; 573239-5; 501584-6; 445853-1; 401267-8.

10. 767716; 614172-4; 511810-4; 438694-6; 383858; 341207-1; 307086-4.

EXERCISE XLVII. PAGE 38.

1. 16, 28, 44, 88 yd. *2.* $121. *3.* $75. *4.* 329 bbl. *5.* 142 lb. *6.* 524 lb. *7.* 15065 lb. *8.* 532 pk. *9.* $7625. *10.* 379 t.

EXERCISE XLVIII. PAGE 39.

1. 480 mi. *2.* 1760 yd. *3.* $13. *4.* 13 hr. *5.* 30 ct. *6.* $32. *7.* 57 persons. *8.* 362 mi. *9.* 11625000 mi. a min.
10. 5642 pence.

EXERCISE XLIX. PAGE 39.

1. 1220-34; 1020-34; 876-58; 768-46; 684-10.

2. 556-17; 420-33; 243; 213; 189-54.

3. 3111; 2501; 2091; 1574-57; 1262-89.

4. 1725-18; 1482-21; 1299-24; 1156-47; 349-194.

5. 1481-255; 741-595; 659-377; 1174-92; 843-664.

6. 1566-216; 941-141; 2346-36; 776-550; 582-774.

7. 1138-529; 965-82; 847-460; 1286-121; 1577-390.

8. 824-106; 672-178; 582-568; 669-124; 494-460.

9. 1168-775; 1042-525; 1213-40; 2896-7; 1737-340.

10. 786-586; 1554-10; 801-424; 872-706; 1985-353.

EXERCISE L. PAGE 39.

1. 1082-870; 1071-192; 1057-416; 1012-954; 1048-438.

2. 929-464; 896-573; 883-482; 880-509; 897-594.

3. 1128-93; 1115-159; 1102-459; 1097-46; 1110-759.

4. 1456-486; 1433-317; 1421-59; 1412-666; 1417-5.

5. 1383-423; 1338-561; 1318-439; 1334-241; 1314-195.

6. 1443-225; 1411-224; 1425-405; 1464-288; 1408-304.

7. 2388-101; 2318-293; 2253-80; 2196-293; 2213-149.

8. 2456-40; 2364-256; 2517-274; 2380-280; 2422-42.

9. 20408-8; 26315-30; 34482-22; 58823-9; 52631-11; 27027-1.

10. 94966-9; 72621-10; 68587-1; 64977-4; 42571-8; 31655-22.

EXERCISE LI. PAGE 40.

1. 11550-5; 9625-5; 8250-5; 6930-5.

2. 36519; 29215-6; 24346; 20868.

3. 21217-3; 17680-48; 17049-24; 15155-3.

4. 29745-20; 23238-29; 20656-29; 18591-5.

5. 27374-8; 25027-31; 20856-24; 16221-42.

6. 25000-7; 21428-31; 12500-7; 11111-16.

7. 13249-16; 9937-6; 7949-36; 6624-46.

8. 13116-18; 11242-38; 9837-18; 8744-18.

9. 9876-41; 4938-41; 3292-41; 2469-41.

10. 1975-185; 1646-85; 1410-685; 1234-485.

EXERCISE LII. PAGE 40.

1. 6540. *2*. 391525. *3*. 489673. *4*. 150287. *5*. 481. *6*. 4798. *7*. 61. *8*. 378. *9*. 9605. *10*. 228689.

EXERCISE LIII. PAGE 40.

1. 134 da. *2*. $2003. *3*. 2640. *4*. 29603 lb. *5*. $658. *6*. 160 sq. rd. *7*. 327 mi. *8*. 275 persons. *9*. 168 lb. *10*. 9 da.

EXERCISE LIV. PAGE 41.

1. 179 mi. *2*. 96934 yd. *3*. 19 mi. *4*. 64 bu. *5*. 51 bu. *6*. 245 bu. *7*. 144. *8*. 75 c. yd. *9*. 207 bbl. *10*. 704 cd.

EXERCISE LV. PAGE 42.

1. 119. *2*. 910. *3*. 83. *4*. 1331424. *5*. 45. *6*. 27. *7*. 14839. *8*. 811. *9*. 41. *10*. 23849.

EXERCISE LVI. PAGE 42.

1. 7469. *2*. 23. *3*. 631229. *4*. 323603. *5*. 15061. *6*. 66. *7*. 245. *8*. 119025. *9*. 767561. *10*. 336.

EXERCISE LVII. PAGE 43.

1. 840 steps. *2*. 14668. *3*. 2380277. *4*. 1249. *5*. 962556. *6*. 179538. *7*. 253440 in. *8*. 20063110804. *9*. 35643910. *10*. 40160 ct.

EXERCISE LVIII. PAGE 43.

1. 91132497. *2*. $288. *3*. 1145348. *4*. 6029. *5*. 30595. *6*. 815. *7*. 99368. *8*. $254. *9*. 1420799. *10*. 2240 ct.

EXERCISE LIX. PAGE 44.

1. $336. *2*. 18 ct. *3*. $5 each. *4*. 2632 ct. *5*. 200 ct. *6*. $50. *7*. $1620. *8*. 485 sheep. *9*. 11657397. *10*. 8 yr.

EXERCISE LX. PAGE 44.

1. 2552. *2*. 18480 ct. *3*. 95 ct. *4*. 42. *5*. 10624. *6*. 960 ct. *7*. 72 ct. *8*. $4860. *9*. 113. *10*. 203.

EXERCISE LXI. PAGE 45.

1. 27 ct. *2*. 16 ct. *3*. 21 mi. *4*. $51. *5*. $150. *6*. 30 yd. *7*. 21 dresses. *8*. 1728 bu. *9*. 11 hr. *10*. 579600 lb.

EXERCISE LXII. PAGE 45.

1. 21 da. *2*. 95 da. *3*. 20 da. *4*. 27 da. *5*. 18 da. *6*. 15 men. *7*. 45 men. *8*. 6 men. *9*. 240 men. *10*. 225 men.

EXERCISE LXIII. PAGE 46.

1. 16, 20. *2*. 20, 30. *3*. 39 bu., 46 bu. *4*. 42 yd., 58 yd. *5*. 44, 50. *6*. 12 yr., 15 yr. *7*. 19, 28. *8*. 111 votes, 129 votes. *9*. 67 mi., 90 mi. *10*. 192, 195.

EXERCISE LXIV. PAGE 46.

1. 8458. *2*. 181357. *3*. $3570. *4*. 264 mi. *5*. $16. *6*. 240 ct. *7*. $6. *8*. $135. *9*. 170. *10*. $46836.

EXERCISE LXV. PAGE 47.

1. 33148. *2*. 5607 ct. *3*. 10 ct. *4*. 17. *5*. 47 ct. *6*. 21450 ct. *7*. 19 gal. *8*. $170. *9*. 2046 ct. *10*. $468.

EXERCISE LXVI. PAGE 48.

1. 120 pieces. *2*. 159 pieces. *3*. 9 ct. *4*. 21. *5*. 1800 ct. *6*. 11 stacks. *7*. 302304 ct. *8*. 500 A. *9*. 60 bu. *10*. 17 hats.

EXERCISE LXVII. PAGE 48.

1. 36405 bu. *2*. 202878 ct. *3*. 23 lb. *4*. $2028. *5*. Gain, $161. *6*. 38400 lb. *7*. 5355 ct. *8*. 4860 ct., 90 ct. *9*. 36 lb. *10*. 1953.

EXERCISE LXVIII. PAGE 49.

1. 1580712. *2*. 148 mi. *3*. $2277. *4*. 1862 ct. *5*. $2688. *6*. 150 boys. *7*. $31104. *8*. 86 A., $42. *9*. 132 head. *10*. 114 mi.

EXERCISE LXIX. PAGE 50.

1. 725c.; 408c.; 505c. *2.* 3004c.; 10090c.; 300000c.

3. 706c.; 1045c.; 8007c.; 10090c.; 2705c.; 40404c.

4. 10001c.; 100000c.; 10181c.; 100001c.; 7000770c.

5. 37175c.; 57549c.; 618472c.; 37884c.; 96709c.

6. \$7; \$36.84; \$449.68. *7.* \$10.01; \$576.48; \$100.

8. \$270.10; \$470.06; \$74000.07. *9.* \$970; \$20000.07; \$47100.71.

10. \$4708.41; \$10101.01; \$206006.09.

EXERCISE LXX. PAGE 51.

1. 32 far.; 496 far.; 1500 far.; 1804 far.; 3136 far.; 7840 far.; 28000 far.

2. 60d.; 852d.; 1200d.; 2892d.; 4428d.; 3564d.; 108108d.

3. 140s.; 340s.; 1800s.; 8380s.; 977s.; 2012s.

4. 264 far.; 400 far.; 504 far.; 579 far.; 948 far.; 9600 far.

5. 6240 far.; 4704 far.; 6752 far.; 5464 far.; 48933 far.

6. 9d.; 180d.; 106¼d.; 225d.; 250d.; 690d.

7. 6s. 8d.; 14s.; 31s. 3d.; 150s. 5d.; 1474s. 1d.; 416s. 8d.

8. £4; £38 4s.; £93 19s.; £228 7s.

9. £3 4s.; £15 7s. 5d.; £30 18s.; £365 4s. 3d.

10. £3 16s. 9d.; £8 0s. 2¼d.; £47 11s. 6d.; £82 4s. 8¾d.

EXERCISE LXXI. PAGE 54.

1. 32 oz.; 80 oz.; 464 oz.; 2048 oz.

2. 64000 oz.; 96800 oz.; 243200 oz.; 224015 oz.

3. 2 lb. 4 oz.; 30 lb.; 36 lb. 9 oz.; 447 lb. 10 oz.

4. 2 t. 216 lb.; 41 t. 161 lb.; 11 t. 949 lb.; 24 t. 1352 lb. 9 oz.

5. 84 in.; 116 in.; 351 in.; 1746 in.

6. 9 yd.; 12 yd. 2 ft. 8 in.; 104 yd. 1 ft. 9 in.; 495 yd. 2 ft. 10 in.

7. 1152 sq. in.; 2548 sq. in.; 6480 sq. in.; 288 sq. in.

8. 8 sq. yd. 6 sq. ft.; 97 sq. yd. 3 sq. ft.; 2 sq. yd. 7 sq. ft. 89 sq. in.; 136 sq. yd. 1 sq. ft. 15 sq. in.

9. 5184 cu. in.; 8640 cu. in.; 15552 cu. in.; 209188 cu. in.

10. 13 cd. 104 cu. ft.; 45 cd. 8 cu. ft.; 76 cd. 36 cu. ft.; 754 cd. 29 cu. ft.

EXERCISE LXXII. PAGE 55.

1. 8 pt.; 14 pt.; 193 pt.; 943 pt.

2. 6 gal. 3 qt.; 190 gal. 1 qt.; 71 gal.; 1220 gal. 5 pt.

3. 79 bu. 28 lb.; 1702 bu. 28 lb.; 1373 bu. 3 lb.

4. 24 pt.; 42 pt.; 62 pt.; 75 pt.

5. 2 gal. 1 pt.; 30 gal. 2 qt. 1 pt.; 961 gal. 0 qt. 1 pt.; 251 gal. 1 qt.

6. 425 sec.; 62100 sec.; 436020 sec.

7. 5 da.; 49 da. 21 hr. 36 min.; 23 da. 3 hr. 33 min.; 9 da. 5 hr. 20 min. 41 sec.

8. 1440″; 46097″; 123319″; 615651″.

9. 11° 40′; 15° 56′; 2° 8′ 9″; 21° 14′ 11″.

10. 1760 and 1856.

EXERCISE LXXIII. PAGE 55.

1. £83 12s. 9d.; £74 18s. 11d.; £810 2s.

2. 74 cwt. 64 lb. 3 oz.; 127 t. 0 cwt. 72 lb.; 56 t. 1 cwt. 28 lb. 3 oz.

3. 82 yd. 1 ft. 9 in.; 18 yd. 2 ft. 4 in.; 58 yd. 0 ft. 9 in.

4. 16 bu. 2 pk. 6 qt.; 32 bu. 3 pk. 5 qt. 1 pt.; 133 gal. 2 qt. 1 pt.

5. 38 gal. 3 qt.; 32 da. 18 hr. 51 min. 56 sec.; 7 wk. 4 da. 2 hr. 45 min. 33 sec.

6. £3 3s. 9d.; £19 9s. 3d.; 4 cwt. 22 lb. 10 oz.

7. 10 t. 15 cwt. 62 lb.; 3 yd. 1 ft. 5 in.; 47 yd. 1 ft. 3 in.

8. 28 bu. 1 pk. 7 qt.; 8 bu. 3 pk. 6 qt.; 2 qt. 1 pt.

9. 2 da. 18 hr. 8 min. 41 sec.; 1 wk. 5 da. 7 hr. 14 min. 45 sec.; 6° 50′ 39″.

10. 30° 53′ 49″; 31 c. yd. 18 c. ft. 1483 c. in.; 51 c. yd. 2 c. ft. 1488 c. in.

EXERCISE LXXIV. PAGE 56.

1. £66 9s. 2d.; £454 14s.; 136 cwt. 67 lb. 1 oz.

2. 61 t. 13 cwt. 31 lb. 8 oz.; 211 yd. 2 ft.; 967 yd. 0 ft. 3 in.

3. 23 bu. 0 pk. 1 qt. 1 pt.; 95 bu. 3 pk. 2 qt.; 34 gal. 0 qt. 1 pt.

4. 187 gal. 2 qt.; 159 da. 11 hr. 33 min.; 46 da. 13 hr. 34 min. 56 sec.

5. 123° 53′ 39″; 422° 17′ 20″; 86 c. yd. 22 c. ft. 967 c. in.

6. £20 9s. 5d.; £5 3s. 11d.; 7 cwt. 85 lb. 14 oz.

7. 2 t. 10 cwt. 64 lb. 9 oz.; 12 yd. 1 ft. 11 in.; 9 yd. 0 ft. 11 in.

8. 4 bu. 3 pk. 6 qt. 1 pt.; 8 bu. 0 pk. 1 gal. 1 qt.; 15 gal. 2 qt. 1 pt.

9. 4 gal. 1 qt. 1 pt.; 164 hr. 26 min. 24 sec.; 187 da. 21 hr. 55 min. 57 sec.

10. 19° 15′ 42″; 125° 9′ 24″; 19 c. yd. 22 c. ft. 969 c. in.

EXERCISE LXXV. PAGE 57.

1. 13; 32. *2.* 400; 35. *3.* 49; 42. *4.* 77; 720. *5.* 325; 64. *6.* 1856; 375. *7.* 90; 35. *8.* 2880; 156. *9.* 78; 1680. *10.* 1120; 303.

EXERCISE LXXVI. PAGE 57.

1. 4 ft. 8 in. *2.* 504 pt. *3.* 744 hr. *4.* 131, 36 oz. *5.* £116 16s. 3d. *6.* £54 13s. 4d. *7.* 11 ft. 6 in.; 6 ft. 6 in. *8.* 107° 40′ 38″. *9.* 353. *10.* $529.20.

EXERCISE LXXVII. PAGE 58.

1. \$27. *2.* 1301. *3.* \$104. *4.* 3080 yd. *5.* \$164475. *6.* 342 min. *7.* 37 lb. 14 oz. *8.* 15 t. *9.* 3000 gal. *10.* \$551760.

EXERCISE LXXVIII. PAGE 58.

1. 45 t. *2.* 60 spoons. *3.* 108. *4.* 15 pages. *5.* 42 mi. 60 rd. *6.* £110 10s. *7.* £9 7s. 6d. *8.* \$54.92. *9.* 60 yd. *10.* 15 ct.

EXERCISE LXXIX. PAGE 59.

1. 95 gal. 2 qt. 1 pt. *2.* 12 gal. 2 qt. 1 pt. *3.* 278 bu. 36 lb. *4.* 23 bu. 13 lb. *5.* 5 min. 15 sec. past 8. *6.* 4 lb. 4 oz. 16 dwt. *7.* £3 17s. 10½d. *8.* 288 lb. *9.* \$2919. *10.* 15 lb.

EXERCISE LXXX. PAGE 61.

1. \$44.70. *2.* \$153. *3.* \$8778.50. *4.* \$134.09. *5.* \$297.45. *6.* \$153.27. *7.* \$17.81. *8.* \$49.97. *9.* \$2.56. *10.* \$58.95.

EXERCISE LXXXI. PAGE 61.

1. \$290. *2.* \$2.79. *3.* \$7.25. *4.* \$311.48. *5.* \$221.81. *6.* \$5.32. *7.* \$21.80. *8.* \$42.70. *9.* \$13.94. *10.* \$71.91. *11.* \$13.54.

EXERCISE LXXXII. PAGE 63.

1. 10 in.; 14 in.; 22 in.; 16 in. *2.* 56 ft. *3.* \$45. *4.* 250 boards. *5.* \$3.40. *6.* \$700. *7.* \$256. *8.* 14080 yd. *9.* \$1440. *10.* 120 ft., 40 ft.

EXERCISE LXXXIII. PAGE 63.

1. 1224 sq. ft.; 1216 sq. ft.; 5616 sq. yd. *2.* 9 A.; 10 A.; 15 A.; 42 A.; 15 A.; 19 A. *3.* 18 sq. ft. *4.* 77 sq. yd. *5.* 2304 sq. yd. *6.* 200 sq. yd. *7.* 112 sq. in. *8.* 1056 sq. ft. *9.* 2000 sq. ft. *10.* 24 sq. ft.

EXERCISE LXXXIV. PAGE 64.

1. 18 ft. *2.* 54 ft. *3.* 40 rd. *4.* \$290. *5.* \$150. *6.* 500. *7.* 72 ft. *8.* \$940. *9.* 360 ft. *10.* \$660.

EXERCISE LXXXV. PAGE 65.

1. 40 yd. *2.* 60 yd. *3.* 32 yd. *4.* 70 yd. *5.* 196 yd. *6.* \$57.60. *7.* \$13. *8.* \$130. *9.* \$24. *10.* \$12.60.

EXERCISE LXXXVI. PAGE 65.

1. 6; 9. *2.* 8. *3.* 16. *4.* 8. *5.* 10. *6.* 72 yd. *7.* 48 yd. *8.* \$58.80. *9.* \$88. *10.* \$110.

EXERCISE LXXXVII. PAGE 66.

1. 80 sq. yd. *2.* 168 sq. yd. *3.* 224 sq. yd. *4.* $9.12.
5. $40.02. *6.* 82 sq. yd.; 91 sq. yd. *7.* $23.60.; $26.20.
8. $21; $23. *9.* $39; $41.50. *10.* $70.20; $76.14.

EXERCISE LXXXVIII. PAGE 67.

1. 160 yd. *2.* 288 yd. *3.* 448 yd. *4.* 96 yd. *5.* 348 yd.
6. 82 yd. *7.* 396 yd. *8.* 144 yd. *9.* $5.85. *10.* $16.25.

EXERCISE LXXXIX. PAGE 68.

1. 9 ft.; 24 ft.; 24 ft.; 24 ft.; 40 ft.; 24 ft.; 12 ft. *2.* 3300 ft.
3. 4200 ft. *4.* 2220 ft. *5.* 7680 ft. *6.* 768 ft. *7.* 1650 ft.
8. 10000 ft. *9.* $300. *10.* $540.

EXERCISE XC. PAGE 69.

1. 2880 c. ft. *2.* 210 c. ft. *3.* 60 c. yd. *4.* 210 c. yd.
5. 4032 lb. *6.* $40.50. *7.* 13600 c. yd. *8.* 109 c. yd. 1 c. ft.
9. 2376 c. ft. *10.* $1380.

EXERCISE XCI. PAGE 69.

1. 30 cd. *2.* 63 cd. *3.* 30 cd. *4.* $84. *5.* $216. *6.* $75.25.
7. 80 ft. *8.* 5 ft. *9.* 8 ft. *10.* 224 ft.

EXERCISE XCII. PAGE 70.

1. 7 ft. *2.* 11 ft. *3.* 24 ft. *4.* 3 ft. *5.* 112 ft. *6.* 18144.
7. 192 ft. *8.* 40 ft. *9.* 30 in. *10.* 17 ft.

EXERCISE XCIII. PAGE 70.

1. 32 ft. *2.* $40.25. *3.* $1080. *4.* 1120 lb. *5.* 32 t.
6. 48 t. *7.* 1760 c. yd. *8.* $792. *9.* $1440. *10.* $72.

EXERCISE XCIV. PAGE 71.

1. $32.64. *2.* 220 yd. *3.* 3600 sods. *4.* 3 ft. *5.* 90 c. ft.
6. 33 mi. *7.* 15 ft. *8.* 9 ft. *9.* 36 sq. yd. *10.* 5 ft.

EXERCISE XCV. PAGE 72.

1. $17; $23. *2.* 19 yd.; 30 yd. *3.* 36; 48. *4.* $48.75; $50.25.
5. $27000; $23000. *6.* $215; $160. *7.* $425; $371.
8. 271 mi.; 237 mi. *9.* 395; 361. *10.* $4500; $1250.

EXERCISE XCVI. PAGE 72.

1. $5120; $1280. *2.* 2064 bu.; 2311 bu. *3.* $2726.
4. 361 yd.; 242 yd. *5.* 297; 315. *6.* 462; 534.
7. 24 cd. 39 c. ft.; 27 cd. 89 c. ft. *8.* 9 A. 115 rd.; 10 A. 45 rd.
9. 4 mi. 275 rd. *10.* 1 t. 8 cwt.; 2 t.

EXERCISE XCVII. PAGE 73.

1. 30; 90. *2.* 254; 508. *3.* 23 yd.; 92 yd. *4.* $2450; $9800. *5.* $75; $168. *6.* $145; $462. *7.* $3450; $10500. *8.* 30 rd.; 90 rd. *9.* $126. *10.* 1240; 3737.

EXERCISE XCVIII. PAGE 73.

1. $36; $48. *2.* 60 bu. oats; 150 bu. peas. *3.* 17. *4.* $7800; $11700. *5.* 275; 330. *6.* $20; $40; $60. *7.* 30; 45; 75. *8.* 6. *9.* $420; $140; $560. *10.* $10940; $3020; $2450.

EXERCISE XCIX. PAGE 74.

1. $8; $10; $13. *2.* 170 lb.; 152 lb.; 134 lb.
3. 150 yd.; 125 yd.; 169 yd. *4.* 2400 lb.; 2200 lb.; 1800 lb.
5. 127 lb.; 100 lb.; 175 lb. *6.* 60 mi.; 25 mi.; 40 mi.
7. 81 rd.; 45 rd.; 123 rd. *8.* 256; 378; 452. *9.* $450; $531; $612.
10. 180 bu.; 233 bu.; 160 bu. *11.* 32 ct.
12. 87 red, 94 blue, 105 white.

EXERCISE C. PAGE 75.

1. 1975. *2.* 3683. *3.* 184019. *4.* 7, 10, 9. *5.* 14, 11. *6.* 43, 16. *7.* 14 lb. *8.* 48 yr. *9.* $119. *10.* $7095.

EXERCISE CI. PAGE 76.

1. 659 lb. 13 oz. *2.* 256 mi. 160 rds. *3.* $204.75. *4.* $17.82. *5.* 35 cwt. 25 lb. *6.* 12 lb. 9 oz. *7.* 33 mi. 96 rd. *8.* 45. *9.* $75. *10.* 24 lb. 10 oz.

EXERCISE CII. PAGE 76.

1. $32. *2.* 7 ct. *3.* 15 carats. *4.* 40 ct. *5.* 6 ct.
6. 200 bu. at 75 ct., 300 bu. at 70 ct. *7.* 4 ft. 3 in. *8.* 139 lb.
9. 1s. 7d. *10.* 4s. 9d.

EXERCISE CIII. PAGE 77.

1. 272 lb., 2448 lb. *2.* $570940. *3.* 2 oz. *4.* 35 ct. *5.* 22 yr. *6.* $25. *7.* 7 ct. *8.* 46 ct. *9.* $6.60. *10.* 12 carats.

EXERCISE CIV. PAGE 78.

1. 2, 2, 3; 2, 2, 2, 7; 2, 2, 3, 7; 2, 3, 5.
2. 2, 2, 7, 7; 3, 7, 11; 2, 2, 3, 3, 7; 2, 2, 2, 2, 2, 2, 2, 3.
3. 2, 2, 3, 73; 2, 2, 3, 79; 2, 2, 263; 3, 353.
4. 3, 5, 73; 3, 7, 53; 7, 7, 23; 2, 2, 17, 17.
5. 2, 601; 2, 607; 2, 3, 7, 29; 5, 311. *6.* 797, 821. *7.* 1187.
8. 2543, 2521. *9.* 2273, 2339, 2417. *10.* 3119.

EXERCISE CV. PAGE 79.

1. 2, 5; 5, 7. *2.* 3, 7; 11, 13. *3.* 7, 17; 2, 2, 3.
4. 2, 2; 2, 2, 3, 3, 3. *5.* 2, 2; 5. *8.* 30; 25; 275; 193; 443.
9. 2, 3, 5; 3, 5, 7; 5, 7, 11; 7, 11, 13; 11, 17, 19.

EXERCISE CVI. PAGE 79.

1. 180. *2.* 5. *3.* 6. *4.* 120. *5.* 4. *6.* 1008. *7.* 2400.
8. 900 bu. *9.* 75 yd. *10.* 40 da.

EXERCISE CVII. PAGE 80.

1. 14; 16; 13. *2.* 35; 46; 42. *3.* 72; 32; 4. *4.* 9; 16; 7.
5. 25; 42; 42. *6.* 9; 4; 73. *7.* 73; 32; 11. *8.* 9; 8; 13.
9. 32; 110; 14. *10.* 91; 143; 323.

EXERCISE CVIII. PAGE 80.

1. 14 ft. *2.* 17 in. *3.* 15. *4.* 7 ft. *5.* 2 ft. 3 in. *6.* 13 ft.
7. 63 bu. *8.* \$1, \$2, \$5, \$10, or \$50. *9.* \$124. *10.* 26880 rails.

EXERCISE CIX. PAGE 81.

1. 18; 7; 251. *2.* 107, 311, 103. *3.* 52, Prime, 601.
4. 31, 947, 233. *5.* Prime, 47, 811. *6.* 8, 616, 884. *7.* 1188, 13, 13.
8. 46, 21, 9. *9.* 897, 1323, 1429. *10.* 315, 19.

EXERCISE CX. PAGE 81.

1. 105; 3, 5, 7, 15, 21, 35, 105. *2.* 2 da. 5 hr. *3.* 1 mi. 37 yd.
4. 3 gal. *5.* 117. *6.* 29. *7.* 25. *8.* 19 men. *9.* 7 oz. *10.* 126.

EXERCISE CXI. PAGE 82.

1. 24; 120; 120. *2.* 75; 600; 720. *3.* 315; 2652; 3705.
4. 360; 1260; 504. *5.* 576; 600; 27720. *6.* 1260; 8415; 16546530.
7. 10890; 17160; 205205. *8.* 14280; 166320; 22770.
9. 1190595; 27945372; 885989. *10.* 12018233514; 110250; 620310.

EXERCISE CXII. PAGE 83.

1. 120. *2.* 360. *3.* 85085. *4.* 6435. *5.* 48. *6.* 143.
7. 1526. *8.* 90. *9.* 120 ft. *10.* 60 gal.

EXERCISE CXIII. PAGE 83.

1. \$20. *2.* 141 lb. *3.* 94 lb. *4.* 30. *5.* 30 mi. *6.* 365.
7. 62, 122, 182. *8.* 600 bu. *9.* 13, 14, 15, 16. *10.* 1008000 gr.

EXERCISE CXIV. PAGE 84.

1. $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$. *2.* $\frac{5}{8}$; $\frac{5}{9}$; $\frac{5}{12}$. *3.* $\frac{6}{7}$; $\frac{6}{13}$; $\frac{6}{17}$. *4.* $\frac{7}{21}$; $\frac{7}{12}$; $\frac{7}{32}$.

5. Two-thirds; three-fifths; four-sevenths; three-elevenths.

6. Seven-twelfths; eleven-twenty firsts; nineteen-twenty seconds; seventeen-twenty fifths.

7. Seventeen-thirty firsts; eleven-twelfths; nineteen-thirty seconds; eleven-fifty firsts.

8. One inch is divided into 3 equal parts and 2 of these parts are represented by $\frac{2}{3}$ in.;

Or, $\frac{2}{3}$ in. represents a third of 2 in.

Or, $\frac{2}{3}$ in. represents the quantity which must be multiplied by 3 to produce 2 in.

(All the other fractions are capable of similar interpretations.)

One foot is divided into 4 equal parts and 3 of these parts are represented by $\frac{3}{4}$ ft.;

One acre is divided into 5 equal parts and 4 of these are represented by $\frac{4}{5}$ A.;

A dollar is divided into 10 equal parts and 7 of these are represented by $\$\frac{7}{10}$.

9. One hour is divided into 9 equal parts and 7 of these are represented by $\frac{7}{9}$ hr.

One is divided in 12 equal parts and 11 of these are represented by $\frac{11}{12}$;

When the fraction is abstract it may have a fourth meaning. Thus, $\frac{11}{12}$ may represent the ratio of 11 to 12 in., or the fraction that 11 in. is of 12 in., or the measure of 11 in. when 12 in. is the unit.

One is divided in 21 equal parts and 13 of these are represented by $\frac{13}{21}$;

One is divided in 25 equal parts and 7 of these are represented by $\frac{7}{25}$.

10. One cord is divided in 7 equal parts and 5 of these are represented by $\frac{5}{7}$ cd.;

One cwt. is divided into 25 equal parts and 17 of these are represented by $\frac{17}{25}$ cwt.;

One pound is divided in 8 equal parts and 7 of these are represented by $\frac{7}{8}$ lb.;

One ton is divided into 12 equal parts and 5 of these are represented by $\frac{5}{12}$ t.

EXERCISE CXV. PAGE 86.

1. $\frac{5}{2}$; $\frac{13}{3}$; $\frac{13}{5}$; $\frac{19}{4}$.
2. $\frac{43}{7}$; $\frac{57}{8}$; $\frac{73}{9}$; $\frac{30}{7}$.
3. $\frac{43}{8}$; $\frac{61}{9}$; $\frac{89}{11}$; $\frac{283}{31}$.
4. $\frac{55}{3}$; $\frac{79}{4}$; $\frac{43}{2}$; $\frac{128}{5}$.
5. $\frac{480}{11}$; $\frac{508}{5}$; $\frac{223}{15}$; $\frac{5273}{203}$.
6. $\frac{2449}{15}$; $\frac{4678}{17}$; $\frac{45664}{99}$; $\frac{8480}{21}$.
7. $\frac{5191}{25}$; $\frac{68861}{81}$; $\frac{35449}{39}$; $\frac{19725}{251}$.
8. $\frac{13337}{43}$; $\frac{44399}{78}$; $\frac{69881}{103}$; $\frac{160589}{230}$.
9. $\frac{17022}{17}$; $\frac{77041}{801}$; $\frac{783827}{97}$; $\frac{707007}{101}$.
10. $\frac{287747}{41}$; $\frac{498803}{707}$; $\frac{288761}{784}$; $\frac{210867}{5698}$.

EXERCISE CXVI. PAGE 86.

1. 4 boys. *2.* 12 girls. *3.* 30 fifths. *4.* 11 persons.
5. 15 boys. *6.* $\frac{75}{15}$; $\frac{105}{15}$. *7.* $\frac{432}{9}$; $\frac{570}{10}$. *8.* $\frac{72}{24}$; $\frac{120}{24}$; $\frac{192}{24}$; $\frac{288}{24}$.
9. 13 mi. *10.* 35 pupils.

EXERCISE CXVII. PAGE 86.

1. $2\frac{1}{2}$; $2\frac{1}{3}$; $3\frac{1}{4}$; 2.
2. $1\frac{7}{8}$; $2\frac{5}{7}$; $5\frac{1}{6}$; $2\frac{11}{12}$.
3. $7\frac{1}{12}$; 5; $5\frac{8}{14}$; $5\frac{6}{17}$.
4. $9\frac{3}{24}$; $16\frac{21}{23}$; $33\frac{19}{23}$; $35\frac{13}{25}$.
5. $5\frac{12}{125}$; $4\frac{49}{123}$; $6\frac{72}{151}$; $13\frac{51}{73}$.
6. $4\frac{960}{1760}$; $260\frac{21}{23}$; $16\frac{63}{323}$; $1\frac{585}{1133}$.
7. $20\frac{119}{240}$; $9\frac{458}{727}$; $3\frac{1104}{1931}$; $34\frac{791}{2237}$.
8. $334\frac{2}{23}$; $100\frac{1}{83}$; $10\frac{11}{607}$; $10\frac{37}{853}$.
9. $20\frac{16}{401}$; $45\frac{7}{9}$ gal.; $53\frac{5}{7}$ da.; $50\frac{3}{5}$ lb.
10. $7\frac{28}{271}$ mi.; $24\frac{20}{31}$ cd.; $21\frac{12}{17}$ yd.; $63\frac{1}{11}$ oz.

EXERCISE CXVIII. PAGE 87.

1. $\$4\frac{1}{4}$. *2.* $92\frac{1}{4}$ bu. *3.* 19 mi. *4.* $16\frac{1}{6}$ lb. *5.* $321\frac{1}{3}$ bu.
6. Latter by $\frac{1}{20}$ lb. *7.* 11 in.; 7 lb.; 25 oz. *8.* $7\frac{1}{2}$ gal.
9. 20 ft.; 16 ft. *10.* \$19.75.

EXERCISE CXIX. PAGE 87.

1. $\frac{6}{12}$; $\frac{4}{12}$; $\frac{3}{12}$. *2.* $\frac{8}{24}$; $\frac{6}{24}$; $\frac{4}{24}$. *3.* $\frac{12}{18}$; $\frac{15}{18}$; $\frac{10}{18}$.
4. $\frac{15}{20}$; $\frac{16}{20}$; $\frac{14}{20}$. *5.* $\frac{12}{30}$; $\frac{20}{30}$; $\frac{9}{30}$. *6.* $\frac{16}{24}$; $\frac{20}{24}$; $\frac{9}{24}$.
7. $\frac{24}{30}$; $\frac{9}{30}$; $\frac{22}{30}$. *8.* $\frac{28}{42}$; $\frac{35}{42}$; $\frac{18}{42}$. *9.* $\frac{30}{48}$; $\frac{28}{48}$; $\frac{9}{48}$.
10. $\frac{50}{60}$; $\frac{42}{60}$; $\frac{55}{60}$.

EXERCISE CXX. PAGE 88.

1. $\frac{1}{4}$; $\frac{1}{4}$; $\frac{3}{4}$. *2.* $\frac{3}{5}$; $\frac{1}{5}$; $\frac{2}{5}$. *3.* $\frac{5}{7}$; $\frac{4}{7}$; $\frac{5}{7}$.
4. $\frac{5}{9}$; $\frac{5}{9}$; $\frac{5}{9}$. *5.* $\frac{5}{11}$; $\frac{7}{11}$; $\frac{7}{11}$. *6.* $\frac{13}{18}$; $\frac{5}{18}$; $\frac{7}{18}$.
7. $\frac{13}{20}$; $\frac{11}{20}$; $\frac{17}{20}$. *8.* $\frac{7}{24}$; $\frac{5}{24}$; $\frac{5}{24}$. *9.* $\frac{22}{31}$; $\frac{5}{31}$; $\frac{13}{31}$.
10. $\frac{17}{52}$; $\frac{13}{52}$; $\frac{23}{52}$.

EXERCISE CXXI. PAGE 88.

1. $\frac{8}{12}$; $\frac{12}{20}$; $\frac{15}{21}$. *2.* $\frac{20}{35}$; $\frac{15}{27}$; $\frac{35}{55}$. *3.* $\frac{25}{40}$; $\frac{36}{78}$; $\frac{25}{85}$.
4. $\frac{3}{4}$; $\frac{3}{5}$; $\frac{2}{4}$. *5.* $\frac{7}{12}$; $\frac{3}{4}$; $\frac{5}{12}$. *6.* $\frac{25}{40}$; $\frac{28}{44}$; $\frac{44}{52}$.
7. $\frac{35}{119}$; $\frac{121}{209}$; $\frac{63}{119}$. *8.* $\frac{154}{168}$; $\frac{72}{204}$; $\frac{81}{261}$. *9.* $\frac{1}{2}$; $\frac{1}{3}$; $\frac{3}{5}$.
10. $\frac{2}{3}$; $\frac{2}{4}$; $\frac{12}{17}$.

EXERCISE CXXII. PAGE 88.

1. $\frac{1}{2}$; $\frac{3}{4}$; $\frac{2}{3}$; $\frac{1}{4}$.
2. $\frac{7}{15}$; $\frac{3}{5}$; $\frac{7}{8}$; $\frac{4}{7}$.
3. $\frac{5}{7}$; $\frac{5}{11}$; $\frac{9}{35}$; $\frac{2}{3}$.
4. $\frac{1}{6}$; $\frac{3}{7}$; $\frac{19}{25}$; $\frac{17}{20}$.
5. $\frac{2}{5}$; $\frac{1}{3}$; $\frac{5}{7}$; $\frac{63}{149}$.
6. $\frac{41}{62}$; $\frac{29}{517}$; $\frac{14}{15}$; $\frac{26}{225}$.
7. $\frac{8}{9}$; $\frac{29}{37}$; $\frac{241}{661}$; $\frac{27}{41}$.
8. $\frac{54}{101}$; $\frac{2}{3}$; $\frac{653}{19820}$; $\frac{9}{13}$.
9. $\frac{8}{11}$; $\frac{11}{37}$; $\frac{2}{7}$; $\frac{6539}{12963}$.
10. $\frac{124}{437}$; $\frac{23}{29}$; $\frac{13}{20}$; $\frac{829}{7337}$.

EXERCISE CXXIII. PAGE 89.

1. $\frac{4}{6}, \frac{5}{6}; \frac{14}{16}, \frac{5}{16}; \frac{10}{24}, \frac{17}{24}$. *2.* $\frac{9}{21}, \frac{8}{21}; \frac{20}{25}, \frac{7}{25}; \frac{63}{77}, \frac{17}{77}$.

3. $\frac{3}{6}, \frac{2}{6}; \frac{5}{15}, \frac{3}{15}; \frac{6}{30}, \frac{5}{30}$. *4.* $\frac{10}{15}, \frac{9}{15}; \frac{27}{63}, \frac{35}{63}; \frac{20}{55}, \frac{33}{55}$.

5. $\frac{63}{72}, \frac{56}{72}; \frac{55}{77}, \frac{21}{77}; \frac{52}{91}, \frac{35}{91}$.

6. $\frac{119}{204}, \frac{96}{204}; \frac{189}{231}, \frac{77}{231}; \frac{138}{299}, \frac{91}{299}$.

7. $\frac{54}{90}, \frac{25}{90}; \frac{64}{304}, \frac{133}{304}; \frac{75}{325}, \frac{91}{325}$.

8. $\frac{15}{36}, \frac{14}{36}; \frac{20}{45}, \frac{33}{45}; \frac{49}{84}, \frac{68}{84}$.

9. $\frac{21}{48}, \frac{38}{48}; \frac{39}{54}, \frac{46}{54}; \frac{66}{210}, \frac{85}{210}$.

10. $\frac{33}{72}, \frac{10}{72}; \frac{75}{140}, \frac{24}{140}; \frac{161}{168}, \frac{118}{168}$.

EXERCISE CXXIV. PAGE 89.

1. $\frac{80}{180}, \frac{75}{180}, \frac{84}{180}; \frac{42}{98}, \frac{35}{98}, \frac{8}{98}$.

2. $\frac{125}{150}, \frac{24}{150}, \frac{35}{150}; \frac{52}{286}, \frac{110}{286}, \frac{77}{286}$.

3. $\frac{84}{144}, \frac{20}{144}, \frac{33}{144}; \frac{18}{114}, \frac{15}{114}, \frac{19}{114}$.

4. $\frac{216}{252}, \frac{147}{252}, \frac{80}{252}, \frac{39}{252}; \frac{36}{90}, \frac{60}{90}, \frac{20}{90}, \frac{63}{90}$.

5. $\frac{30}{72}, \frac{68}{72}, \frac{69}{72}, \frac{14}{72}; \frac{63}{84}, \frac{60}{84}, \frac{32}{84}, \frac{34}{84}$.

6. $\frac{420}{720}, \frac{405}{720}, \frac{468}{720}, \frac{440}{720}; \frac{88}{120}, \frac{102}{120}, \frac{70}{120}, \frac{95}{120}$.

7. $\frac{500}{1200}, \frac{850}{1200}, \frac{1080}{1200}, \frac{192}{1200}; \frac{119}{168}, \frac{104}{168}, \frac{132}{168}, \frac{90}{168}$.

8. $\frac{64}{120}, \frac{78}{120}, \frac{70}{120}, \frac{55}{120}; \frac{102}{120}, \frac{50}{120}, \frac{65}{120}, \frac{84}{120}$.

9. $\frac{84}{108}, \frac{30}{108}, \frac{14}{108}, \frac{33}{108}; \frac{105}{231}, \frac{165}{231}, \frac{121}{231}, \frac{39}{231}$.

10. $\frac{22}{99}, \frac{63}{99}, \frac{120}{99}, \frac{198}{99}; \frac{324}{450}, \frac{230}{450}, \frac{625}{450}, \frac{3150}{450}$.

EXERCISE CXXV. PAGE 89.

1. Greatest, $\frac{4}{5}$; least, $\frac{2}{3}$; greatest, $\frac{5}{6}$; least, $\frac{3}{4}$.

2. Greatest, $\frac{6}{7}$; least, $\frac{17}{21}$; greatest, $\frac{2}{3}$; least, $\frac{7}{12}$.

3. Greatest, $\frac{3}{4}$; least, $\frac{8}{11}$; greatest, $\frac{4}{7}$; least, $\frac{4}{9}$.

4. Greatest, $\frac{9}{28}$; least, $\frac{7}{24}$; greatest, $\frac{9}{10}$; least, $\frac{21}{25}$.

The smallest being placed first the following is the order:

5. $\frac{12}{35}, \frac{7}{20}, \frac{11}{30}, \frac{17}{42}; \frac{7}{36}, \frac{11}{54}, \frac{5}{24}, \frac{15}{64}$.

6. $\frac{2}{7}, \frac{3}{8}, \frac{5}{13}, \frac{36}{91}; \frac{4}{9}, \frac{13}{21}, \frac{11}{20}, \frac{7}{12}$.

7. $\frac{5}{16}, \frac{13}{40}, \frac{7}{20}, \frac{23}{60}; \frac{7}{18}, \frac{5}{12}, \frac{11}{25}, \frac{17}{30}$.

8. $\frac{49}{60}$. *9.* $\frac{71}{84}$. *10.* $\frac{35}{72}$.

EXERCISE CXXVI. PAGE 90.

1. $\frac{3}{11}; \frac{4}{17}; \frac{2}{9}$. *2.* $\frac{4}{7}; \frac{6}{11}; \frac{10}{13}$. *3.* $\frac{6}{17}; \frac{20}{29}; \frac{28}{41}$.

4. $\frac{1}{15}; \frac{1}{22}; \frac{7}{40}$. *5.* $\frac{10}{21}; \frac{15}{77}; \frac{105}{152}$. *6.* $3; 2\frac{17}{20}; 3$.

7. 16; 49; 30. *8.* $\frac{1}{24}$; $\frac{2}{5}$; $\frac{3}{16}$. *9.* $\frac{1}{2}$; $\frac{2}{15}$; $\frac{5}{52}$.
10. 288, 1024.

EXERCISE CXXVII. PAGE 90.

1. $\frac{9}{11}$; $\frac{6}{7}$; $\frac{8}{9}$. *2.* $\frac{13}{12}$; $\frac{15}{17}$; $\frac{9}{13}$. *3.* $\frac{14}{17}$; $\frac{19}{25}$; $\frac{18}{19}$.
4. 2; 1; 2. *5.* 1; 2; $1\frac{9}{14}$. *6.* $1\frac{35}{72}$; $1\frac{67}{90}$; $2\frac{1}{12}$.
7. $1\frac{59}{84}$; $1\frac{7}{12}$; $2\frac{5}{12}$. *8.* $2\frac{101}{420}$; $1\frac{3}{7}$; $1\frac{173}{216}$.
9. $\frac{119}{132}$; $2\frac{11}{84}$; $2\frac{129}{440}$. *10.* $1\frac{104}{315}$; $2\frac{17}{120}$; $1\frac{431}{660}$.

EXERCISE CXXVIII. PAGE 91.

1. $\frac{7}{12}$; $\frac{9}{20}$; $\frac{12}{35}$. *2.* $\frac{14}{45}$; $\frac{18}{77}$; $\frac{22}{117}$.
3. $1\frac{61}{99}$; $1\frac{19}{45}$; $1\frac{59}{117}$. *4.* $1\frac{17}{63}$; $1\frac{13}{35}$; $\frac{72}{77}$.
5. $1\frac{31}{77}$; $1\frac{29}{91}$; $1\frac{1}{143}$. *6.* $1\frac{79}{110}$; $1\frac{19}{35}$; $1\frac{37}{80}$.
7. $1\frac{29}{195}$; $\frac{32}{35}$; $\frac{272}{285}$. *8.* $1\frac{43}{90}$; $1\frac{11}{24}$; $1\frac{5}{16}$.
9. $1\frac{23}{42}$; $1\frac{19}{56}$; $1\frac{1}{99}$. *10.* $1\frac{119}{156}$; $1\frac{13}{20}$; $1\frac{13}{7\text{[illegible]}}$.

EXERCISE CXXIX. PAGE 91.

1. $1\frac{17}{60}$. *2.* $3\frac{19}{30}$. *3.* $1\frac{73}{336}$. *4.* $4\frac{109}{360}$. *5.* $2\frac{197}{2160}$.
6. $24\frac{3}{4}$. *7.* $35\frac{7}{11}$. *8.* $17\frac{7}{20}$. *9.* $23\frac{3}{4}$. *10.* $26\frac{19}{20}$.

EXERCISE CXXX. PAGE 91.

1. $\frac{13}{24}$. *2.* $29\frac{13}{24}$ A. *3.* $111\frac{37}{40}$ lb. *4.* $5\frac{73}{120}$. *5.* $\frac{11}{12}$.
6. $178\frac{7}{40}$. *7.* $8\frac{3}{7}$. *8.* $21\frac{25}{36}$ mi. *9.* $114\frac{7}{12}$ gal.
10. $132\frac{1}{2}$ mi.

EXERCISE CXXXI. PAGE 92.

1. $\frac{3}{5}$; $\frac{2}{7}$; $\frac{5}{9}$. *2.* $\frac{2}{7}$; $\frac{8}{15}$; 1. *3.* $\frac{1}{2}$; $\frac{9}{16}$; $\frac{2}{9}$.
4. $\frac{2}{5}$; $\frac{17}{42}$; $\frac{13}{63}$. *5.* $\frac{3}{8}$; $\frac{1}{6}$; $\frac{1}{21}$. *6.* $\frac{6}{35}$; $\frac{3}{56}$; $\frac{14}{143}$.
7. $\frac{3}{50}$; $\frac{1}{36}$; $\frac{7}{24}$. *8.* $\frac{283}{576}$; $\frac{29}{48}$; $\frac{1}{42}$. *9.* $\frac{17}{150}$; $\frac{17}{45}$; $\frac{29}{42}$.
10. $1\frac{2}{3}$; $2\frac{5}{24}$; $4\frac{3}{7}$.

EXERCISE CXXXII. PAGE 92.

1. $\frac{1}{6}$; $\frac{1}{20}$; $\frac{1}{56}$. *2.* $\frac{4}{63}$; $\frac{6}{143}$; $\frac{5}{72}$. *3.* $\frac{5}{18}$; $\frac{7}{60}$; $\frac{11}{72}$.
4. $\frac{14}{75}$; $\frac{13}{40}$; $\frac{28}{75}$. *5.* $\frac{11}{84}$; $\frac{29}{156}$; $\frac{35}{192}$. *6.* $\frac{5}{36}$; $\frac{10}{143}$; $\frac{14}{99}$.
7. $\frac{13}{240}$; $\frac{9}{112}$; $\frac{42}{575}$. *8.* $\frac{16}{63}$; $\frac{4}{363}$; $\frac{19}{80}$. *9.* $\frac{10}{1089}$; $\frac{20}{693}$; $\frac{13}{252}$.
10. $\frac{169}{375}$; $\frac{7}{450}$; $\frac{193}{1008}$.

EXERCISE CXXXIII. PAGE 93.

1. $\$\frac{7}{20}$. *2.* $\frac{13}{35}$. *3.* $\frac{59}{200}$. *4.* $\frac{23}{60}$. *5.* $\frac{31}{700}$. *6.* $\frac{5}{24}$.
7. $\frac{11}{120}$. *8.* $\frac{17}{30}$. *9.* $\frac{71}{300}$. *10.* $\frac{7}{20}$.

EXERCISE CXXXIV. PAGE 93.

1. $3\frac{1}{6}$; $3\frac{5}{11}$; $5\frac{8}{13}$. *2.* $\frac{3}{8}$; $1\frac{1}{14}$; $\frac{17}{144}$.
3. $\frac{5}{48}$; $5\frac{1}{28}$; $6\frac{4}{363}$. *4.* $3\frac{15}{17}$; $4\frac{9}{11}$; $4\frac{9}{13}$.
5. $2\frac{1}{2}$; $2\frac{17}{27}$; $1\frac{263}{288}$. *6.* $14\frac{2}{7}$; $23\frac{7}{11}$; $20\frac{2}{15}$.
7. $1\frac{48}{539}$; $\frac{13}{84}$; $10\frac{133}{140}$. *8.* $26\frac{41}{60}$; $20\frac{25}{36}$; $10\frac{7}{72}$.
9. $17\frac{35}{72}$; $38\frac{53}{60}$; $31\frac{19}{42}$. *10.* $23\frac{13}{24}$; $12\frac{4}{143}$; $13\frac{14}{75}$.

EXERCISE CXXXV. PAGE 93.

1. $16\frac{5}{8}$ gal. *2.* $13\frac{5}{6}$ yd. *3.* $16\frac{19}{24}$ mi. *4.* $30\frac{5}{8}$.
5. $\$95\frac{59}{60}$. *6.* $159\frac{43}{60}$ A. *7.* $\$68\frac{3}{4}$. *8.* $13\frac{1}{10}$ *9.* $15\frac{59}{75}$.
10. $164\frac{13}{20}$ t.

EXERCISE CXXXVI. PAGE 94.

1. $\frac{43}{90}$; $1\frac{5}{16}$. *2.* $3\frac{1}{8}$; $8\frac{23}{36}$. *3.* $\frac{101}{110}$; $4\frac{1}{9}$. *4.* 6; $9\frac{7}{18}$.
5. $4\frac{4}{7}$; $5\frac{19}{27}$. *6.* $1\frac{20}{21}$; $34\frac{8}{9}$. *7.* $11\frac{7}{8}$; $27\frac{41}{54}$. *8.* $2\frac{11}{14}$; $7\frac{5}{16}$.
9. 10; $11\frac{5}{11}$. *10.* $8\frac{17}{22}$; $1\frac{67}{112}$.

EXERCISE CXXXVII. PAGE 94.

1. $97\frac{19}{24}$ lb. *2.* $16\frac{1}{2}$ gal. *3.* 54 lb. *4.* $193\frac{41}{56}$.
5. $33\frac{1}{2}$ mi. *6.* Gained $\$1\frac{3}{20}$. *7.* $\$23\frac{1}{3}$. *8.* $\$94\frac{23}{48}$.
9. $18\frac{1}{3}$ yd. *10.* $37\frac{23}{24}$ yd.

EXERCISE CXXXVIII. PAGE 95.

1. $1\frac{1}{2}$; $2\frac{2}{3}$; $1\frac{1}{5}$. *2.* $3\frac{2}{11}$; $3\frac{1}{9}$; $4\frac{2}{7}$. *3.* $\$1\frac{1}{4}$; $\$4\frac{1}{3}$; $\$1\frac{4}{5}$.
4. 15; 8; 21. *5.* 6; 7; 12. *6.* 16; $8\frac{1}{2}$; $3\frac{3}{4}$.
7. 10; 15; 30. *8.* 15; $8\frac{1}{3}$; 21. *9.* $4\frac{1}{2}$; $11\frac{2}{3}$; $9\frac{3}{4}$.
10. 23; 68; 93.

EXERCISE CXXXIX. PAGE 95.

1. $\frac{4}{9}$; $\frac{4}{25}$; $\frac{7}{47}$. *2.* $\frac{2}{23}$; $\frac{6}{47}$; $\frac{8}{101}$. *3.* $2\frac{1}{4}$; $5\frac{3}{17}$; $4\frac{2}{19}$.
4. $2\frac{5}{86}$; $3\frac{2}{23}$; $9\frac{4}{25}$. *5.* $\frac{3}{4}$; $\frac{3}{7}$; $\frac{2}{3}$. *6.* $\frac{4}{7}$; $1\frac{1}{8}$; $2\frac{1}{10}$.
7. $\frac{1}{6}$; $\frac{2}{15}$; $\frac{3}{20}$. *8.* $\frac{5}{63}$; $\frac{7}{66}$; $\frac{8}{65}$. *9.* $\frac{3}{14}$; $\frac{19}{24}$; $\frac{76}{117}$.
10. $8\frac{29}{32}$; $7\frac{1}{42}$; $5\frac{2}{11}$.

EXERCISE CXL. PAGE 96.

1. 4; 9; 8. *2.* $\frac{8}{15}$; $\frac{9}{20}$; $\frac{12}{35}$. *3.* $\frac{25}{42}$; $\frac{12}{35}$; $\frac{21}{32}$.
4. $\frac{20}{63}$; $\frac{28}{143}$; $\frac{40}{63}$. *5.* $\frac{3}{19}$; $\frac{4}{7}$; $4\frac{1}{2}$. *6.* $\frac{6}{7}$; $6\frac{2}{3}$; $5\frac{5}{7}$.
7. $\frac{3}{40}$; $\frac{3}{4}$; $\frac{15}{32}$. *8.* $\frac{9}{10}$; $1\frac{1}{20}$; $\frac{11}{24}$. *9.* $\frac{3}{8}$; $\frac{6}{7}$; $\frac{10}{21}$.
10. $\frac{5}{6}$; $\frac{94}{103}$; 35.

EXERCISE CXLI. PAGE 96.

1. $20\frac{1}{4}$; $30\frac{3}{16}$; $56\frac{7}{64}$. *2.* $30\frac{2}{9}$; $42\frac{10}{49}$; $72\frac{12}{49}$.
3. $42\frac{20}{81}$; $56\frac{6}{25}$; $156\frac{15}{64}$. *4.* $\frac{1}{4}$; $\frac{8}{13}$; $\frac{4}{11}$. *5.* 15; $16\frac{1}{3}$; $9\frac{4}{5}$.
6. $1\frac{1}{4}$; 11; 20. *7.* $1\frac{1}{5}$; 195; $16\frac{1}{2}$. *8.* $31\frac{1}{9}$; $2\frac{3}{11}$; 1.
9. $\$\frac{7}{12}$; $\$\frac{1}{2}$; 555 mi. *10.* \$558; 324 oz.; 3 hr.

EXERCISE CXLII. PAGE 96.

1. $\$39\frac{9}{10}$. *2.* $\$170\frac{2}{5}$. *3.* $71\frac{7}{8}$c. *4.* $8\frac{1}{4}$ mi. *5.* $\$13\frac{1}{2}$. *6.* $12\frac{3}{5}$ bu.
7. $\$1733\frac{1}{3}$. *8.* 36 bu. *9.* 47 A. *10.* \$4.

EXERCISE CXLIII. PAGE 97.

1. \$1.68. *2.* \$30000. *3.* 85 mi. *4.* $37\frac{1}{2}$ ft. *5.* 385 mi.
6. \$105000. *7.* $257\frac{2}{5}$. *8.* 39100 men. *9.* 210 pages. *10.* \$65600.

EXERCISE CXLIV. PAGE 97.

1. $2\frac{1}{2}$; $24\frac{1}{2}$. *2.* 18; $\frac{21}{32}$. *3.* $17\frac{2}{3}$; 7. *4.* $3\frac{3}{7}$; $\frac{70}{99}$. *5.* $\frac{5}{11}$; $3\frac{1}{5}$.
6. 7; 20. *7.* 10; 40. *8.* $2\frac{5}{8}$; $\frac{1}{10}$. *9.* $35\frac{3}{5}$; $\frac{1}{2}$. *10.* 1; $2\frac{3}{19}$.

EXERCISE CXLV. PAGE 98.

1. $185\frac{5}{8}$ mi. *2.* \$1012. *3.* $59\frac{3}{8}$. *4.* $32\frac{13}{14}$ bu. *5.* $\$17086\frac{2}{3}$.
6. 10. *7.* $37\frac{53}{105}$. *8.* \$3125. *9.* $\$43\frac{3}{8}$. *10.* Gain, \$12.

EXERCISE CXLVI. PAGE 98.

1. 14, 16, 15. *2.* 36, 55, 44. *3.* $8\frac{3}{4}$, $14\frac{2}{5}$, $24\frac{3}{4}$. *4.* $17\frac{1}{7}$, $31\frac{1}{4}$, $30\frac{6}{7}$.
5. $4\frac{4}{7}$, $5\frac{5}{14}$, $6\frac{6}{23}$. *6.* $6\frac{12}{19}$, $7\frac{19}{23}$, $11\frac{21}{25}$. *7.* $5\frac{5}{9}$, 4, $7\frac{1}{2}$. *8.* $1\frac{1}{20}$, $1\frac{1}{35}$, $\frac{35}{36}$.
9. $1\frac{5}{27}$, $2\frac{1}{4}$, $1\frac{3}{5}$. *10.* 3, $\frac{1}{2}$, $1\frac{1}{8}$.

EXERCISE CXLVII. PAGE 99.

1. $1\frac{2}{3}$; $\frac{12}{35}$, $1\frac{1}{6}$. *2.* $1\frac{1}{8}$, $\frac{15}{16}$, $\frac{3}{10}$. *3.* $1\frac{1}{27}$, $1\frac{1}{2}$, $1\frac{2}{7}$. *4.* $\frac{2}{5}$, $\frac{2}{3}$, $\frac{5}{7}$.
5. $\frac{10}{11}$, $1\frac{3}{7}$, $\frac{3}{10}$. *6.* $3\frac{1}{18}$, $1\frac{5}{7}$, $1\frac{1}{54}$. *7.* $2\frac{1}{2}$, 6, 8. *8.* $3\frac{1}{2}$, $\frac{9}{11}$, $3\frac{1}{8}$.
9. $1\frac{29}{40}$, $\frac{8}{4}$, 2. *10.* $7\frac{5}{7}$, $1\frac{5}{11}$, $5\frac{1}{4}$.

EXERCISE CXLVIII. PAGE 99.

1 $37\frac{1}{2}$ yd. *2.* $8\frac{2}{5}$ mi. *3.* $\$26\frac{1}{4}$. *4.* $17\frac{2}{5}$ mi. *5.* 10 bags.
6. $12\frac{1}{2}$ ct. *7.* $28\frac{2}{3}$ hr. *8.* $8\frac{23}{29}$. *9.* $217\frac{1}{2}$ A. *10.* $370\frac{1}{2}$ A.
11. $15\frac{1}{8}$ ct. *12.* 32 da. *13.* \$3.60. *14.* 20.

EXERCISE CXLIX. PAGE 100.

1. $7\frac{4}{7}$, 4. *2.* $3\frac{1}{2}$, $3\frac{3}{5}$. *3.* $2\frac{18}{41}$, 12. *4.* $\frac{7}{8}$, $\frac{7}{29}$. *5.* $8\frac{1}{7}$, $\frac{25}{399}$.
6. $\frac{5}{7}$, $2\frac{26}{35}$. *7.* $4\frac{1}{2}$, $\frac{8}{115}$. *8.* $23\frac{5}{8}$, 16. *9.* $\frac{1}{60}$, $\frac{1}{16}$. *10.* $36\frac{27}{68}$, $\frac{16}{187}$.

EXERCISE CL. PAGE 100.

1. $\frac{2}{15}$, $3\frac{1}{3}$. *2.* $3\frac{1}{3}$, 30. *3.* 54, $66\frac{2}{3}$. *4.* 6, $\frac{15}{56}$. *5.* $\frac{23}{90}$, $\frac{7}{64}$.
6. $\frac{9}{10}$, $2\frac{4}{7}$. *7.* $\frac{1}{6}$, $5\frac{1}{3}$. *8.* $10\frac{1}{12}$, $\frac{3}{4}$. *9.* $5\frac{17}{44}$, $\frac{8}{4}$. *10.* $\frac{45}{88}$, $\frac{55}{512}$.

EXERCISE CLI. PAGE 101.

1. $5\frac{5}{8}$, $\frac{1}{8}$. *2.* $1\frac{1}{3}$, $1\frac{1}{2}$. *3.* $1\frac{7}{13}$, $1\frac{1}{2}$. *4.* $\frac{20}{21}$, $\frac{13}{31}$. *5.* $\frac{11}{91}$, $\frac{3}{17}$. *6.* 1, $\frac{14}{17}$. *7.* $2\frac{1}{5}$, $3\frac{5}{9}$. *8.* 7, $\frac{1}{6}$. *9.* $1\frac{1}{2}$, 40. *10.* $\frac{1}{3}$, 2.

EXERCISE CLII. PAGE 101.

1. $144\frac{3}{8}$. *2.* 2. *3.* $1\frac{4}{93}$. *4.* $\frac{2}{5}$. *5.* $7\frac{1}{2}$. *6.* $4\frac{7}{10}$. *7.* $15\frac{1}{2}$. *8.* $43\frac{5}{12}$. *9.* $\frac{6}{25}$. *10.* 4.

EXERCISE CLIII. PAGE 102.

1. $\frac{11}{12}$. *2.* 4. *3.* $6\frac{13}{75}$. *4.* $\frac{19}{20}$. *5.* 2. *6.* 35. *7.* $10\frac{2}{3}$. *8.* $1\frac{1}{2}$. *9.* $1\frac{13}{36}$; $\frac{19}{36}$; $\frac{85}{216}$; $2\frac{4}{15}$. *10.* $\frac{3}{5}$.

EXERCISE CLIV. PAGE 102.

1. $\frac{5}{23}$; $\frac{30}{23}$. *2.* $\frac{7}{135}$; $\frac{28}{9}$. *3.* $\frac{4}{45}$; 60. *4.* $2\frac{3}{10}$ ft. *5.* $5\frac{5}{9}$ ft.; 19. *6.* 60. *7.* \$$112\frac{1}{2}$. *8.* $3\frac{3}{8}$ bu., 51 bags. *9.* 59.
10. $6\frac{2}{3}$ da.; A, 10 times; B, 15 times; C, 8 times.

EXERCISE CLV. PAGE 103.

1. 16s.; $3\frac{1}{2}$d.; $4\frac{1}{2}$d. *2.* $87\frac{1}{2}$ ct.; 80 ct.; 75 ct.
3. 17 cwt. 50 lb.; 7 lb. 8 oz.; 14 oz.
4. 213 rd. 1 yd. 2 ft. 6 in.; 3 yd. 2 in.; 2 ft. 6 in.
5. 128 sq. rd.; 22 sq. yd.; 2 sq. ft. 117 sq. in.
6. 96 c. ft.; 11 c. ft. 432 c. in.; 504 c. in.
7. 2 pk. 1 gal.; 1 gal. $1\frac{1}{3}$ qt.; 3 qt. *8.* 3 qt. 1 pt.; $1\frac{1}{2}$ pt.; $3\frac{1}{5}$ qt.
9. 4 da. 21 hr. 36 min.; 7 hr.; 25 min. *10.* 40′; 288″; 16′ 40″.

EXERCISE CLVI. PAGE 104.

1. 444546 in.; 126744 in.
2. 1 mi. 131 rd. 4 yd. 2 ft. 6 in.; 1 mi. 179 rd. 1 ft. 6 in.
3. 237600 in.; $1559\frac{1}{4}$ in. *4.* 31637628 sq. in.; 3860136 sq. in.
5. 1 A. 52 sq. rd. 19 sq. yd. 8 sq. ft.; 2 A. 47 sq. rd. 9 sq. yd. 3 sq. ft. 36 sq. in.
6. 16727040 sq. in.; 131868 sq. in. *7.* 9600 lb.; 3800 oz.
8. 145 rd. 2 yd. 1 ft. 6 in.; 93 sq. rd. 10 sq. yd. 108 sq. in.
9. 9 A. 120 sq. rd. 22 sq. yd.; 1 A. 16 sq. rd. 9 sq. yd., 3 sq. ft.
10. 200 gal., 35 t.

EXERCISE CLVII. PAGE 104.

1. \$82.11; \$28.80. *2.* £1 5s. $3\frac{3}{4}$d.; £1 2s. 8d.
3. 3 t. 12 cwt. 4 lb.; 1 t. 11 cwt. 48 lb.
4. 2 mi. 197 rd. 5 yd. 9 in.; 2 mi. 270 rd. $4\frac{4}{5}$ in.
5. 1 A. 135 rd. 22 yd. 8 ft. $5\frac{1}{7}$ in.; 2 A. 16 rd.
6. 2 cd. 59 cu. ft.; 10 cd. 60 cu. ft.
7. 3 bu. $\frac{1}{9}$ gal.; 2 bu. 0 pk. 4 qt.
8. 5 gal.; 13 gal. 1 pt. *9.* 23 hr. 20 min.; 5 hr. 26 min. 40 sec.
10. 10° 30′ 12″; 14° 30′ 30″.

EXERCISE CLVIII. PAGE 104.

1. $\frac{3}{400}$ cwt. *2.* £$\frac{1}{360}$. *3.* $\frac{1}{12}$ bu. *4.* $\frac{4}{55}$ rd. *5.* $\frac{1}{396}$ rd.
6. $\frac{3}{5}$ pt. *7.* $\frac{3}{2}$ oz. *8.* $\frac{10}{13}$. *9.* $\frac{5}{11}$. *10.* $\frac{1}{3}$ ft.

EXERCISE CLIX. PAGE 105.

1. $\frac{5}{9}$ lb. *2.* $\frac{1}{128}$. *3.* $\frac{103}{440}$ mi. *4.* $\frac{4}{13}$. *5.* $\frac{41}{180}$. *6.* $\frac{5}{18}$ cu. yd.
7. $\frac{71}{84}$. *8.* $\frac{1}{9}$ mi. *9.* $\frac{144}{175}$. *10.* $\frac{7}{13}$ A.

EXERCISE CLX. PAGE 105.

1. 34940585. *2.* 525 lb. *3.* 169 qt. *4.* 55756$\frac{4}{5}$. *5.* 4320.
6. 5940. *7.* \$251.25. *8.* \$1837$\frac{1}{2}$. *9.* \$44.20. *10.* \$300.

EXERCISE CLXI. PAGE 105.

1. 118188 lb. *2.* 31$\frac{1}{2}$ cd. *3.* \$150. *4.* \$3.84. *5.* \$633.60.
6. 267 d. *7.* 129600 min. *8.* 1440 min. *9.* \$21.51. *10.* 2024 poles.

EXERCISE CLXII. PAGE 106.

1. 47 lb. *2.* 36112 pt. *3.* 5392 oz. *4.* 31 t. 5 cwt. *5.* 1800.
6. \$499.95. *7.* 1248 oz. *8.* 40 ct. *9.* 189. *10.* 46772 lb.

EXERCISE CLXIII. PAGE 106.

1. 22506 gr. *2.* 161 gr. *3.* 5$\frac{17}{120}$ oz. *4.* 5184 gr. *5.* 11088 gr.
6. 10 lb. 1 oz. 12 dwt. 8 gr. *7.* 25. *8.* 37$\frac{1}{2}$ lb.
9. 701 lb. 6 oz. 6 dwt. 5 gr. *10.* 114$\frac{27}{32}$ lb.

EXERCISE CLXIV. PAGE 107.

1. 63360 in. *2.* 8064 in. *3.* 2 mi. *4.* \$462. *5.* \$422.40.
6. \$500. *7.* 10 hr. 17 min. *8.* 1584000 in. *9.* 506898 in.
10. 440 mi.

EXERCISE CLXV. PAGE 107.

1. \$72. *2.* \$31.50. *3.* \$1.95. *4.* 438 baskets. *5.* 335 pt.
6. \$1.05. *7.* 4. *8.* \$510.30. *9.* 2 qt. *10.* \$137.74$\frac{1}{2}$.

EXERCISE CLXVI. PAGE 108.

1. 32 da. *2.* 264 ft. *3.* 9. *4.* 44 ft. *5.* 9 mi. *6.* 320 hr.
7. 500 da. *8.* \$57.24. *9.* \$47.25. *10.* \$19.44.

EXERCISE CLXVII. PAGE 108.

1. 68 ct. *2.* 5 ft. *3.* 8184. *4.* 15. *5.* 43 mi. *6.* 48 hr.
7. 4620 paces. *8.* 43. *9.* 24320. *10.* 8$\frac{35}{44}$ mi.

EXERCISE CLXVIII. PAGE 109.

1. 6 lb. 4 oz. 8 dwt. 12 gr. *2.* £200. *3.* 10 cwt. 80 lb. 1 oz.
4. 24 ft. *5.* 990 yds. *6.* 4 min. 16 sec.; 980 yd. *7.* 32 yd.
8. 54. *9.* 989 lb. *10.* 1271.

EXERCISE CLXIX. PAGE 109.

1. 84 in. *2.* 7 da. *3.* 506 da. *4.* 66 mi. *5.* 75 ct. *6.* \$8050.50. *7.* $49\frac{7}{9}$ yd. *8.* 60 ct. *9.* 12 da. 20 hr. 20 min. *10.* 4 yd. 1 ft. 3 in.

EXERCISE CLXX. PAGE 110.

1. 3 lb. 11 oz. 11 dwt. 6 gr. *2.* 9. *3.* 12 hr. *4.* 110 da. *5.* 36 hr. *6.* 42. *7.* 92928. *8.* 12 spoons. *9.* $5\frac{1}{22}$ mi. *10.* £43 1s. 3d.; £6 8s. 9d.

EXERCISE CLXXI. PAGE 110.

1. 209 bu. 1 pk. 3 qt. *2.* 1569 bbl. *3.* A, $13\frac{3}{4}$ yd.; B, $19\frac{1}{4}$ yd. *4.* 36 mi. 298 rd. 1 ft. 6 in. *5.* 48 rd. *6.* $8\frac{4}{7}$ yd. *7.* \$62. *8.* \$13.50. *9.* \$432. *10.* $14\frac{7}{8}$ t.

EXERCISE CLXXII. PAGE 111.

1. \$7.50. *2.* \$1.76. *3.* \$96. *4.* \$49. *5.* \$19.12. *6.* $27\frac{1}{5}$ yd. *7.* 27 lb. *8.* 10 hr. *9.* \$8.92½. *10.* \$94.40, \$660.80.

EXERCISE CLXXIII. PAGE 111.

1. 48, 8 lb. *2.* 32 lb. *3.* 42 mi. *4.* 24 yd. *5.* 450 sheep. *6.* 176 ft. *7.* $58\frac{2}{5}$ A. *8.* 60 ct. *9.* \$12000. *10.* $\frac{39}{40}$ A.

EXERCISE CLXXIV. PAGE 112.

1. 288. *2.* \$68. *3.* $160\frac{2}{3}$ bu., $321\frac{1}{3}$ bu. *4.* \$20¼. *5.* \$6000. *6.* $8\frac{1}{4}$ ft., $2079\frac{11}{16}$ gal. *7.* \$23.76. *8.* 100 A. *9.* \$4070. *10.* \$6250.

EXERCISE CLXXV. PAGE 113.

1. 50 A. 156 rd. *2.* 7 ft. 9 in. *3.* $8\frac{2}{3}$ ft. *4.* 3 doz. *5.* $181\frac{17}{80}$ A., $203\frac{47}{80}$ A. *6.* 38 A. 91¼ rd. *7.* 42240. *8.* \$66.01. *9.* 396 lots. *10.* \$13459.56; James, \$3364.89; William, \$4486.52; daughter, \$1121.63; wife, \$4486.52.

EXERCISE CLXXVI. PAGE 113.

1. $\frac{9}{20}$. *2.* $\frac{5}{12}$. *3.* $1\frac{1}{3}$ da. *4.* 36 da. *5.* $16\frac{4}{5}$ da. *6.* $41\frac{1}{4}$ da. *7.* 10 da. *8.* $1\frac{13}{47}$ da. *9.* $2\frac{6}{17}$ da. *10.* $1\frac{26}{59}$ da. *11.* $21\frac{3}{17}$ da. *12.* $5\frac{1}{3}$ hr. *13.* $28\frac{4}{7}$ hr. *14.* $6\frac{15}{29}$ da. *15.* $2\frac{4}{9}$ da. *16.* $3\frac{28}{31}$ da.

EXERCISE CLXXVII. PAGE 116.

1. Nine-tenths; forty-five hundredths; seventy-five hundredths; eight hundredths.

2. Two hundred and seventy-five thousandths; eighty-seven thousandths; six thousandths; two hundred and nine thousandths.

3. Two, and six hundredths; seven, and one thousandth; ten, and seven hundredths; nine, and two hundred and seven thousandths.

4. Thr e thousand eight hundred and seventy-five ten-thousandths; five hundred and sixty-two ten-thousandths; eighty-three ten-thousandths; six ten-thousandths.

5. One, and six hundred and thirty-one thousandths; one, and three hundred and fifteen thousandths; forty-eight, and seven thousandths; eighty-seven, and six ten-thousandths.

6. Two hundred and one, and two hundred and one thousandths; seventy-eight, and five hundred and sixty-seven thousandths; one hundred, and one thousandth; seven hundred and nine, and two hundred and twenty-four thousandths.

7. Six hundred and twelve, and six hundred and twelve thousandths; thirteen, and one hundred and eight ten-thousandths; seven hundred, and six hundred and twenty-five thousandths; five, and six thousand and six ten-thousandths.

8. Ten thousand, and one thousandth; one thousand, and one ten-thousandth; one hundred thousand, and one hundredth; one million, and one tenth.

9. Two hundred, and two thousand and six ten-thousandths; two thousand and two, and six thousandths; twenty thousand and twenty, and six hundredths; twenty, and two thousand and six hundred-thousandths.

10. Seventy-eight thousand nine hundred and sixty-five, and four hundred and thirty-two thousandths; seven hundred and eighty-nine, and sixty-five thousand four hundred and thirty-two hundred-thousandths; seven thousand eight hundred and ninety-six, and five thousand four hundred and thirty-two ten-thousandths; seven hundred and eighty-nine thousand six hundred and fifty-four, and thirty-two hundredths.

EXERCISE CLXXVIII. PAGE 116.

1. 325.7. *2.* 465.14. *3.* 93.07. *4.* .213. *5.* 1000.0006.
6. 37.072. *7.* .0718. *8.* 240000.406. *9.* 56000000.000056.
10. 70000000.000007.

EXERCISE CLXXIX. PAGE 117.

1. 961.15; 509.031; 1119.1817; 227.05. *2.* 1605.50494.
3. 405.970973. *4.* 9932.1237939. *5.* 7449.093. *6.* 392.842547.
7. 131.8562. *8.* 170.63676. *9.* $508.35. *10.* $913.829.

EXERCISE CLXXX. PAGE 117.

1. 1006.6106. *2.* 251.8474. *3.* 843.086. *4.* 1065.0771.
5. 8574.1378. *6.* 4679.1364. *7.* 226.35187. *8.* 4196.924.
9. 222.91522. *10.* 4533.246.

EXERCISE CLXXXI. PAGE 117.

1. 282.405 A. *2.* 307222.086446. *3.* 363.536487.
4. 115.8125 yd. *5.* 24.2675 t. *6.* 25269.111505. *7.* .42951303.
8. 6.5 t. *9.* 138.122427. *10.* 48095.139833.

EXERCISE CLXXXII. PAGE 118.

1. 4.17; 53.93; 14.221; 69.497.
2. 37.6211; 3.322; 4.10001; 19.7233.
3. 4.753; 1.8742; .0058; 5.655. *4.* 5.2311; 4.122.
5. .453; 55.0316. *6.* 12.211; 16.9993. *7.* 99.9999; 9.99.
8. 81.4211; .12346. *9.* 37.344; 3.405. *10.* 384.9924; 993.9994.

EXERCISE CLXXXIII. PAGE 119.

1. 3.544; 12.66; 10.565. *2.* .7141; .2703; 4.3155.
3. 3.09; .7632; 2.786. *4.* 70.4625; 2.524; 7.9933.
5. 4.2517; 1.255; .9999. *6.* 9.9995; .125; 21.325.
7. .675; 777.15; 784.215. *8.* 593.34; 4.725; .825.
9. .1235; 25.1205; 2.46. *10.* 96.855; 4.4445; .99646.

EXERCISE CLXXXIV. PAGE 119.

1. 72.927. *2.* .1993. *3.* 364.9953. *4.* 6999.9996045.
5. 999999.999999. *6.* .017481. *7.* 2999999.900001. *8.* .000999.
9. 43.08997. *10.* 6929.95993.

EXERCISE CLXXXV. PAGE 119.

1. 30.280965. *2.* 75.0665. *3.* 15.635563. *4.* 16.6799884.
5. 49.156. *6.* 4.152. *7.* 81.143. *8.* 9.7597. *9.* 80.025.
10. 9.0897.

EXERCISE CLXXXVI. PAGE 120.

1. 228.475 A. *2.* $327.065. *3.* 199.35 mi. *4.* $963.68.
5. 262.35 A. *6.* .08 in. *7.* $111.58. *8.* 29569.92 A. *9.* 123.315 lb.
10. 73.73 ft. *11.* .3. *12.* 30.338 in.

EXERCISE CLXXXVII. PAGE 121.

1. 20.16; 30.24; 56.09. *2.* 12.25; 156.21; 110.16.
3. 16.11; 1.591; 19.6875. *4.* 60.125; .0945; .000243.
5. 76.845; .00063; 156.25. *6.* 4.38496; .0372161; 152.2756.
7. .00000081; .007224; .005621.
8. .00004216; .00015625; .005625.
9. 28.3743; 5673.024; 60.025.
10. 49.098049; .0010001; .27648.

EXERCISE CLXXXVIII. PAGE 121.

1. .435; .0676. *2.* 506.4463; 35.4367519. *3.* .013272; 32.5779.
4. 36.9; 1.7005. *5.* 3.8; 1860.867. *6.* 525; 92.7.
7. 11221.1; 54.706. *8.* 9.06453; 279.29475. *9.* .000000027; 4.8.
10. .1728; .032.

EXERCISE CLXXXIX. PAGE 121.

1. 1234.56; 12345.6; 123456. *2.* 33.5175. *3.* .000019737. *4.* .00365. *5.* .005162. *6.* 45; 450; 4500; 45000. *7.* 421000. *8.* 1144.90001605. *9.* .68785. *10.* .0000000000884.

EXERCISE CXC. PAGE 122.

1. 204.375 mi. *2.* 453.125 lb. *3.* \$7000. *4.* 270.465 ft. *5.* 864.86 ft. *6.* 23.625 yr. *7.* 436247.424 gr. *8.* 590.628 mi. *9.* 993.125 gal. *10.* 297.25375.

EXERCISE CXCI. PAGE 122.

1. .4; 40; 50. *2.* 200; 400; 30. *3.* .016; 7.5; 8. *4.* 34; 2.5; 15. *5.* .2; 200; .0016. *6.* 2480; .00025; 4000. *7.* 42.8; 73.21; 2.78. *8.* .85; .234; 4602. *9.* 60000; .0075; .000436. *10.* 42250; 4600000; 548.8.

EXERCISE CXCII. PAGE 123.

1. 14; 19.28. *2.* 185000; .00002. *3.* 52.416; .013352. *4.* 40000; 24106.25. *5.* 24.488; 31.8. *6.* .0015; .0672. *7.* .128; 115312.5. *8.* 192; 325.26. *9.* 1.099; 160. *10.* .043; .135.

EXERCISE CXCIII. PAGE 123.

1. 15.24. *2.* 350000. *3.* .413. *4.* 330000. *5.* .024. *6.* 64753000000. *7.* 12.345, 1.2345, .12345, .012345. *8.* 10.5. *9.* 127.4. *10.* .075, .0075, .00075.

EXERCISE CXCIV. PAGE 123.

1. 6.165 rd. *2.* 320 rd. *3.* 42 bu. *4.* 144 bbl. *5.* 87.5 bbl. *6.* \$128.50. *7.* \$27.5. *8.* \$10800. *9.* 15.75 bu. *10.* 5.2 hr.

EXERCISE CXCV. PAGE 124.

1. $\frac{7}{20}$; $\frac{12}{25}$; $\frac{1}{4}$; $\frac{1}{8}$. *2.* $\frac{5}{8}$; $\frac{3}{4}$; $\frac{3}{8}$; $\frac{16}{25}$.
3. $\frac{2}{125}$; $\frac{9}{40}$; $\frac{7}{8}$; $\frac{7}{200}$. *4.* $\frac{11}{40}$; $\frac{23}{40}$; $\frac{3}{80}$; $\frac{1}{200}$.
5. $\frac{17}{250}$; $\frac{3}{125}$; $\frac{7}{40}$; $\frac{7}{400}$. *6.* $\frac{99}{100}$; $\frac{21}{1000}$; $\frac{123}{1000}$; $\frac{3}{1000}$.
7. $2\frac{3}{4}$; $4\frac{19}{25}$; $7\frac{9}{20}$; $5\frac{9}{25}$. *8.* $3\frac{1}{4}$; $9\frac{3}{4}$; $12\frac{29}{40}$; $5\frac{8}{125}$.
9. $6\frac{1}{80}$; $16\frac{3}{40}$; $7\frac{7}{8}$; $11\frac{5}{8}$. *10.* $5\frac{5}{16}$; $2\frac{3}{16}$; $7\frac{15}{16}$; $9\frac{11}{16}$.

EXERCISE CXCVI. PAGE 124.

1. .25; .75; .125; .375. *2.* .625; .875; .1875; .3125.
3. .4375; .5625; .6875; .8125. *4.* 16; .24; .44; .72.
5. .175; .425; .925; .825. *6.* .144; .084; .888; .868.
7. 4.28; 3.725; 5.775; 7.975. *8.* .072; 3.068; 5.04; 7.125.
9. .036; 5.04; 7.9375; 8.36. *10.* .2192; .1616; .7056; .5936.

EXERCISE CXCVII. PAGE 125.

1. 15s. $9\frac{3}{4}$d.; 15s. $3\frac{3}{4}$d.; £1 16s. $2\frac{1}{4}$d.

2. 83 lb. 4 oz.; 1 cwt. 74 lb. 14.4 oz.; 3 t. 17 cwt. 95 lb.

3. 252 rd.; 80 rd. 4 yd. 1.2 ft.; 21 mi. 118 rd.

4. 145 sq. rd. 6 sq. yd. 64.8 sq. in.; 141 sq. rd.; 13 A. 70 sq. rd.

5. 25 cu. ft. 450 cu. in.; 7cd. 112 cu. ft.; 9 cu. yd. 20 cu. ft. 432 cu. in.

6. 3 pk. 1 gal.; 2 bu. 1 pk. 1 gal.; 5 bu. 1 pk. 1 gal. 3.2 qt.

7. 3 qt. 1 pt.; 5 gal. 1.4 pt.; 7 gal. 3 qt.

8. 11 hr. 52 min. 48 sec.; 7 wk. 6 da. 3 hr.; 9 wk. 6 da. 15 hr 36 min.

9. $58' 30''$; $2° 50' 51''$; $17° 23' 15''$.

10. 3 ft. 9 in.; 17 rm. 17 qr. 12 sh.; 7 gro. 9 doz.

EXERCISE CXCVIII. PAGE 125.

1. £1.525; £7.68125.
2. 9.854 t.; 5.70375 t.
3. 7.1109375 mi.; 6.089 mi.
4. 9.3 A.; 7.283 A.
5. 7.875 cd.; 7.6875 cu. yd.
6. 7.8125 bu.; 5.890625 bu.
7. 27.875 gal.; 14.375 gal.
8. 3.55875 da.; 2.7725 wk.
9. 3.8775°; 17.12375°.
10. 5.8875 rm.; 24.75 grs.

EXERCISE CXCIX. PAGE 125.

1. \$3380. *2.* 10.3826 ft. *3.* \$59.375. *4.* The latter, 8.875 cu. in. *5.* 3.2 lb. *6.* \$24.80. *7.* 83.75 yd. *8.* \$322.28. *9.* 199218.75 oz. *10.* \$186.15.

EXERCISE CC. PAGE 126.

1. \$414. *2.* \$109.0625. *3.* 75 sheep. *4.* \$161.655. *5.* 50. *6.* \$492.61875. *7.* 3720. *8.* \$53.25. *9.* \$4. *10.* 13.378 A.

EXERCISE CCI. PAGE 126.

1. 67.75 A. *2.* 28 yr. *3.* \$26484. *4.* 7.39, 8.95. *5.* \$10.32. *6.* Gain, \$846.875, \$12.50. *7.* A., \$75.87; B., \$57.39. *8.* 16.5. *9.* 1585584 cu. ft. *10.* 8.6328 mi., 9.1872 mi.

EXERCISE CCII. PAGE 127.

1. \$5. *2.* \$96.768. *3.* 29.83 in. *4.* 2.7 ft. *5.* 2.2968. *6.* .5625; 33.75. *7.* 15360. *8.* 58 lb. *9.* \$24.60. *10.* \$192.78.

EXERCISE CCIII. PAGE 128.

1. 42.34. *2.* 17.3. *3.* 8.45, 5.65. *4.* 26.1 cd., 24.65 cd. *5.* 64 mi., 56.25 mi. *6.* \$340.05. *7.* \$18.8325. *8.* .7875. *9.* 4 ft. 3.2 in. *10.* 92.4 yd. *11.* \$181.50, \$77, \$44. *12.* 660 ft. *13.* 2446.875 lb. *14.* $4\frac{2}{13}$ da.

EXERCISE CCIV. PAGE 129.

1. $\frac{5}{100}$; $\frac{7}{100}$; $\frac{10}{100}$; $\frac{15}{100}$; $\frac{27}{100}$.

2. $\frac{7\frac{1}{2}}{100}$; $\frac{6\frac{1}{4}}{100}$; $\frac{12\frac{1}{2}}{100}$; $\frac{20}{100}$; $\frac{50}{100}$.

3. 5%; 7%; 10%; 17%; 25%.

4. $5\frac{1}{2}$%; $7\frac{1}{4}$%; $10\frac{1}{2}$%; $12\frac{1}{2}$%; $17\frac{1}{4}$%.

5. (1) .05; .08; .1; .125; .2.
(2) $\frac{1}{20}$; $\frac{2}{25}$; $\frac{1}{10}$; $\frac{1}{8}$; $\frac{1}{5}$.

6. (1) $.11\frac{1}{9}$; $.16\frac{2}{3}$; .375; .75; .875.
(2) $\frac{1}{9}$; $\frac{1}{6}$; $\frac{3}{8}$; $\frac{3}{4}$; $\frac{7}{8}$.

7. (1) $\frac{25}{100}$; $\frac{75}{100}$; $\frac{40}{100}$; $\frac{70}{100}$; $\frac{16}{100}$.
(2) 25%; 75%; 40%; 70%; 16%.

8. (1) $\frac{33\frac{1}{3}}{100}$; $\frac{66\frac{2}{3}}{100}$; $\frac{16\frac{2}{3}}{100}$; $\frac{12\frac{1}{2}}{100}$; $\frac{37\frac{1}{2}}{100}$.
(2) $33\frac{1}{3}$%; $66\frac{2}{3}$%; $16\frac{2}{3}$%; $12\frac{1}{2}$%; $37\frac{1}{2}$%.

9. $\frac{1}{25}$; $\frac{2}{5}$; $\frac{4}{5}$; $\frac{3}{20}$; $\frac{3}{10}$.

10. $\frac{1}{3}$; $\frac{1}{16}$; $\frac{1}{12}$; $\frac{1}{18}$; $\frac{5}{8}$.

EXERCISE CCV. PAGE 130.

1. \$3; \$20; \$35.
2. 16 A.; 20 A.; 12 A.
3. 15 words; 33 words; 54 cows.
4. 50 boys; \$374.50; 15 yd.
5. \$50; $187\frac{1}{2}$ mi.; 45 hens.
6. 2 men; 2 yd.; 40c.
7. \$65; 50 A.; 55 men.
8. 30 yd.; 250 sheep; 30 cows.
9. \$302; 415 men; 216 books.
10. 79; 1456 yd.; \$1602.

EXERCISE CCVI. PAGE 130.

1. 20%; 10%; 25%. *2.* 10%; 5%; $12\frac{1}{2}$%. *3.* 50%; $12\frac{1}{2}$%; $4\frac{4}{9}$%. *4.* 5%; 7%; $5\frac{5}{9}$%. *5.* 12%; 7%; 5%. *6.* 5%; $62\frac{1}{2}$%; $12\frac{1}{2}$%. *7.* \$10; 5 A. *8.* 24 lb.; 4 gal. *9.* \$42; \$107. *10.* \$90; 55 mi.

EXERCISE CCVII. PAGE 130.

1. 444 sheep. *2.* \$103. *3.* 225 bales. *4.* 288 boxes. *5.* \$216. *6.* 480 A. *7.* 120 lb. *8.* 200 A. *9.* 300. *10.* 30 bu.

EXERCISE CCVIII. PAGE 131.

1. \$1400. *2.* \$7760. *3.* 120 A. *4.* \$3375. *5.* \$29500. *6.* 4930 sheep. *7.* 231 girls. *8.* 20277. *9.* \$717.25. *10.* 73%, 876 sheep.

EXERCISE CCIX. PAGE 131.

1. $66\frac{2}{3}$%. *2.* 98%. *3.* 80%. *4.* 50%. *5.* 20%. *6.* 12%. *7.* 25%. *8.* 8%. *9.* $33\frac{1}{3}$%. *10.* 40%; 60%.

EXERCISE CCX. PAGE 132.

1. 70. *2.* \$2500. *3.* \$500. *4.* 800 A. *5.* 450 sheep. *6.* \$4500. *7.* 150 lines. *8.* \$1300. *9.* 4500 sheep. *10.* 450 pupils.

EXERCISE CCXI. PAGE 132.

1. 3290 bu. *2.* 6%. *3.* \$72. *4.* \$20. *5.* \$12932.50. *6.* 48992 lb. *7.* \$125. *8.* \$5400. *9.* \$9000. *10.* Lost, $12\frac{1}{2}$%.

EXERCISE CCXII. PAGE 133.

1. 330 rd. *2.* \$372. *3.* 10 lb. *4.* 48 mi. *5.* 2800 books. *6.* 224 men. *7.* 1880, 2021. *8.* $11\frac{1}{3}$%. *9.* 13752. *10.* $9\frac{1}{11}$%.

EXERCISE CCXIII. PAGE 134.

1. \$112.50; \$216. *2.* \$292; \$504. *3.* \$108. *4.* \$105. *5.* \$180. *6.* \$150. *7.* $4\frac{1}{2}$%; $37\frac{1}{2}$%. *8.* \$360. *9.* \$800. *10.* \$3.75.

EXERCISE CCXIV. PAGE 135.

1. \$270, \$540, \$547.20, \$1051.65. *2.* \$557.46. *3.* \$10.26. *4.* \$3.20. *5.* \$25. *6.* \$$6\frac{2}{3}$. *7.* \$500. *8.* \$2.50. *9.* $32\frac{1}{2}$%. *10.* 46%.

EXERCISE CCXV. PAGE 135.

1. \$25.50. *2.* \$3.20. *3.* 28%. *4.* 46.19%. *5.* \$20. *6.* \$7.56. *7.* \$490.96. *8.* \$$33\frac{1}{3}$%. *9.* \$461.70. *10.* \$357, $47\frac{1}{2}$%.

EXERCISE CCXVI. PAGE 136.

1. Profit, \$2; profit, \$8; loss, \$1.50; profit, \$1.50.

2. Gain, $33\frac{1}{3}$%; gain, $56\frac{1}{4}$%; gain, 20%; loss, 5%.

3. Profit, \$10; loss, \$40; gain, \$5.20; profit, \$150.

4. \$43; \$75; \$102; \$100. *5.* \$100; \$200; \$60; \$72; \$60; \$96.

6. \$30; \$160; \$25; \$700. *7.* 25%. *8.* 15%. *9.* 4%. *10.* $42\frac{6}{7}$%.

EXERCISE CCXVII. PAGE 137.

1. \$1.89. *2.* \$926.10. *3.* \$38.28. *4.* \$3612.50. *5.* \$238. *6.* \$2719.50. *7.* \$60. *8.* \$900. *9.* 9000 bu. *10.* \$4.20.

EXERCISE CCXVIII. PAGE 137.

1. \$112. *2.* \$8600. *3.* \$204.70. *4.* \$1007.50. *5.* \$75. *6.* $12\frac{1}{2}$%. *7.* \$330. *8.* 40%. *9.* $35\frac{5}{17}$. *10.* \$160, \$208.

EXERCISE CCXIX. PAGE 138.

1. \$18; \$8.40; \$24.15; \$56.70; \$209; \$87.50.

2. \$12.50; \$16.10; \$24.40; \$40.50; \$42; \$10.50.

3. \$168.75. *4.* \$115.92. *5.* \$12.50. *6.* \$19.38. *7.* \$13.50. *8.* 3%. *9.* $1\frac{1}{2}$%. *10.* $3\frac{2}{5}$%.

EXERCISE CCXX. PAGE 139.

1. 3%. *2.* $2\frac{1}{2}$%. *3.* $3\frac{1}{3}$%. *4.* \$14400. *5.* \$8400. *6.* \$2592.80. *7.* 84 ct. *8.* \$7200, \$216. *9.* 156 members. *10.* \$3264.

EXERCISE CCXXI. PAGE 139.

1. \$150. *2.* \$3650, \$146. *3.* \$270. *4.* 25000 yd. *5.* 60000 lb. *6.* 4100 lb. *7.* 52000 yd. *8.* \$3500. *9.* \$897.75. *10.* \$4000.

EXERCISE CCXXII. PAGE 140.

1. \$8.50; \$9; \$6; \$12; \$24; \$60; \$42; \$44; \$44. *2.* \$80. *3.* \$66.25. *4.* \$225. *5.* \$675, \$74325. *6.* \$22.50. *7.* $1\frac{3}{4}$%. *8.* $\frac{3}{4}$%. *9.* $\frac{7}{8}$%. *10.* $\frac{7}{9}$%.

EXERCISE CCXXIII. PAGE 141.

1. \$16000. *2.* \$3000. *3.* \$20000. *4.* \$14000. *5.* \$420. *6.* \$34737.50. *7.* \$40000. *8.* \$2040, \$5960. *9.* \$75. *10.* \$2500, \$3500, \$4000.

EXERCISE CCXXIV. PAGE 142.

1. \$54. *2.* \$150. *3.* \$19.50. *4.* \$525. *5.* \$652.50. *6.* \$22. *7.* \$18. *8.* 14 mills. *9.* $12\frac{1}{2}$ mills. *10.* $17\frac{1}{2}$ mills.

EXERCISE CCXXV. PAGE 143.

1. \$2800. *2.* \$2500. *3.* \$544000. *4.* \$101.60. *5.* \$2500. *6.* \$13000. *7.* \$5675. *8.* $6\frac{1}{4}$ mills. *9.* \$25. *10.* \$114.20.

EXERCISE CCXXVI. PAGE 144.

1. \$6, \$8, \$3, \$7, \$7, \$9. *2.* \$9. *3.* \$56. *4.* \$340. *5.* \$10. *6.* \$5.39. *7.* \$15.12. *8.* \$38.88. *9.* \$13.20. *10.* \$4.6116.

EXERCISE CCXXVII. PAGE 144.

1. \$915.60. *2.* \$1012.65. *3.* \$2084.02. *4.* \$674.01. *5.* \$531.65. *6.* \$425.656. *7.* \$1859.04. *8.* \$66. *9.* \$1910.20. *10.* \$1154.64.

EXERCISE CCXXVIII. PAGE 145.

1. 6%. *2.* 7%. *3.* $4\frac{1}{2}$%. *4.* $6\frac{2}{3}$%. *5.* $6\frac{1}{4}$%. *6.* $5\frac{1}{2}$%. *7.* $7\frac{1}{2}$%. *8.* $4\frac{1}{2}$%. *9.* 6%. *10.* 6%.

EXERCISE CCXXIX. PAGE 145.

1. 1 yr. *2.* $1\frac{1}{2}$ yr. *3.* $1\frac{1}{4}$ yr. *4.* 5 mo. *5.* 9 mo. *6.* 8 mo. *7.* 115 da. *8.* $1\frac{1}{4}$ yr. *9.* 150 da. *10.* 75 da.

EXERCISE CCXXX. PAGE 145.

1. $540. *2.* $655. *3.* $850. *4.* $360. *5.* $840. *6.* $750. *7.* $876. *8.* $1825. *9.* $219. *10.* $2920.

EXERCISE CCXXXI. PAGE 146.

1. $250. *2.* $465. *3.* $540. *4.* $572.50. *5.* $1320. *6.* $408. *7.* $750. *8.* $1752. *9.* $3650. *10.* $3212.

EXERCISE CCXXXII. PAGE 146.

1. $126.10, $926.10. *2.* $477.54, $2977.54. *3.* $76.51, $1326.51. *4.* $1724.05, $9724.05. *5.* $11255.09. *6.* $5250. *7.* $960. *8.* $10000. *9.* $4.32. *10.* $124.05.

EXERCISE CCXXXIII. PAGE 148.

1. (1) $250. TORONTO, Oct. 27, 1900.

Three months after date, for value received, I promise to pay John Smith, or bearer, the sum of two hundred and fifty dollars with interest at six per cent. per annum.

A. B.

(2) In the note under (1) substitute *or order* for *or bearer*.

(3) In the note under (1) omit the words *or bearer*.

2. $250. TORONTO, Oct. 27, 1900.

On demand, I promise to pay John Smith, or order, the sum of two hundred and fifty dollars for value received.

3. $250. TORONTO, Oct. 27, 1900.

Three months after date, I promise to pay Thomas Harris, or order, the sum of two hundred and fifty dollars for value received, with interest at four per cent. per annum.

JAMES JONES.

4. TORONTO, Oct. 27, 1900.

Three months after date, at the Bank of Commerce, here, I promise to pay Wm. Meadows, or order, the sum of fifty dollars, for value received, with interest at five per cent. per annum.

THOMAS JONES.

5. $751.875. *6.* $1485.15.

7. $350. TORONTO, Oct. 27, 1900.

At sight, pay to the order of the Bank of Commerce, the sum of three hundred and fifty dollars, and charge the same to the account of

MYSELF.

To PORTER AND JONES,
Montreal.

EXERCISE CCXXXIV. PAGE 148.

1.	Day, July 20;	Term of Discount, 80 da.;	Proceeds, $645.48.
2.	" Feb. 23;	" " 70 da.;	" $792.22.
3.	" March 18;	" " 80 da.;	" $180.50.
4.	" Sept. 14;	" " 45 da.;	" $5579.42.
5.	" Nov. 24;	" " 30 da.;	" $508.69.
6.	" Aug. 22;	" " 55 da.;	" $115.48.

7. $325. *8.* $576. *9.* $1926.70. *10.* $1053.77.

EXERCISE CCXXXV. PAGE 150.

1. $7828. *2.* $6760. *3.* $7032.50. *4.* $1248. *5.* $2669. *6.* $4750. *7.* $3262.50. *8.* $9750. *9.* $20400. *10.* $2600.

EXERCISE CCXXXVI. PAGE 150.

1. $700. *2.* $2500. *3.* $3500. *4.* $5600. *5.* $2200. *6.* 5. *7.* 10. *8.* 6. *9.* 28. *10.* 15.

EXERCISE CCXXXVII. PAGE 150.

1. $200. *2.* $240. *3.* $200. *4.* $100. *5.* $112. *6.* $5\frac{3}{5}\%$. *7.* $4\frac{4}{9}\%$. *8.* 5%. *9.* $5\frac{1}{3}\%$.

EXERCISE CCXXXVIII. PAGE 151.

1. $9100. *2.* $13310. *3.* $11340. *4.* $15570. *5.* $11495. *6.* 60. *7.* 70. *8.* 220. *9.* 500.

EXERCISE CCXXXIX. PAGE 151.

1. $16000. *2.* 124. *3.* $18000; 720. *4.* 8%. *5.* 7%. *6.* $3\frac{1}{2}\%$. *7.* $100. *8.* $540. *9.* $4925.

EXERCISE CCXL. PAGE 152.

1. $114\frac{2}{7}$. *2.* $83\frac{1}{3}$. *3.* $266\frac{2}{3}$. *4.* $5\frac{1}{3}\%$. *5.* $2400. *6.* \$$1426\frac{1}{4}$. *7.* 50. *8.* $29400. *9.* 5%. *10.* $6400.

EXERCISE CCXLI. PAGE 152.

1. $1000; $500; $200. *2.* $3000; $1500; $1000; $600; $375; $300. *3.* 16 da.; 5 da.; 4 da.; 2 da. *4.* 3 mo. *5.* 10 mo. *6.* 8 da. *7.* 60 days after the debt is due. *8.* 6 mo. *9.* 62 da.

EXERCISE CCXLII. PAGE 153.

1. 6 mo.; 3 mo.; 4 mo. *2.* 120 da. *3.* 4 mo. *4.* 5 mo. *5.* Ap. 23. *6.* $112\frac{1}{2}$ da. *7.* $3\frac{1}{2}$ mo. *8.* 70 da. *9.* 8 mo.

EXERCISE CCXLIII. PAGE 154.

1. B, \$450; A, \$600. *2.* A, \$189; B, \$243. *3.* A, \$135; B, \$125.
4. A, \$85; B, \$130; C, \$60. *5.* A, \$864; B, \$408; C, \$1152.
6. A, \$2600; B, \$3400. *7.* B, \$650; C, \$925.
8. A, \$1750; B, \$2125; C, \$1125. *9.* \$720.
10. A, \$2662; B, \$2420; C, \$2200; D, \$2000.

EXERCISE CCXLIV. PAGE 155.

1. A, \$17.50; B, \$22.50. *2.* A, \$285; B, \$450.
3. A, \$25; B, \$21; C, \$54. *4.* A, \$75; B, \$175; C, \$330. *5.* Equally.
6. A, \$2200; B, \$3000. *7.* A, \$1225; B, \$875; C, \$1050.
8. 4 mo. *9.* \$1480. *10.* A, \$8445.94; B, \$4054.05. *11.* \$4800.
12. C, \$1605$\frac{15}{17}$; D, \$1544$\frac{2}{17}$; E, \$1050.

EXERCISE CCXLV. PAGE 156.

1. $2^3=8$; $2^4=16$; $3^5=243$. *2.* $5^3=125$; $5^4=625$; $5^5=3125$.
3. $(\frac{1}{2})^3=\frac{1}{8}$; $(\frac{1}{3})^4=\frac{1}{81}$; $(\frac{2}{5})^5=\frac{32}{3125}$.
4. $(2\frac{1}{2})^2=6\frac{1}{4}$; $(3\frac{1}{4})^3=34\frac{21}{64}$; $(4\frac{1}{3})^4=352\frac{49}{81}$.
5. $(.1)^3=.001$; $(.03)^3=.000027$; $(.05)^4=.00000625$.
6. $(.12)^3=.001728$; $(.15)^3=.003375$; $(2.5)^4=39.0625$.
7. $2 \,.\, 2 \,.\, 2=8$; $3 \,.\, 3 \,.\, 3 \,.\, 3=81$; $4 \,.\, 4 \,.\, 4=64$.
8. $\frac{2}{3} \,.\, \frac{2}{3}=\frac{4}{9}$; $\frac{3}{4} \,.\, \frac{3}{4} \,.\, \frac{3}{4}=\frac{27}{64}$; $\frac{4}{5} \,.\, \frac{4}{5} \,.\, \frac{4}{5} \,.\, \frac{4}{5}=\frac{256}{625}$.
9. $(.5)\ (.5)\ (.5)=.125$; $(.05)\ (.05)\ (.05)=.000125$;
$.001 \times .001 \times .001 \times .001 \times .001=.000000000000001$.
10. $2\frac{1}{4} \times 2\frac{1}{4}=5\frac{1}{16}$; $3\frac{1}{4} \times 3\frac{1}{4} \times 3\frac{1}{4}=34\frac{21}{64}$; $5\frac{1}{4} \times 5\frac{1}{4} \times 5\frac{1}{4} \times 5\frac{1}{4}=759\frac{177}{256}$.

EXERCISE CCXLVI. PAGE 157.

1. 361; 2025; 1296; 8649. *2.* 10201; 254016; 820836; 501264.
3. 5715.36; 2190.24; 9506.25; 63.3616. *4.* $\frac{1}{9}$; $\frac{16}{49}$; $\frac{25}{64}$; $\frac{49}{256}$.
5. $20\frac{1}{4}$; $50\frac{49}{64}$; $90\frac{1}{4}$; $160\frac{4}{9}$. *6.* 729; 6859; 68921; 753571.
7. 1030301; 1331000; 129554216; 751089429.
8. 28372.625; 94.196375; 481890.304; 338.608873.
9. $\frac{8}{27}$; $\frac{125}{4096}$; $\frac{1331}{1728}$; $\frac{2744}{3375}$. *10.* $15\frac{5}{8}$; $144\frac{45}{64}$; $56\frac{71}{216}$; $376\frac{521}{729}$.

EXERCISE CCXLVII. PAGE 157.

1. 81; 125; 625; 4096. *2.* .0081; .0225; 15.625; .035937.
3. $\frac{27}{64}$; $\frac{16}{81}$; $15\frac{5}{8}$; $34\frac{21}{64}$. *4.* 2025; 13824; 2085136; $\frac{144}{169}$.
5. 32768; 40353607; 429981696; $\frac{1}{32}$. *6.* 12; 64; 216; 243.
7. 80; 15625; .512; 343. *8.* 6th. *9.* 256. *10.* 24th.
11. 49126081. *12.* 128787625000. *13.* .000000000016.
14. $36^2=(2 \,.\, 2 \,.\, 3 \,.\, 3)^2=2 \,.\, 2 \,.\, 2 \,.\, 2 \,.\, 3 \,.\, 3 \,.\, 3 \,.\, 3=2^4 \times 3^4$. *15.* 10 A.

EXERCISE CCXLVIII. PAGE 158.

1. 9; 11; 12; 15. *2.* 37; 56; 64; 81. *3.* 108; 314; 256; 527.
4. $\frac{5}{6}$; $\frac{11}{19}$; $\frac{16}{17}$; $\frac{35}{74}$. *5.* $\frac{36}{41}$; $\frac{43}{99}$; $\frac{79}{101}$; $\frac{87}{103}$. *6.* .1; .05; .03; .29.
7. .247; .035; .058; .16. *8.* 12; $\frac{3}{7}$; $\frac{13}{27}$; $\frac{1}{6}$.
9. 3.05; .102; .571; .00563. *10.* 2.236; .707; .948; .094.

EXERCISE CCXLIX. PAGE 158.

1. 490. *2.* 49$\frac{1}{2}$. *3.* 3202. *4.* 24 yd. *5.* 440 yd. *6.* 200 rd.
7. 272 ft. *8.* $160. *9.* 144 rd. *10.* $324.

EXERCISE CCL. PAGE 159.

1. 75 yd., 25 yd. *2.* 220 yd., 44 yd. *3.* 1$\frac{1}{2}$ in. *4.* 17 rd.
5. 1440 rails. *6.* 280 rd. *7.* 8030 yd. *8.* 15 ft. *9.* 221 yd.
10. 7712 yd.

EXERCISE CCLI. PAGE 159.

1. 6; 8; 9; 12. *2.* 15; 16; 18; 21. *3.* 25; 24; 28; 33.
4. 35; 49; 55; 56. *5.* 17; 23; 29; 37. *6.* 53; 59; 67; 71.
7. 101; 111; 169; 189. *8.* $\frac{7}{13}$; $\frac{9}{19}$; $\frac{14}{26}$; $\frac{11}{39}$. *9.* .1; .06; 11.6; 36.3.
10. .928; .4308; .2; 1.709.

EXERCISE CCLII. PAGE 160.

1. 25; .25; 1.23; 10.04. *2.* 21; 9$\frac{1}{3}$; 7$\frac{1}{5}$; 7$\frac{5}{6}$. *3.* 35.3.
4. 5476 sq. ft. *5.* 56454 sq. ft. *6.* 17 ft. *7.* 60 in. *8.* 9.2 in.
9. 12 ft. *10.* 15.75 in. *11.* 37 ft. *12.* 5 ft. *13.* 12. *14.* 36 ft.

EXERCISE CCLIII. PAGE 162.

1. 1521 sq. yd. *2.* 29 sq. ft. 56 sq. in. *3.* 1144 sq. ft.
4. 210 ft. *5.* 2 ch. 40 l. *6.* (*a*) 229$\frac{1}{2}$ sq. in. (*b*) .54 A. (*c*) 26$\frac{2}{3}$ sq. ft.
7. 5$\frac{5}{12}$ sq. ft. *8.* 16 rd. *9.* 80 rd. *10.* $50.

EXERCISE CCLIV. PAGE 163.

1. (*a*) 186 sq. ft. (*b*) 318$\frac{55}{72}$ sq. ft. (*c*) 5$\frac{9}{40}$ A. (*d*) 3$\frac{13}{20}$ A.
2. 50 in. *3.* 57 in. *4.* 92 rd. *5.* 4300 sq. ft. *6.* 2145 sq. yd.
7. 1715 sq. ft. *8.* 554$\frac{2}{3}$ sq. yd. *9.* 203 sq. ft. *10.* 56 ft.

EXERCISE CCLV. PAGE 163.

1. 44 in.; 66 in.; 11 ft.; 51$\frac{1}{3}$ ft.; 73$\frac{1}{3}$ ft.; 14 ch. 96 l.
2. 7 in.; 17$\frac{1}{2}$ in.; 19$\frac{1}{4}$ ft.; 1 ch. 71$\frac{1}{2}$ l.; 7 yd.; 21 yd. 7 in.
3. 24$\frac{1}{2}$ min. *4.* 111 rd. 1 ft. 10 in. *5.* 5 ft. *6.* 7. *7.* 41$\frac{1}{7}$ ft.
8. 84 yd. *9.* 12 ft. 10 in.
10. 22 in.; 33 in.; 55 in.; 66 in.; 110 in.; 121 in.

EXERCISE CCLVI. PAGE 164.

1. $962\frac{1}{2}$ sq. in.; 1386 sq. in.; 3850 sq. in.; 5544 sq. in.; $8662\frac{1}{2}$ sq. in.;
154 sq. in.; $346\frac{1}{2}$ sq. in.; $471\frac{5}{8}$ sq. in.; $962\frac{1}{2}$ sq. in.; 7546 sq. ft.; 154 sq. in.; $38\frac{1}{2}$ sq. ft.; $68\frac{4}{9}$ sq. ft.; $86\frac{5}{8}$ sq. ft.; 1386 sq. ch.
2. $26\frac{53}{72}$ sq. yd. *3.* 88 rd. *4.* 59.4 in. *5.* 308 sq. in. *6.* 62.04 yd.
7. 246.66 yd. *8.* $277\frac{1}{5}$. *9.* 110 sq. in. *10.* $\$838\frac{4}{9}$.

EXERCISE CCLVII. PAGE 165.

1. (*a*) 85; (*b*) 650; (*c*) 1730; (*d*) 698.
2. (*a*) 528; (*b*) 1230; (*c*) 44; (*d*) 144.
3. (*a*) 456; (*b*) 782; (*c*) 390; (*d*) 225. *4.* 29 mi. *5.* 35 ft.
6. 274 ft. *7.* 20.78 in. *8.* 11.18 ft. *9.* 10 ft. *10.* $54.60.

EXERCISE CCLVIII. PAGE 165.

1. $91\frac{1}{3}$ cu. in.; $34\frac{21}{64}$ cu. in.; 14 cu. ft., 197 cu. in.; 68 cu. ft., 145 cu. in.
2. $121\frac{1}{2}$ sq. in.; $63\frac{3}{8}$ sq. in.; $35\frac{1}{24}$ sq. ft.; $100\frac{1}{24}$ sq. ft.
3. $31\frac{1}{2}$ cu. ft.; $99\frac{1}{2}$ sq. ft. *4.* $131\frac{1}{4}$ cu. ft.; $181\frac{1}{4}$ sq. ft.
5. 20 in.; $4\frac{17}{27}$ cu. ft. *6.* $\$41\frac{1}{4}$. *7.* $16\frac{4}{7}$ ft. *8.* $25\frac{1}{2}$ ft.; 17 ft.
9. 28.09 in. *10.* 1650 gal.

EXERCISE CCLIX. PAGE 166.

1. (*a*) $8\frac{5}{9}$ cu. ft.; (*b*) $115\frac{1}{2}$ cu. ft.; (*c*) 385 cu. ft.; (*d*) $133\frac{49}{72}$ cu. ft.
2. (*a*) $29\frac{1}{3}$ sq. ft.; (*b*) 132 sq. ft.; (*c*) 220 sq. ft.; (*d*) $183\frac{1}{3}$ sq. ft.
3. $5\frac{25}{72}$ cu. yd. *4.* $66. *5.* (*a*) $31\frac{17}{36}$ sq. ft.; (*b*) $151\frac{1}{4}$ sq. ft.; (*c*) 297 sq. ft.; (*d*) 196 sq. ft., 101 sq. in. *6.* $91\frac{3}{8}$ sq. ft.
7. $24.75. *8.* 1 ft. *9.* 1 yd. *10.* $14\frac{3}{8}$ cu. ft.

EXERCISE CCLX. PAGE 166.

1. 108 cu. ft. *2.* 2760 cu. in. *3.* 75 sq. ft. *4.* 33 sq. ft.
5. $33.75. *6.* 19250 cu. in. *7.* (1) $86\frac{5}{8}$ sq. ft.; (2) 33 ft.
8. 89.51 yd. *9.* $89\frac{5}{6}$ cu. ft. *10.* 61 in.

EXERCISE CCLXI. PAGE 167.

1. (*a*) 154 sq. in. (*b*) $38\frac{1}{2}$ sq. in. (*c*) 616 sq. in. (*d*) 2464 sq. in.
2. $24.64. *3.* 14 ft. *4.* 132 ft.
5. (*a*) $905\frac{1}{7}$ cu. in. (*b*) $7241\frac{1}{7}$ cu. in. (*c*) $1437\frac{1}{3}$ cu. in. (*d*) $179\frac{2}{3}$ cu. in.
6. 4851 cu. ft. *7.* $12\frac{6}{7}$ cu. ft. *8.* 4 in. *9.* 110592. *10.* 2541.

EXERCISE CCLXII. PAGE 167.

1. 92.9 ft. *2.* 88 yd. *3.* 68 yd.; 867 sq. yd. *4.* 391 ft.
5. 75 ft. *6.* 43.3 sq. ft. *7.* 32 ft. *8.* 98 yd. *9.* 820 in.
10. 1120 yd.

EXERCISE CCLXIII. PAGE 168.

1. 43 ft. *2.* 242 yd. *3.* \$$22\frac{2}{9}$. *4.* 25. *5.* $3\frac{3}{8}$ mi. *6.* 4.24 ft. 7. 140 rd. *8.* 4 ft. *9.* $8063\frac{1}{3}$ lb. *10.* \$3.30.

EXERCISE CCLXIV. PAGE 169.

1. 14 ft. *2.* 3 in. tiles. *3.* 126 ft. *4.* 704 sq. ft. *5.* 170 ft. *6.* 14175 sq. ft. *7.* 5544 sq. ft. *8.* 79194 sq. ft. *9.* 140 sq. ft. *10.* $346\frac{1}{2}$ sq. in.

EXERCISE CCLXV. PAGE 169.

1. 80 ft. 90 ft. *2.* 5 A. *3.* 8232 sq. ft. *4.* 28 sq. ft. *5.* 80 rd.; 120 rd.; 89.44 rd. *6.* 1560s q. in. *7.* 65 ft., 68 ft.; 60 ft. *8.* 72 ft. *9.* 2646 sq. in.; 4374 sq. in. *10.* $112\frac{1}{2}$ ft.

EXERCISE CCLXVI. PAGE 170.

1. 63 ft. *2.* 308 sq. ft.; 385 sq. ft. *3.* 246.66 yd. *4.* 3630 men. *5.* 4800 ft. *6.* 40 rd., 20 rd. *7.* 2.73 ft. *8.* 12 ft. *9.* \$28.32. *10.* \$32.

EXERCISE CCLXVII. PAGE 170.

1. 52 ft. *2.* 10.39 yd. *3.* 23.54 ch. *4.* $3\frac{1}{3}$ ch. *5.* 56 yd. *6.* 42 ft. *7.* 414.12 ft. *8.* 21.213 ft. *9.* $292\frac{1}{2}$ sq. ft. *10.* $48\frac{1}{3}$ c. ft.

EXERCISE CCLXVIII. PAGE 171.

1. $38\frac{1}{2}$ sq. ft. *2.* 34.64 in. *3.* 27.71 in. *4.* $37\frac{1}{2}$ ft.; 30 ft. *5.* 1925 lb. *6.* 864. *7.* 192000 shot. *8.* 243 doz. *9.* 12 in. *10.* 48 oz.

EXERCISE CCLXIX. PAGE 173.

1. Thirty-seven, and five-tenths metres; twelve, and seven-tenths centimetres; forty-eight, and three-tenths millimetres; seven, and eight hundred and sixty-five dekametres; nine, and seven-tenths decimetres.

2. (1) 456 cm., (2) 45.60 dm., (3) 4.560 m.

3. (1) 18.7 m., (2) 18700 mm., (3) 1870 cm. *4.* 1897009.434 m.

5. 780.43 m.; 11.03 mm. *6.* 92.208 m.; 299.18892 m.

7. 640 leaves. *8.* 27600 times. *9.* \$76.56. *10.* 7.6375 mi.

11. \$555705. *12.* 28000 times.

EXERCISE CCLXX. PAGE 174.

1. 287.6345 hectares. *2.* 37856 ares.

3. 57536 sq. cm.; 5753600 sq. mm. *4.* 2341700 sq. m. *5.* 542 sq. cm.

6. 3030303 sq. m. *7.* 112 ares. *8.* 32.64 sq. m. *9.* \$2.50.

10. 280500 bricks.

EXERCISE CCLXXI. PAGE 174.

1. 72.5 st. *2.* \$432.50. *3.* 25800 l. *4.* 23.375 c. m. *5.* 2.604 m. *6.* 1.40 c. m. *7.* 30 cm. *8.* 300 Hl. *9.* \$275.31. *10.* 8 ct.

EXERCISE CCLXXII. PAGE 175.

1. 5037 g.; 503.7 Dg. *2.* 3075 mg.; 30.75 dg. *3.* 8070006.5 cg. *4.* 2.756 Kg.; 275600 cg. *5.* 7.28 l. *6.* \$750. *7.* 264.6 Kg. *8.* \$420.15625. *9.* 310 mg. *10.* .00504.

EXERCISE CCLXXIII. PAGE 176.

1. (*a*) 70.25 m. (*b*) 808.7 l. (*c*) 15.908 a. (*d*) 27000.37 g. *2.* 594 Kg. *3.* 47250 Kg. *4.* 2100 coins. *5.* 388 g. *6.* 110 st. *7.* 225 francs. *8.* 33 Hl. *9.* 650 g. *10.* 200 m. *11.* 50 ct. *12.* 190 mi. *13.* 477 times. *14.* 13.44 rolls.

EXERCISE CCLXXIV. PAGE 177.

1. .3; .45; .96; .765. *2.* .4; 142857; .285714; .428571.
3. .571428; .714285; .857142; .076923.
4. .230769; .384615; .692307; .63. *5.* .83; .583; .9318; .9285714.
6. .9428571; .954; .89285714; .15990.
7. .7083; .4196428571; .54861; .164772.
8. .44230769; .721153846; .1392045; .380952.
9. .20238095; .8270; .140765; .5050675.
10. .170138; .686789772; .35267856142; .20145.

EXERCISE CCLXXV. PAGE 178.

1. $\frac{5}{9}$; $\frac{17}{99}$; $\frac{4}{11}$; $\frac{8}{37}$. *2.* $\frac{2}{111}$; $\frac{1}{37}$; $\frac{1}{111}$; $\frac{16}{111}$.
3. $\frac{2}{37}$; $\frac{12}{37}$; $\frac{10}{27}$; $\frac{8}{27}$. *4.* $\frac{35}{37}$; $\frac{47}{111}$; $\frac{22}{37}$; $\frac{29}{111}$.
5. $\frac{99}{101}$; $\frac{12}{13}$; $\frac{1}{7}$; $\frac{2}{7}$. *6.* $\frac{21}{22}$; $\frac{19}{36}$; $\frac{21}{55}$; $\frac{10}{11}$.
7. $\frac{523}{666}$; $\frac{57}{550}$; $\frac{11}{825}$; $\frac{41}{3000}$. *8.* $\frac{29}{44}$; $\frac{1}{148}$; $\frac{329}{1350}$; $\frac{27}{70}$.
9. $\frac{35}{44}$; $\frac{43}{925}$; $2\frac{11}{37}$; $7\frac{25}{37}$. *10.* $4\frac{1}{150}$; $5\frac{1}{165}$; $7\frac{2}{333}$; $5\frac{5}{18}$.

EXERCISE CCLXXVI. PAGE 178.

1. 6.60244. *2.* 2.3060941229258. *3.* 9.4869; 14.0636727. *4.* .32457; .32147. *5.* .73231777. *6.* 1.590; 6.893. *7.* 339; .429. *8.* .4675; .350649. *9.* 63; 10. *10.* .54175; 7.805.

EXERCISE CCLXXVII. PAGE 179.

1. 10.5. *3.* .42. *4.* .161; .03. *6.* 33.142857. *7.* £2 0s. 8$\frac{3}{8}$. *8.* 3 t. 8 cwt. 35.3 lb. *9.* 6 mi. 92 rd. 1.6 yd. *10.* 69 t. 888.8 lb.

EXERCISE CCLXXVIII. PAGE 179.

1. Those which in their lowest terms have 2's and 5's, or 2's or 5's as the prime factors of their denominators.

2. When reduced to lowest terms, each denominator has only 2's and 5's, or 2's or 5's as its prime factors.

3. $\frac{7}{10}$, 1; $\frac{5}{8}$, 3; $\frac{17}{32}$, 5; $\frac{21}{200}$, 3.

4. Because their denominators have other factors than 2 or 5

5. $\frac{5}{6}$, 1; $\frac{17}{18}$, 1; $\frac{13}{30}$, 1; $\frac{11}{24}$, 3; $\frac{17}{96}$, 5.

6. 7.456. *7.* $(.005 \times .01) \times .(005 \times .01) = .0002 \times .0002$.

8. To multiply by 10, 100, 1000, &c., move the point 1, 2, 3, &c., places to the right.

To divide by 10, 100, 1000, &c., move the point 1, 2, 3, &c., places to the left.

9. $.\dot{1}3$.

EXERCISE CCLXXIX. PAGE 179.

1. \$3780. *2.* 9215 cu. ft. *3.* \$132, \$247.50, \$275. *4.* \$100.
5. .0078125. *6.* 33 mi. *7.* .405 hr. *8.* $16\frac{1}{31}$%. *9.* $.00\dot{1}2\dot{9}$.

EXERCISE CCLXXX. PAGE 180.

1. $\frac{8}{15}$. *2.* $\frac{1}{9}$ da. *3.* $3\frac{1}{3}$ da. *4.* 14.7 da. *5.* $\frac{7}{8}$.
6. A, 27 da.; B, 54 da. *7.* A, $19\frac{3}{5}$ da.; B, $9\frac{4}{5}$ da.; C, $39\frac{1}{5}$ da. *8.* $4\frac{1}{2}$ da.
9. 18 da. *10.* $4\frac{1}{6}$ hr. *11.* A, $36\frac{2}{3}$ da.; B, $73\frac{1}{3}$ da.; C, 110 da.
12. 15 da. *13.* 32 hr. *14.* $1\frac{1}{2}$ hr. *15.* 8 horses.
16. A, $3\frac{1}{15}$ da.; B, $3\frac{17}{30}$ da.; C, $5\frac{1}{15}$ da. *17.* A, \$1.10; B, \$1.

EXERCISE CCLXXXI. PAGE 181.

1. 12 min.; 24 min.; 30 min. *2.* 18 min. *3.* $16\frac{4}{11}$ min. past 3.
4. $16\frac{4}{11}$ min. past 6. *5.* $32\frac{8}{11}$ min. past 6. *6.* 24 min. past 5.
7. 24 min. past 6 and $41\frac{5}{11}$ min. past 6.
8. (*a*) $43\frac{7}{11}$ min. past 8, (*b*) $27\frac{3}{11}$ min. past 8, (*c*) $10\frac{10}{11}$ min. past 8.
9. $49\frac{1}{11}$ min. past 9; $54\frac{6}{11}$ min. past 10; 12 o'clock.
10. $23\frac{1}{13}$ min. past 5.

EXERCISE CCLXXXII. PAGE 182.

1. $13\frac{1}{11}$ min. past 4. *2.* $19\frac{7}{11}$ min. past 3. *3.* $31\frac{7}{23}$ min. past 6.
4. 120 da.; 44 min. past 11; 14 min. past 12. *5.* 75 da.
6. 36 min. past 6 a.m., Friday. *7.* $5\frac{5}{59}$ min. past 5.
8. $5\frac{1}{2}$ min. past 9 p.m. on Saturday, and $57\frac{1}{4}$ min. past 8 p.m.
9. 3 p.m., May 3. *10.* $43\frac{3667}{4331}$ min. past 9 p.m.

EXERCISE CCLXXXIII. PAGE 182.

1. 44 ft. *2.* $34\frac{1}{11}$ mi. *3.* $22\frac{1}{2}$. *4.* $7\frac{1}{2}$ sec. *5.* 14 min. 24 sec.
6. 8 hr. *7.* $33\frac{3}{4}$ mi. *8.* $7\frac{1}{3}$ yd. *9.* $32\frac{63}{80}$. *10.* Gains $10\frac{10}{143}$ min.

EXERCISE CCLXXXIV. PAGE 183.

1. 224, 141. *2.* 4 mi. *3.* $\frac{2}{3}$ mi. *4.* $\frac{1}{2}$ hr. *5.* $4\frac{7}{8}$ mi. *6.* 2 mi. 7. 45 min. *8.* 3 mi. *9.* 5 to 1. *10.* $2\frac{1}{4}$ hr.

EXERCISE CCLXXXV. PAGE 184.

1. $1\frac{2}{5}$ mi. *2.* 45 mi., 36 mi. *3.* 114 yd. *4.* 45 mi. *5.* 11 min. *6.* 396 ft. *7.* $94\frac{4}{9}$ yd. *8.* 22 mi. *9.* $23\frac{1}{5}$ mi. *10.* $6\frac{2}{3}$ mi. *11.* $6\frac{2}{3}$ hr.

EXERCISE CCLXXXVI. PAGE 185.

1. \$1.92. *2.* 325, 175, 125. *3.* \$620. *4.* 74; 3s. $8\frac{1}{2}$d. *5.* 17. *6.* 23. *7.* £16 6s. *8.* 1980. *9.* 2 hr. 24 min. *10.* 200 lb.

EXERCISE CCLXXXVII. PAGE 185.

1. 144 min. *2.* Men, \$12.50; women, \$8; boys, \$5.25. *3.* 7 bbl. *4.* 1022. *5.* 75 yd. *6.* 32 da. *7.* 17 mi. *8.* \$80. *9.* 110 sheep; 140 pigs. *10.* \$51.

EXERCISE CCLXXXVIII. PAGE 186.

1. \$12800. *2.* 123 gal. *3.* \$672. *4.* 720 apples. *5.* \$75. *6.* £140. *7.* \$97.50. *8.* 223 sec. *9.* John, \$880; Thomas, \$176; Henry, \$22. *10.* 56 ct.

EXERCISE CCLXXXIX. PAGE 187.

1. \$27. *2.* 26 ft. 6 in. *3.* 4 ft. 1 in. *4.* 15 ct. *5.* 2688 rails. *6.* 20, 21, 22, 23. *7.* 182. *8.* \$60. *9.* 7875 shingles. *10.* 150 lb.

EXERCISE CCXC. PAGE 187.

1. \$90, \$60. *2.* 3 mi. *3.* 87 da. *4.* Latter, 51 ct. *5.* \$20.0246. *6.* \$8460. *7.* \$870, 6%. *8.* \$2.50. *9.* 19 da.

EXERCISE CCXCI. PAGE 188.

1. \$900, \$1350, \$1800. *2.* \$36. *3.* Gain, 72 ct. *4.* 230400 A. *5.* 49.90 rd., 37.43 rd. *6.* \$2.50. *7.* 100 ft. by 76 ft. *8.* \$321.25. *9.* \$354. *10.* \$7.50.

EXERCISE CCXCII. PAGE 189.

1. A, 30 ct.; B, 36 ct.; C, 40 ct. *2.* 160 leaps. *3.* \$2450. *4.* $130\frac{1}{2}$ sec. *5.* \$500. *6.* 80c. *7.* $1\frac{1}{4}$ yd. *8.* 900 lb. at 7 ct.; 1100 at 10 ct. *9.* 248 yd.; 62 yd. *10.* \$440.

EXERCISE CCXCIII. PAGE 189.

1. 396 ft. *2.* \$329. *3.* Divisor, 547; Quot., 3233. *4.* $\$4\frac{1}{6}$. *5.* \$23.50. *6.* A's rate is to B's as 79 to 60. *7.* 409036320 post-holes. *8.* A, \$432; B, \$216; C, \$1296. *9.* \$1680. *10.* \$300; \$450.

EXERCISE CCXCIV. PAGE 190.

1. 294; 84. *2.* \$900, \$750. *3.* \$200. *4.* $\frac{33}{101}$. *5.* \$400.
6. \$240; 5 yr. *7.* 4 in. *8.* 300 leaps. *9.* 9 hr. *10.* 24 mi.

EXERCISE CCXCV. PAGE 191.

1. \$4.80. *2.* 8%, 9%. *3.* $31\frac{9}{11}$%. *4.* $3060\frac{5}{12}$ bu. *5.* 224.
6. 60 mi. per hr. *7.* $16\frac{1}{3}$ da. *8.* $821\frac{1}{3}$ cu. yd. *9.* $5\frac{1}{30}$ da.
10. A, \$776.16; B, \$693; C, \$630; D, \$600.

EXERCISE CCXCVI. PAGE 192.

1. 405 bu. *2.* 24 ft. by 18 ft. by 12 ft. *3.* 4.5 cu. ft.
4. Gain, \$14. *5.* 8000 oranges. *6.* $5\frac{1}{3}$s.; 56 oz. silver; 160 oz. gold.
7. \$2480. *8.* $1\frac{1}{3}$ mi. per hr. *9.* 20 min. past 4 a.m. *10.* \$217.80.

EXERCISE CCXCVII. PAGE 193.

1. 1152 sq. ft. *2.* 4%. *3.* $\frac{81}{256}$. *4.* 14.625 in. *5.* $\frac{25}{37}$; $.\dot{6}7\dot{5}$.
6. 14.288 to 1. *7.* $48\frac{3}{4}$ ct. *8.* \$$3\frac{1}{3}$. *9.* \$1.80. *10.* 190.

EXERCISE CCXCVIII. PAGE 193.

1. 272 rd. *2.* $77\frac{1}{3}$. *3.* 24 ct.; loss, 4%.
4. Length, 34 ft.; width, 26 ft.; height, 12 ft. *5.* $4\frac{3}{8}$ mi.
6. $3675\frac{3}{8}$ t. *7.* \$441. *8.* 63 yr.; 35 yr. *9.* 567 leaves. *10.* 104 da.

EXERCISE CCXCIX. PAGE 194.

1. 213 plants. *2.* \$4500. *3.* $2\frac{1}{4}$ mi. *4.* 100 lb. *5.* 28 ct.
6. \$2625. *7.* \$4000.
8. .384 in. (Note, their length and width being equal.)
9. 164.7114 in. *10.* A, \$1035; B, \$1656; C, \$2025.

EXERCISE CCC. PAGE 195.

1. 2 to 5. *2.* $13\frac{7}{11}$ min. past 4, or 30 min. past 4. *3.* \$40.
4. 5 yd. *5.* \$1.02; 80 ct. *6.* \$3250; \$2600. *7.* $233\frac{11}{12}$ ft.
8. 3 to 2. *9.* $2\frac{1}{4}$ ft. *10.* \$1600; 15 mo.

EXERCISE CCCI. PAGE 196.

1. $\frac{1}{45}$. *2.* B, \$$2.90\frac{5}{9}$; A, \$$5.09\frac{4}{9}$. *3.* 10 mo. *4.* 10 in. *5.* \$3.
6. 90. *7.* 3 hr. *8.* 525 lb., 495 lb., 390 lb.
9. \$2100, \$1750, \$1050, \$700. *10.* B, by 50 yd. in $4\frac{4}{5}$ min.

EXERCISE CCCII. PAGE 197.

1. 141.4 sq. ft. *2.* A, \$600; B, \$780; C, \$180; D, \$2000.
3. \$2000; \$1296. *4.* 4%; $4\frac{1}{2}$%. *5.* 60 ct. *6.* 32 men. *7.* $15\frac{1}{3}$ mi.
8. \$62500. *9.* $11\frac{3}{8}$ min. past 12. *10.* \$$9\frac{8}{13}$; \$$9\frac{4}{5}$.

PART II.

Solution of Certain Problems.

NOTE.—In the following solutions the multiplier is usually written to the left of the multiplicand and is to be read TIMES.

EXERCISE LX. PAGE 44.

1. The factors of 35 are 5 and 7.

As the last remainder is 6 the second divisor must be 7;

$$\text{First quotient} = (7\times72)+6 = 510;$$
$$\text{Number} = (5\times510)+2 = 2552.$$

3. Number of yards in first piece $= \frac{5600}{70} = 80$;

Price of 1 yard of second piece $= \frac{7600}{80}$c. $=$ 95c.

EXERCISE LXII. PAGE 45.

8. Number of men to do the work in 45 da. $=60$;

$\therefore$ " " " " " 1 da. $=45\times60$;

$\therefore$ " " " " " 50 da. $=\dfrac{45\times60}{50}=54$.

Number discharged $=(60-54)$ men $=6$ men.

10. Number of men to do 1 work in 36 da. $=25$;

$\therefore$ " " " 1 " 1 da. $=36\times25$;

" " " 1 " 12 da. $=\dfrac{36\times25}{12}$;

" " " 3 " 12 da. $=\dfrac{3\times36\times25}{12}=225$.

EXERCISE LXVII. PAGE 48.

3. It is clear that the quantity to be bought is the least number of pounds which added to 17385 pounds will make the result contain 68 pounds.

$$17385\div68=255 \text{ and } 45 \text{ remainder.}$$

Quantity to be bought $=(68-45)$ lb. $=23$ lb.

EXERCISE LXXVI. PAGE 57.

10. Weight of wool from 1 sheep $=7$ lb. 8 oz.;
" " " 2 " $=2\times(7$ lb. 8 oz.$)=15$ lb.;
" " " 360 " $=(\frac{360}{2}\times 15)$ lb.;
$=2700$ lb. $=27$ cwt.
Value of 1 cwt. of wool $=\$19.60$;
∴ " 27 " $=\$(27\times 19.60)=\$529.20$.

EXERCISE LXXVIII. PAGE 58.

7. Cost of building $(35-30)$ yd., or 5 yd. $=12$s. 6d.;
∴ " " 1 yd. $=\frac{12\text{s. }6\text{d.}}{5}=2$s. 6d.;
∴ " " 75 yd. $=75\times(2\text{s. }6\text{d.})$;
$=£9$ 7s. 6d.

EXERCISE LXXIX. PAGE 59.

5. Between noon on Monday and 8 p.m. on Wednesday are 56 hours.

Time gained in 8 hr. $=45$ sec.;
∴ " " 1 hr. $=\frac{45}{8}$ sec.;
∴ " " 56 hr. $=(56\times\frac{45}{8})$ sec. $=5$ min. 15 sec.
The clock will show 8 hr. 5 min. 15 sec.

9. Quantity of hay eaten $=(120\times 20\times 7\times 20)$ lb.;
$=\frac{120\times 20\times 7\times 20}{2000}$ t. $=168$ t.;
Quantity of oats eaten $=(120\times 20\times 7\times 10)$ qt.;
$=\frac{120\times 20\times 7\times 10}{32}$ bu. $=5250$ bu.;
Cost of 168 t. hay $=\$(168\times 8)=\$1344$;
Cost of 5250 bu. oats $=\$(5250\times\frac{30}{100})=\$1575$;
Cost of hay and oats $=\$2919$.

EXERCISE LXXXII. PAGE 63.

5. Perimeter of lot $=(2\times 60+2\times 144)$ ft. $=408$ ft.;
Number of post holes $=\frac{408}{6}=68$;
Cost of digging 1 hole $=5$c.;
∴ " " 68 " $=(68\times 5)$c. $=\$3.40$.

7. Number of rods in 1 mi.$=320$;

" " " fence$=2\times320=640$;

Length of wire used$=(5\times640)$ rods;

Weight of wire used$=(5\times640)$ lb.;

Cost of 1 lb.$=8$c.;

Cost of (5×640) lb.$=(5\times640\times8)$c.$=\$256$.

EXERCISE LXXXIII. PAGE 63.

9. Area of surface within the outside boundary of the path

$=(140\times70)$ sq. ft. $=9800$ sq. ft.;

Area of rectangular plot$=(130\times60)$ sq. ft. $=7800$ sq. ft.;

$\therefore$ area of path$=(9800-7800)$ sq. ft.$=2000$ sq. ft.

10. Average width of board$=\dfrac{19\times13}{2}$in.$=16$ in.;

Area of board$=(18\times12\times16)$ sq. in.;

$=\dfrac{18\times12\times16}{144}$sq. ft.$=24$ sq. ft.

EXERCISE LXXXIV. PAGE 64.

5. Width of lot$=\dfrac{1600\times9}{240}$ ft.$=60$ ft.;

Perimeter of lot$=(2\times240+2\times60)$ ft.$=600$ ft.;

Cost of fencing 1 ft.$=25$c.;

$\therefore$ " " 600 ft.$=(600\times25)$c.$=\$150$.

10. Area of farm$=\frac{2000}{25}$a.$=80$a.;

Width of farm$=\dfrac{80\times160}{200}$rd.$=64$ rd.;

Perimeter of farm$=(2\times200+2\times64)$ rd.$=528$ rd.;

Cost of fencing 1 rd.$=\$1.25$;

$\therefore$ " " 528 rd.$=528\times\$1.25=\$660$.

EXERCISE LXXXV. PAGE 65.

1. Area of room$=(24\times15)$ sq. ft.$=(24\times15\times144)$ sq. in.;

$\therefore$ Length of carpet$=\dfrac{24\times15\times144}{36}$in. $=\dfrac{24\times15\times144}{3\times12\times36}$ yd.$=40$ yd.

Or, Area of room$=(24\times15)$ sq. ft.;

Length of carpet to cover (3×3) sq. ft.$=1$ yd.;

$\therefore$ " " " (24×15) sq. ft.$=\dfrac{24\times15}{3\times3}\times1$ yd.$=40$ yd.

EXERCISE LXXXVI. PAGE 65.

1. Width of room$=(18\times 12)$ in.;

No. of strips in width$=\frac{18\times 12}{36}=6$.

Length of room$=(27\times 12)$ in.;

No. of strips in length$=\frac{27\times 12}{36}=9$.

EXERCISE LXXXVII. PAGE 66.

4. Area of ceiling$=(24\times 18)$ sq. ft. $=\frac{24\times 18}{9}$ sq. yd.;

48 sq. yd.;

Cost of plastering 1 sq. yd.$=19$c.;

∴ " " 48 sq. yd.$=(48\times 19)$c.$=\$9.12$.

6. Perimeter of room$=(2\times 27+2\times 18)$ ft. $=90$ ft.;

Area of entire walls$=(90\times 10)$ sq. ft. $=900$ sq. ft.;

Area of 3 doors$=(3\times 7\times 4)$ sq. ft. $=84$ sq. ft.;

Area of 3 windows$=(3\times 6\times 3)$ sq. ft. $=54$ sq. ft.;

Area of 1 window$=(1\times 6\times 4)$ sq. ft. $=24$ sq. ft.

Area of doors and windows$=(84+54+24)$ sq. ft. $=162$ sq. ft.;

(1) Area to be plastered$=(900-162)$ sq. ft. $=82$ sq. yd.;

(2) " " " $=(900-81)$ sq. ft. $=91$ sq. yd.

EXERCISE XC. PAGE 69.

8. By drawing a diagram of the foundation it will be seen that two of the walls are 60 ft. long and two of them 32 ft. long.

Length of walls$=(2\times 60+2\times 32)$ ft.$=184$ ft.;

Cubic content of walls$=(2\times 8\times 184)$ c. ft. $=2944$ c. ft.;

$=109$ c. yd. 1 c. ft.

EXERCISE XCI. PAGE 69.

10. No. of cords in the pile$=\frac{23625}{225}=105$.

Cubic content of pile$=(105\times 128)$ c. ft.;

Length of pile$=\frac{105\times 128}{5\times 12}$ ft.$=224$ ft.

EXERCISE XCII. PAGE 70.

8. Length of wall=192 ft.;

By drawing a diagram of the foundation the following will be readily seen:

Length of the two side-walls=(2×60) ft.;

Remaining length of wall=(192—120) ft.=72 ft.;

Length of end-wall=($\frac{72}{2}$+4) ft. =40 ft.;

∴ Width of barn=40 ft.

EXERCISE XCVII. PAGE 73.

7. Value of first+(3 times value of first+\$150)=\$13950;

∴ 4 times value of first=\$(13950—150);

∴ " " =\$$\frac{13800}{4}$ =\$3450;

∴ " second=\$(3×3450+150)=\$10500.

9. It is easily found that A owns 72 a. and B 48 a.

Value of A's land=\$(72×84);

" B's " =\$(72×84);

∴ value of 1 a.=\$$\frac{72\times 84}{48}$=\$126.

EXERCISE XCVIII. PAGE 73.

6. \$1+\$2+\$3=\$6.

Every time \$6 is divided, the first gets \$1; the second, \$2; and the third, \$3.

No. of times \$6 is divided=$\frac{\$120}{\$6}$=20;

Share of first=20×\$1=\$20;

" second=20×\$2=\$40;

" third=20×\$3=\$60.

9. When B receives \$1, A receives \$3, and C \$4 out of \$8.

Share of A=$\frac{1120}{8}$×\$3=\$420;

" B=$\frac{1120}{8}$×\$1=\$140;

" C=$\frac{1120}{8}$×\$4=\$560.

10. Value of first house=2 times value of other two;

∴ 3 times value of other two houses=\$16410;

∴ " " " =$\frac{\$16410}{3}$=\$5470;

value of first house=2×\$5470=\$10940.

(Value of third+\$570)+value of third=\$5470;

∴ 2 times value of third=\$(5470−570)=\$4900;

∴ " " $=\frac{\$4900}{2}=\$2450$;

∴ " second=\$(2450+570)=\$3020.

EXERCISE XCIX. PAGE 74.

1. Value of first=value of first;

" second= " " +\$2;

" third= " " +\$2+\$3;

∴ " all=3 times value of first+\$7;

∴ 3 times value of first+\$7=\$31;

∴ 3 " " " =\$(31−7)=\$24;

∴ " " $=\frac{\$24}{3}$ = \$8;

" second =\$(8+2) =\$10;

" third =\$(10+3)=\$13.

3. Length of second=length of second;

" first= " " +25 yd.;

" third= " " +25 yd.+19 yd.;

" all=3 times length of second+69 yd.;

∴ 3 times length of second+69 yd.=444 yd.;

∴ 3 " " " =(444−69) yd.=375 yd.;

∴ " " $=\frac{375}{3}$ yd.=125 yd.;

" first =(125+25) yd.=150 yd.;

" third =(150+19) yd.=169 yd.

11. If instead of 5 bu. of oats, 5 bu. of wheat had been bought, the cost would have been (5×33)c. more.

Hence, cost of 13 bu. of wheat=\$(6.80+1.65)=\$8.45;

∴ " 1 " " $=\frac{845}{13}$c. =65c.;

" 1 " oats=(65−33)c. =32c.

12. (Red+White) marbles=192;

(Blue+White) " =199;

∴ (Red+Blue+White+White) " =391;

and (Red+Blue+White) " =286;

∴ White " =105;

Red " =192−105=87;

Blue " =199−105=94.

EXERCISE CII. PAGE 76.

6. Cost of 500 bu. $=(500\times 72)$c.$=36000$c.;
" 500 bu. @ 70c.$=(500\times 70)$c.$=35000$c.;
Difference from real cost$=$ 1000c.;

Every bushel of the dearer kind increases the cost by 5c.
∴ Quantity of the dearer kind$=\frac{1000}{5}$ bu. $=200$ bu.;
" " cheaper " $=(500-200)$ bu.$=300$ bu.

EXERCISE CIII. PAGE 77.

4. Cost of 4 lb. @ 32c.$=128$c.;
" 3 lb. @ 35c.$=105$c.;
" 2 lb. @ 41c.$=$ 82c.;
" 9 lb. $=315$c.;
∴ " 1 lb. $=\frac{315}{9}$c.$=35$c.

9. As the number of men are in proportion to 1, 4, and 10, we may suppose 1 man receives \$13 per week; 4 men, \$9; and 10 men, \$5.

Hence, wages of 1 man @ \$13=\$13;
" 4 men @ \$9=\$36;
" 10 " @ \$5=\$50;
" 15 " =\$99;
∴ " 1 " $=\$\frac{99}{15}=\$6.60$.

EXERCISE CVIII. PAGE 80.

7. As the loads are to be as few as possible, they must be as large as possible.

$441=3\times 3\times 7\times 7$;
$567=3\times 3\times 3\times 3\times 7$;
$315=3\times 3\times 5\times 7$;
Hence, G.C.M.$=3\times 3\times 7=63$;
Number of bushels in a load$=63$.

EXERCISE CIX. PAGE 81.

10. Find the G.C.M. of 13230 and 44100;

```
13230)44100
      39690
       4410)13230
            13230
```

Next find the G.C.M. of 4410 and 118125;
This is readily found to be 315.

Find the G.C.M. of 720100 and 913330.

720100	1	913330
772920	4	720100
52820	4	193230
36100	2	211280
16720	1	18050
15960	12	16720
760	2	1330
760	4	1520
		190

The G.C.M. of 190 and 211109 is easily found to be 19.

EXERCISE CX. PAGE 81.

1.

$$1365=3\times5\times7\times13;$$
$$1785=3\times5\times7\times17;$$
$$\text{G.C.M.}=3\times5\times7=105.$$

To find all the common measures, form as many series as there are different prime factors, making 1 the first term of each series and multiply these together.

1, 3
1, 5
1, 3, 5, 15
1, 7
1, 3, 5, 15, 7, 21, 35, 105.

5. If the number will divide 2000 and leave 11, it will be contained exactly in 2000—11, or 1989.

Similarly, the number will be contained exactly in 2708—17, or 2691.

The G.C.M. of 1989 and 2691 is 117.

EXERCISE CXIII. PAGE 83.

9. $43680=2\times2\times2\times2\times2\times3\times5\times7\times13$.

These can be readily arranged into four consecutive numbers as follows:

$13;\ 7\times2;\ 5\times3;\ 2\times2\times2\times2$, or 13, 14, 15, 16.

EXERCISE CXLV. PAGE 98.

2. Cost of 500 cords $=\$(500\times2\tfrac{7}{8}) \quad =\$1437\tfrac{1}{2};$
Sum received for $75\tfrac{3}{4}$ cords $=\$(75\tfrac{3}{4}\times4\tfrac{1}{3}) \quad =\$328\tfrac{1}{4};$
$(500-75\tfrac{3}{4})$ cords $=424\tfrac{1}{4}$ cords;

Sum received for $424\frac{1}{4}$ cords $=\$(424\frac{1}{4}\times 5) = \$2121\frac{1}{4}$;

Total received $=\$(328\frac{1}{4}+2121\frac{1}{4}) = \$2449\frac{1}{2}$;

Gain $=\$(2449\frac{1}{2}-1437\frac{1}{2})=\$1012$.

3. No. of yards sold at a gain $=\frac{2}{3}$ of $150=100$;

" " " loss $=\frac{1}{3}$ of $150=50$;

Gain on 100 yd. $=\$(100\times\frac{5}{8}) = \$62\frac{1}{2}$;

Loss on 50 yd. $=\$(50\times\frac{1}{16}) = \$3\frac{1}{8}$;

Net gain $=\$(62\frac{1}{2}-3\frac{1}{8})=\$59\frac{3}{8}$.

8. Part owned by the third $=(1-\frac{3}{10}-\frac{1}{5})=\frac{1}{2}$;

Share of third $=\frac{1}{2}$ of $\$6250=\$3125$.

EXERCISE CL. PAGE 100.

The following is the generally received usage respecting the signs of operation in arithmetic:

I. The operations indicated by "of", $\times$, and $\div$ should be performed before adding or subtracting.

II. The operations indicated by $\times$ and $\div$ should be performed in the order in which they occur.

III. The operation indicated by "of" should be performed before that indicated by $\div$. In this case custom makes a distinction between $\times$ and "of."

EXERCISE CLIV. PAGE 102.

8. Reduce $33\frac{3}{4}$, $67\frac{1}{2}$, $70\frac{7}{8}$ to equivalent fractions with a common denominator.

$33\frac{3}{4}=\frac{270}{8}$; $67\frac{1}{2}=\frac{540}{8}$; $70\frac{7}{8}=\frac{567}{8}$;

As the grain is to go into the smallest number of bags, the bags must be the largest possible.

The G.C.M. of 270, 540, and 567 is 27.

Hence, the G.C.M. of $\frac{270}{8}$; $\frac{540}{8}$, and $\frac{567}{8}$ is $\frac{27}{8}$.

Each will contain $\frac{27}{8}$ bu., or $3\frac{3}{8}$ bu.

$33\frac{3}{4}\div 3\frac{3}{8}=10$; $67\frac{1}{2}\div 3\frac{3}{8}=20$; $70\frac{7}{8}\div 3\frac{3}{8}=21$;

Hence, number of bags $=10+20+21=51$.

10. $\frac{2}{3}=\frac{12}{18}$; $\frac{4}{9}=\frac{8}{18}$; $\frac{5}{6}=\frac{15}{18}$;

L.C.M. of $\frac{12}{18}$, $\frac{8}{18}$, and $\frac{15}{18}$ is $\frac{120}{18}$, or $6\frac{2}{3}$;

Number of times A goes round $=6\frac{2}{3}\div\frac{2}{3}=10$:

" " B " $=6\frac{2}{3}\div\frac{4}{9}=15$;

" " C " $=6\frac{2}{3}\div\frac{5}{6}=8$.

EXERCISE CLIX. PAGE 105.

9. Weight of 1 lb. Avoirdupois = 7000 gr.;
" 1 lb. Troy = 5760 gr.;
1 gr. is $\frac{1}{7000}$ of 7000 gr.;
$\therefore$ 5760 gr. is $\frac{5760}{7000}$ of 7000 gr.;
$\frac{5760}{7000} = \frac{144}{175}$.

EXERCISE CLXIV. PAGE 107.

7. 61 mi. 224 rd. = 19744 rd.;
Time to walk 32 rd. = 1 min.;
$\therefore$ " " 1 rd. = $\frac{1}{32}$ min.;
$\therefore$ " " 19744 rd. = $(19744 \times \frac{1}{32})$ min. = 10 hr. 17 min.

EXERCISE CLXVI. PAGE 108.

4. 1 hr. = (60×60) sec.;
30 mi. = (30×5280) ft.;
Distance the train goes in (60×60) sec. = 30×5280 ft.;
$\therefore$ " " " " 1 sec. = $\frac{30 \times 5280}{60 \times 60}$ ft. = 44 ft.

EXERCISE CLXVIII. PAGE 109.

7. 1 hr. = 60 min.;
42 mi. = (42×1760) yd.;
Distance moved over in 60 min. = (42×1760) yd.;
$\therefore$ " " " 1 min. = $\frac{42 \times 1760}{60}$ yd. = 1232 yd.;
Again, " " " 1 sec. = 20 yd.;
$\therefore$ " " " 1 min. = (60×20) yd. = 1200 yd.;
The former is faster by $(1232 - 1200)$ yd., or 32 yd.

EXERCISE CLXXI. PAGE 110.

3. B's part exceeds A's by $(\frac{7}{12} - \frac{5}{12})$, or $\frac{2}{12}$ of web;
$\frac{2}{12}$ of web, or $\frac{1}{6}$ of web = $5\frac{1}{2}$ yd.;
$\therefore$ the web = $(6 \times 5\frac{1}{2})$ yd. = 33 yd.;
A's part = $(\frac{5}{12}$ of 33) yd. = $13\frac{3}{4}$ yd.;
B's part = $(\frac{7}{12}$ of 33) yd. = $19\frac{1}{4}$ yd.

5. Width of first field = $\frac{15 \times 160}{80}$ rd. = 30 rd.;

Length of second " = $\frac{9 \times 160}{30}$ rd. = 48 rd.

EXERCISE CLXXIII. PAGE 111.

5. Number sold to drover $=\frac{2}{3}$ of sheep;
" remaining $=\frac{1}{3}$ of sheep;
" sold to neighbor $=\frac{1}{3}$ of $\frac{1}{3}$ of sheep;
" remaining now $=\frac{2}{3}$ of $\frac{1}{3}$ of sheep $=\frac{2}{9}$ of sheep;
$\therefore\ \frac{2}{9}$ of sheep $=100$ sheep;
$\therefore$ sheep $=\frac{9}{2}$ of 100 sheep $=450$ sheep.

6. Length of road $=(2\times5280)$ ft.;
Rise of road in 10 ft. $=\frac{1}{6}$ ft.;
$\therefore$ Rise of road in (2×5280) ft. $=(2\times528\times\frac{1}{6})$ ft. $=176$ ft.

9. Part received by wife $=\frac{3}{4}$ of estate;
" remaining $=\frac{1}{4}$ "
" received by eldest son $=\frac{2}{3}$ of $\frac{1}{4}$ of estate;
" remaining now $=\frac{1}{3}$ of $\frac{1}{4}$ " $=\frac{1}{12}$ estate;
" received by daughter $=\frac{3}{4}$ of $\frac{1}{12}$ " $=\frac{1}{16}$ "
$\therefore\ \frac{1}{16}$ estate $=\$750$;
$\therefore$ estate $=\$(16\times750)=\$12000$.

EXERCISE CLXXIV. PAGE 112.

3. If the first field yields 1 share of 482 bu.,
the second field yields 2 shares of 482 bu.;
$\therefore$ 3 shares $=482$ bu.;
$\therefore$ 1 " $=\frac{482}{3}$ bu. $=160\frac{2}{3}$ bu. = yield of first;
and 2 " $=(2\times160\frac{2}{3})$ bu. $=321\frac{1}{3}$ bu. = yield of second.

8. Share of first $=12\frac{1}{2}$ a.;
" second $=\frac{3}{8}$ of the land;
" third $=12\frac{1}{2}$ a. $+\frac{3}{8}$ of the land;
" all $=25$ a. $+\frac{3}{4}$ of the land;
$\therefore\ \frac{1}{4}$ of the land $=25$ a.;
$\therefore$ the land $=(4\times25)$ a. $=100$ a.

EXERCISE CLXXV. PAGE 113.

9. As the lots are to be square, the length of each must be the G.C.M. of $201\frac{2}{3}$ rd. and $41\frac{1}{4}$ rd.

The G.C.M. of $201\frac{2}{3}$ rd. and $41\frac{1}{4}$ rd. is $\frac{55}{12}$ rd.;

Number of lots on the length $=\dfrac{201\frac{2}{3}}{\frac{55}{12}}=44$;

" " " breadth $=\dfrac{41\frac{1}{4}}{\frac{55}{12}}=9$;

Total number of lots $=44\times9=396$.

EXERCISE CLXXVI. PAGE 113.

9. Part A does in 1 da. $=\frac{1}{6}$ of work;
$\therefore$ " A " 2 da. $=\frac{1}{3}$ "
" B " 1 da. $=\frac{1}{8}$ "
$\therefore$ " B " 2 da. $=\frac{1}{4}$ "
" A and B do in 2 da. $=(\frac{1}{3}+\frac{1}{4})$ of work $=\frac{7}{12}$ of work;
Work to be done $=(1-\frac{7}{12})$ of work $=\frac{5}{12}$ of work.

10. Part A and B do in 1 da. $=(\frac{1}{8}+\frac{1}{6})$ of work
$=\frac{7}{24}$ of work;
Part yet to be done $=(1-\frac{7}{24})$ of work
$=\frac{17}{24}$ of work;
Part of work A, B and C do in 1 da. $=(\frac{1}{8}+\frac{1}{6}+\frac{1}{5})$ of work
$=\frac{59}{120}$ of work;
Time for A, B and C to do $\frac{59}{120}$ of work $=1$ da.;
$\therefore$ " " " " 1 " $=\frac{120}{59}$ da.;
$\therefore$ " " " " $\frac{17}{24}$ " $=(\frac{17}{24}\times\frac{120}{59})$ da. $=1\frac{26}{59}$ da.

11. Part A and B do in 1 da. $=\frac{1}{10}$ of work;
" A " C " " $=\frac{1}{9}$ "
" B " C " " $=\frac{1}{12}$ "
" 2 men of A's strength, 2 men of B's strength, and 2 men of C's strength do in 1 day
$=(\frac{1}{10}+\frac{1}{9}+\frac{1}{12})$ of work;
$\therefore$ " A, B and C do in 1 da. $=\frac{1}{2}$ of $(\frac{1}{10}+\frac{1}{9}+\frac{1}{12})$, or $\frac{53}{360}$ of work;
" C does in 1 da. $=(\frac{53}{360}-\frac{1}{10})$, or $\frac{17}{360}$ of work;
Time for C to do $\frac{17}{360}$ of work $=1$ da.;
$\therefore$ " C " $\frac{1}{360}$ " $=\frac{1}{17}$ da.;
$\therefore$ " C " $\frac{360}{360}$ " $=(360\times\frac{1}{17})$ da. $=21\frac{3}{17}$ da.

13. Time for 5 men to do the work $=20$ hr.;
$\therefore$ " 1 man " " $=100$ hr.;
$\therefore$ part 1 " does in 1 hr. $=\frac{1}{100}$ of work;
Similarly, " 1 woman " " $=\frac{1}{160}$ "
" " 1 boy " " $=\frac{1}{240}$ "
Hence, part 1 man, 2 women, and 3 boys do in 1 hr.
$=(\frac{1}{100}+\frac{2}{160}+\frac{3}{240})$ of work
$=\frac{7}{200}$ of work;
Time to do $\frac{7}{200}$ of work $=1$ hr.;
$\therefore$ " " $\frac{1}{200}$ " $=\frac{1}{7}$ hr.;
$\therefore$ " " $\frac{200}{200}$ " $=(200\times\frac{1}{7})$ hr. $=28\frac{4}{7}$ hr.

EXERCISE CXCVII. PAGE 125.

1.

£.790625	.790625 of £1=.790625 of 20s.
20	=15.8125 s.;
s. 15.812500	.8125 of 1s.=.8125 of 12 d.
12	=9.75 d.;
d. 9.750000	.75 of 1d.=.75 of 4 qr.
4	=3 qr.;
qr. 3.000000	Hence, £.790625=15s. 9¾d.

EXERCISE CXCVIII. PAGE 125.

1.

12)6 d.	$6\text{d.}=\frac{6}{12}\text{s.} \quad =.5\text{s.};$
20)10.5s.	∴ 10s. 6d.=10.5s.;
£1.525.	$10.5\text{s.}=£\frac{10.5}{20}=£.525;$

Hence, £1 10s. 6d.=£1.525.

EXERCISE CC. PAGE 126.

9. Let 1 man get 1 share, then
1 youth gets .525 shares;
Shares of 6 men and 11 youths=6×1 share+11×.525 share;
=11.775 shares;
Hence, 11.775 shares=$47.10;

$$\therefore 1 \text{ share } =\$\frac{47.10}{11.775}=\$4.$$

EXERCISE CCI. PAGE 126.

9. Distance the stream flows in 60 min.=(2.5×5280) ft.
=13200 ft.;
" " " " 12 " $=\frac{1}{5}$ of 13200 ft.
=2640 ft.;

Cubic content of stream 2640 ft. long, 125.125 ft. wide, and 4.8 ft. deep=(2640×125.125×4.8) cu. ft.=1585584 cu. ft.

EXERCISE CCII. PAGE 127.

6. Expressing each of the numbers as ten-thousandths, we must find the G.C.M. of 22500, 33750, and 28125 ten-thousandths.

5)	22500,	33750,	28125
5)	4500,	6750,	5625
5)	900,	1350,	1125
5)	180,	270,	225
9)	36,	54,	45
	4,	6,	5

Hence, the G.C.M. is $(5\times5\times5\times5\times9)$ ten-thousandths, or .5625.
The L.C.M. is found in a similar way.

EXERCISE CCIII. PAGE 128.

11. Let C receive 1 share;
then B receives 1.75 shares;
and A " $[1.5\times(1+1.75)]$ shares, or 4.125 shares;
All " $(1+1.75+4.125)$ " or 6.875 "
$\therefore$ 6.875 shares=\$302.50;

$$\therefore \quad 1 \text{ share} = \$\frac{302.50}{6.875} = \$44 \quad = \text{C's share};$$
$$1.75 \text{ of } \$44 = \$77 \quad = \text{B's} \quad \text{"}$$
$$1.5 \text{ of } \$(44+77) = \$181.50 = \text{A's} \quad \text{"}$$

13. Cubic contents of first stick$=(32.5\times2.5\times1.125)$ c. ft.;
" " second " $=(21.75\times1.875\times1.25)$ c. ft.;
Weight of $(32.5\times2.5\times1.125)$ c.ft.=4387.5 lb.;

$$\therefore \quad \text{"} \quad 1 \text{ c. ft.} = \frac{4387.5}{32.5\times2.5\times1.125} \text{ lb.};$$
$$\therefore \quad \text{"} \; (21.75\times1.875\times1.25) \text{ c.ft.} = \frac{21.75\times1.875\times1.25\times4387.5}{32.5\times2.5\times1.125} \text{ lb.}$$
$$= 2446.875 \text{ lb.}$$

EXERCISE CCXI. PAGE 132.

5. $\frac{15}{100}$ of 350 a.=52.5 a.;
$\frac{25}{100}$ of 350 a.=87.5 a.;
$\frac{60}{100}$ of 350 a.=210 a.;
Cost of 52.5 a. @ \$28=\$1470.00;
" 87.5 a. @ \$35=\$3062.50;
" 210 a. @ \$40=\$8400.00;
Total cost=\$13932.50.

7. Selling price of 1st piano$=\frac{130}{100}$ of \$250 =\$325;
" " 2nd " $=\frac{140}{100}$ of \$250 =\$350;
" " 3rd " $=\frac{80}{100}$ of \$250 =\$200;
Selling price of all=\$875.
Net gain=\$(875−750)=\$125.

9. Sum deposited $=\frac{80}{100}$ of fortune;
" withdrawn $=\frac{20}{100}$ of ($\frac{80}{100}$ of fortune);
$\therefore$ " remaining $=\frac{80}{100}$ of $\frac{80}{100}$ of fortune$=\frac{64}{100}$ of fortune;
Hence, $\frac{64}{100}$ of fortune=\$5760;
$\therefore$ " $=\frac{100}{64}$ of \$5760=\$9000.

EXERCISE CCXII. PAGE 133.

7. First number $=\frac{100}{100}$ of first number;

Second " $=\frac{107\frac{1}{2}}{100}$ " "

$\therefore$ sum of both " $=\frac{207\frac{1}{2}}{100}$ " "

Hence, $\frac{207\frac{1}{2}}{100}$ of first " $=3901$;

$\therefore$ " " $=\frac{100}{207\frac{1}{2}}$ of $3901=1880$;

and second " $=\frac{107\frac{1}{2}}{100}$ of $1880=2021$.

EXERCISE CCXIV. PAGE 135.

1. Invoice price $=\$375$;
First discount @ 20% $=\ \ 75$;
$\$300$;
Second " @ 10% $=\ \ 30$;
Net price $=\$270$.

Or,

First remainder $=\frac{4}{5}$ of \$375;
Second " $=\frac{9}{10}$ of $\frac{4}{5}$ of $\$375=\$270$.

4. Part of marked price remaining after deducting the discount is $\frac{85}{100}$ of marked price.

Hence, $\frac{85}{100}$ of marked price $=\$2.72$;
$\therefore$ " " $=\frac{100}{85}$ of $\$2.72=\$3.20$.

9. Part of price remaining after first discount
$=\frac{75}{100}$ of cat. price;
" " " " second discount
$=\frac{90}{100}$ of $\frac{75}{100}$ of cat. price
$=\frac{67.5}{100}$ of cat. price;

Hence, single discount $=\left(\frac{100}{100}-\frac{67.5}{100}\right)$ of cat. price
$=\frac{32.5}{100}$ of cat. price
$=32\frac{1}{2}\%$.

EXERCISE CCXV. PAGE 135.

2. Selling price $=\frac{75}{100}$ of marked price;

But " " $=\frac{120}{100}$ of \$2;

$\therefore$ $\frac{75}{100}$ of marked price $=\frac{120}{100}$ of \$2;

$\therefore$ " " $=\frac{100}{75}$ of $\frac{120}{100}$ of \$2 = \$3.20.

6. Gain $=\frac{25}{100}$ of cost = \$1.40;

$\therefore$ " $=\frac{100}{25}$ of \$1.40 = \$5.60;

Selling price to gain 35% $=\frac{135}{100}$ of \$5.60 = \$7.56.

EXERCISE CCXVII. PAGE 137.

6. Loss $=\frac{2}{100}$ of cost = \$55.50;

$\therefore$ " $=\frac{100}{2}$ of \$55.50 = \$2775;

$\therefore$ Selling price = \$(2775—55.50) = \$2719.50.

9. Loss on 1 bu. $=\frac{20}{100}$ of 72c. $=\frac{72}{5}$c.;

Quantity on which $\frac{72}{5}$c. is loss = 1 bu.;

" " " 1c. " $=\frac{5}{72}$ bu.;

$\therefore$ " " " 129600c. " $=\dfrac{129600\times 5}{72}$ bu. = 9000 bu.

EXERCISE CCXVIII. PAGE 137.

5. $\frac{115}{100}$ of cost of 10 cows = \$690;

$\therefore$ " 1 cow $=\frac{1}{10}$ of $\frac{100}{115}$ of \$690;

$\therefore$ Selling price to gain 25% $=\frac{125}{100}$ of $\frac{1}{10}$ of $\frac{100}{115}$ of \$690 = \$75.

9. Buying price $=\frac{85}{100}$ of market price;

Selling " $=\frac{115}{100}$ " "

$\therefore$ gain on $\frac{85}{100}$ of market price $=\frac{30}{100}$ " "

$\therefore$ " 1 " " $=\frac{100}{85}$ of $\frac{30}{100}$ of market price;

$\therefore$ " 100 " " $=\dfrac{100\times 100\times 30}{85\times 100}$ " "

$=35\frac{5}{17}$;

Gain per cent. $=35\frac{5}{17}$.

Or, if the market price is \$100, the buying price is \$85 and the selling price is \$115;

$\therefore$ gain on \$85 = \$30;

$\therefore$ " \$100 $=\$\dfrac{100\times 30}{85}=\$35\frac{5}{17}$;

$\therefore$ gain $=35\frac{5}{17}$%.

EXERCISE CCXIX. PAGE 138.

7. Quantity of goods bought $=(6\times50\times30)$ m.;
Value " " $=(6\times50\times30\times25)$c.;
Commission $=(\frac{3}{500}$ of $6\times50\times30\times25)$c. $=\$13.50$.

10. Commission on $\$15750=\$535.50$;

$\therefore$ " " $\$1=\$\frac{535.50}{15750}$;

$\therefore$ " " $\$100=\$\frac{53550}{15750}=\$3\frac{2}{5}$;

$\therefore$ rate $=3\frac{2}{5}\%$.

EXERCISE CCXX. PAGE 139.

3. Sum received for the goods $=(500\times21\times23)$c. $=\$2415$;
Commission $=\$(2415-2334.50)=\$80.50$;
Commission on $\$2415=\$80.50$;
" " $\$100=\$\frac{100\times80.50}{2415}=\$3\frac{1}{3}$;
$\therefore$ rate $=3\frac{1}{3}\%$.

8. Commission $=\frac{3}{100}$ of sum received;
Sum sent to owner $=\frac{97}{100}$ " "
$\therefore$ $\frac{97}{100}$ of sum received $=\$6984$;
$\therefore$ " " $=\$\frac{100\times6984}{97}=\$7200$.
Agent's commission $=\frac{3}{100}$ of $\$7200=\$216$.

EXERCISE CCXXI. PAGE 139.

5. If sum invested $=\$100$
Commission $=\$\ \ 2$
$\$102$;
$\therefore$ sum invested out of $\$102=\$100$;
$\therefore$ " " $\$2040=\$\frac{2040\times100}{102}=\$2000$.

Quantity bought for $3\frac{1}{3}$c. $=1$ lb.;
$\therefore$ " " " 1c. $=\frac{3}{10}$ lb.;
$\therefore$ " " " 200000c. $=\frac{200000\times3}{10}$ lb. $=60000$ lb.

9. Sum collected $=\frac{75}{100}$ of $\$1260=\$945$;
Commission $=\frac{5}{100}$ of $\$945=\$47.25$;
Sum received by creditor $=\$(945-47.25)=\$897.75$.

10. Auctioneer's commission $=\frac{9}{100}$ of value of goods;
Sum remaining $=\frac{91}{100}$ " " "

Com. at 4% on \$3500=\$140;

∴ sum remaining after the auctioneer took out his first commission =\$3640;

∴ $\frac{91}{100}$ of value of goods=\$3640;

∴ " " $=\$\frac{100\times3640}{91}=\$4000.$

EXERCISE CCXXII. PAGE 140.

9. Sum insured=$\frac{4}{5}$ of \$37560=\$30048;

Premium on \$30048=\$262.92;

∴ " " $\$100=\$\frac{100\times262.92}{30048}=\$\frac{7}{8}$;

∴ rate=$\frac{7}{8}$%.

EXERCISE CCXXIII. PAGE 141.

4. $\frac{7}{800}$ of sum insured=\$87.50;

∴ " " $=\$\frac{800\times87.50}{7}=\$10000$;

Again, $\frac{5}{400}$ of " " =\$50;

∴ " " $=\$\frac{400\times50}{5}=\$4000$;

Sums received=\$14000.

6. Premium received=$\frac{3}{400}$ of \$35000=\$262.50;

Loss by company=\$(35000−262.50)=\$34737.50.

10. \$25000+\$35000+\$40000=\$100000;

Liability of 1st=$\frac{25000}{100000}$ of \$10000=\$2500;

" " 2nd=$\frac{35000}{100000}$ of \$10000=\$3500;

" " 3rd=$\frac{40000}{100000}$ of \$10000=\$4000.

EXERCISE CCXXV. PAGE 143.

3. Tax=$\frac{3\frac{1}{4}}{1000}$ of value of property;

∴ $\frac{3\frac{1}{4}}{1000}$ " " " =\$1768;

∴ " " " $=\$\frac{1000\times1768}{3\frac{1}{4}}=\$544000.$

6. Sum remaining after deducting cost of collecting =$\frac{95}{100}$ of sum levied;

∴ $\frac{95}{100}$ " " =\$12350;

∴ " " $=\$\frac{100\times12350}{95}=\$13000.$

9. Taxes paid on \$60000=\$375;

$\therefore$ " " \$4000=$\$\frac{4000\times 375}{60000}$=\$25.

EXERCISE CCXXIX. PAGE 145.

8. Time for which the interest is $\frac{9}{200}$ principal=1 yr.;

$\therefore$ " " " " 1 " $=\frac{200}{9}$ yr.;

$\therefore$ " " " " $\frac{9}{160}$ " $=\frac{9}{160}$ of $\frac{200}{9}$ yr.

$=1\frac{1}{4}$ yr.

EXERCISE CCXXXII. PAGE 146.

10. Principal=\$8000

Interest 1st half yr.=\$ 400

Amount=\$8400

Interest 2nd half yr.=\$ 420

Amount=\$8820

Interest 3rd half yr.=\$ 441

Amount=\$9261

Interest 4th half yr.=\$ 463.05

\$9724.05

\$8000.00

Compound interest=\$1724.05

Simple " $=2\times\frac{10}{100}\times$\$8000=\$1600;

The compound interest is greater by \$(1724.05−1600), or \$124.05.

EXERCISE CCXXXIII. PAGE 148.

5. Face of draft=\$750

$\frac{1}{400}$ of \$750=\$ 1.875

Cost of draft=\$751.875.

6. \$101 will purchase a draft for \$100;

$\therefore$ \$1500 " " " $\$\frac{1500\times 100}{101}$, or for \$1485.1485.

EXERCISE CCXXXIV. PAGE 148.

1. 3 mo. from April 17 is July 17;

Add 3 days of grace, " 20=date of maturity;

No. of days between May 1 and July 20 is 80=term of discount;

Interest on \$657 for 80 days at 8% $=\frac{80}{365}$ of $\frac{8}{100}$ of \$657

=\$11.52;

Proceeds of note=\$(657−11.52)=\$645.48.

7. Bank discount $=\frac{1}{3}$ of $\frac{6}{100}$ of face value $=\frac{1}{50}$ of face value;
Net proceeds $=\frac{49}{50}$ of face value $=\$318.50$;
$\therefore$ face value $=\frac{50}{49}$ of $\$318.50=\$325$.

9. The note falls due on Nov. 29.
No. of days from Aug. 26 to Nov. 29=95;
Interest on \$1962.45 for 95 da. @ 7%=\$35.75 . . .;
Proceeds=\$1962.45—\$35.75=\$1926.70.

10. The note falls due on July 21.
No. of days from May 8 to July 21=74;
Interest on \$1066.74 for 74 da. @ 6%=\$12.97 . . .;
Proceeds of note=\$1066.74—\$12.97=\$1053.77.

EXERCISE CCXXXV. PAGE 150.

1. \$100 stock is worth \$103;
$\therefore$ 1 " " " $\$\frac{103}{100}$;
$\therefore$ \$7600 " " " $\$\frac{7600\times103}{100}$, or \$7828.

8. No. of dollars of stock in 75 shares $=\$(75\times50)=\$3750$;
Value of \$100 stock=\$260;
$\therefore$ " \$3750 " $=\frac{3750}{100}$ of \$260=\$9750.

EXERCISE CCXXXVI. PAGE 150.

1. \$84 money will buy \$100 stock;
\$588 " " $\frac{588}{84}$ of \$100 stock, or \$700 stock.

7. \$260 money will buy \$100 stock;
\$5200 " " $\frac{5200}{260}$ of \$100 stock, or \$2000 stock.
Now \$200 stock=1 share;
$\therefore$ \$2000 " $=\frac{2000}{200}$ of 1 share=10 shares.

EXERCISE CCXXXVII. PAGE 150.

1. Income from investing \$130=5;
$\therefore$ " " " \$5200 $=\frac{5200}{130}$ of \$5=\$200.

6. Income from investing \$125=\$7;
$\therefore$ " " " \$100 $=\frac{100}{125}$ of \$7 $=\$5\frac{3}{5}$;
$\therefore$ rate $=5\frac{3}{5}\%$.

EXERCISE CCXXXVIII. PAGE 151.

1. Sum invested to produce \$5 income=\$130;
$\therefore$ " " " \$350 " $=\frac{350}{5}$ of \$130=\$9100.

6. \$12 income is obtained from \$100 stock:

$\therefore$ \$360 " " " $\frac{360}{12}$ of \$100 stock, or \$3000.

Now, \$50 stock=1 share;

$\therefore$ \$3000 " $=\frac{3000}{50}$ of 1 share=60 shares.

EXERCISE CCXXXIX. PAGE 151.

3. $\frac{7}{100}$ of stock=\$1260;

$\therefore$ " $=\frac{100}{7}$ of \$1260=\$18000;

No. of shares$=\frac{18000}{25}=720$.

9. Amount of stock$=\frac{4}{100}$ of \$200=\$5000;

Value of \$100 " $=\$98\frac{1}{2}$;

$\therefore$ \$5000 " $=\frac{5000}{100}\times\$98\frac{1}{2}=\$4925$.

EXERCISE CCXL. PAGE 152.

9. Income from \$125=\$5;

$\therefore$ " " $\$1=\$\frac{5}{125}=\$.04$;

" " $\$115=\$4\frac{1}{2}$;

$\therefore$ " " $\$1=\$\frac{4\frac{1}{2}}{115}=\$.039\ldots$

Hence, the first is the better investment.

10. A's income$=\frac{7}{100}$ of \$5000=\$350;

B's " =\$350+\$50 =\$400.

Now, $\frac{6\frac{1}{4}}{100}$ of B's stock=\$400;

$\therefore$ " " $=\frac{100}{6\frac{1}{4}}$ of \$400 =\$6400.

EXERCISE CCXLII. PAGE 153.

1. (a)

$100\times2=$	200
$300\times4=$	1200
$200\times7=$	1400
$400\times8=$	3200
1000	)6000
	6

The interest on \$100 for 2 mo. equals the interest on \$200 for 1 mo. The interest on \$300 for 4 mo. equals the interest on \$1200 for 1 mo., etc.

Equated time=6 mo.

Hence, the interest on \$1000 for the equated time equals the interest on \$6000 for 1 mo.

The principle upon which this solution depends is, that the interest of the money, the payment of which is delayed beyond the time it is due, is equal to the interest of that which is paid before it becomes due.

2. The debt is due in 90 days.

$200 is paid 30 days before it is due.

Hence, the remaining $200 should be retained for 30 days after the debt is due, or should be paid in 120 days.

7.
$$\begin{array}{l}
\frac{1}{3}\times 0=0\\
\frac{1}{4}\times 4=1\\
\frac{5}{12}\times 6=2\frac{1}{2}\\
\overline{1}\qquad)\overline{3\frac{1}{2}}\\
\qquad\quad 3\frac{1}{2}
\end{array}$$

Equated time$=3\frac{1}{2}$ mo.

EXERCISE CCXLIII. PAGE 154.

10. D's share=D's share $=$ D's share;

C's " $=\frac{11}{10}$ of D's share $=\frac{11}{10}$ D's "

B's " $=\frac{11}{10}$ of $\frac{11}{10}$ of D's share $=\frac{121}{100}$ D's "

A's " $=\frac{11}{10}$ of $\frac{11}{10}$ of $\frac{11}{10}$ of D's share $=\frac{1331}{1000}$ D's "

Shares of all$=(1+\frac{11}{10}+\frac{121}{100}+\frac{1331}{1000})$ D's share$=\frac{4641}{1000}$ of D's share

$=\$9282$;

$\therefore$ D's share$=\frac{1000}{4641}$ of $9282

$=\$2000$.

EXERCISE CCXLIV. PAGE 155.

3.
$$\begin{array}{r}
25\times 5=125\\
35\times 3=105\\
45\times 6=\underline{270}\\
500
\end{array}$$

The cost of pasturing 25 cows for 5 mo. is the same as that of 125 cows for 1 mo., etc.

The rent should be paid in proportion to 125, 105, and 270.

A's share of rent$=\frac{125}{500}$ of $100=$25;

B's " " $=\frac{105}{500}$ of $100=$21;

C's " " $=\frac{270}{500}$ of $100=$54.

5.
$$\left.\begin{array}{r}
4000\times 12=48000\\
3000\times 10=30000
\end{array}\right\}=78000$$
$$\left.\begin{array}{r}
5000\times 12=60000\\
2000\times 9\ =18000
\end{array}\right\}=\underline{78000}$$
$$156000$$

A's share$=\frac{78000}{156000}$ of $1200=$600;

B's " $=\frac{78000}{156000}$ of $1200=$600.

7.
$$\begin{array}{r}
10500\times 12=126000\\
7500\times 12=\ 90000\\
\underline{13500}\times\ 8=\underline{108000}\\
31500\qquad 324000
\end{array}$$

A's share$=\frac{126000}{324000}$ of \$3150=\$1225;
B's " $=\frac{90000}{324000}$ of \$3150=\$875;
C's " $=\frac{108000}{324000}$ of \$3150=\$1050.

8. $2250\times12=27000$.

The product of B's capital and his time must be the same as A's, or 27000.

B's time$=\frac{27000}{6750}=4$.

Hence, B joined A 4 mo. before the end of the year.

12. The partnership was formed on April 1, 1898.

$$\left.\begin{array}{l}4500\times15=67500\\1500\times\ \ 7=10500\end{array}\right\}=78000$$
$$\left.\begin{array}{l}5500\times10=55000\\4000\times\ \ 5=20000\end{array}\right\}=75000$$
$$\left.\begin{array}{l}3000\times12=36000\\5000\times\ \ 3=15000\end{array}\right\}=\underline{51000}$$
$$204000$$

C's share$=\frac{78000}{204000}$ of \$4200=\$$1605\frac{15}{17}$;
D's " $=\frac{75000}{204000}$ of \$4200=\$$1544\frac{2}{17}$;
E's " $=\frac{51000}{204000}$ of \$4200=\$1050.

EXERCISE CCL. PAGE 159.

6. Area of field$=(122.5\times160)$ sq. rods$=(1225\times16)$ sq. rods,

Hence, $4\times$(No. of units of width)$^2=1225\times16$;

(No. of units of width)$^2=1225\times4$;

No. of units of width$=\sqrt{1225\times4}=35\times2=70$;

$\therefore$ length$=4\times70$ rd.$=280$ rd.

EXERCISE CCLII. PAGE 160.

14. Width=width;

Height$=\frac{2}{3}$ width;

Length$=\frac{3}{2}$ width;

$\frac{3}{2}\times\frac{2}{3}\times1$ (No. of units of width)$^3=13824$;

No. of units of width$=\sqrt[3]{13824}=24$;

$\therefore$ width$=24$ ft;

and length$=\frac{3}{2}$ of 24 ft.$=36$ ft.

EXERCISE CCLIII. PAGE 162.

3. Length of plot including walk$=(75+8)$ ft.;
Width " " " $=(60+8)$ ft.;
Area " " " $=(83\times68)$ sq. ft.$=5644$ sq. ft.;

Area of plot $=(75\times 60)$ sq. ft. $=4500$ sq. ft.;
$\therefore$ area of walk $=(5644-4500$ sq. ft.
$=1144$ sq. ft.

EXERCISE CCLV. PAGE 163.

4. Circumference of larger wheel $=(\frac{22}{7}\times 42)$ in. $=132$ in.;
" " smaller " $=(\frac{22}{7}\times 35)$ in. $=110$ in.
Distance gained in 1 revolution $=22$ in.;
$\therefore$ " " 1000 " $=22000$ in.;
12)22000 in.;
3)1833 ft.—4 in.;
$5\frac{1}{2}$)611 yd.—0 ft.;
111 rd.—$\frac{1}{2}$ yd.;
111 rd. $\frac{1}{2}$ yd. 0 ft. 4 in. $=111$ rd. 1 ft. 10 in.

EXERCISE CCLVI. PAGE 164.

3. Area of circle $=(3.85\times 160)$ sq. rd. $=616$ sq. rd.;
$\therefore \frac{22}{7}\times r^2=616$;
$\therefore$ $r=14$, and diameter $=28$ rd.
$\therefore$ circumference $=(\frac{22}{7}\times 28)$ rd. $=88$ rd.

9. Area of larger circle $=(\frac{22}{7}\times 18^2)$ sq. in.;
" smaller " $=(\frac{22}{7}\times 17^2)$ sq. in.;
$\therefore$ difference of area $=(\frac{22}{7}\times 35\times 1)$ sq. in. $=110$ sq. in.

EXERCISE CCLXII. PAGE 167.

9. Length of side of 1st square $=\frac{748}{4}$ in. $=187$ in;
" " " 2nd " $=\frac{336}{4}$ in. $=84$ in;
Sum of areas $=(187^2+84^2)$ sq. in.
$=42025$ sq. in.;
Side of square $=\sqrt{42025}$ in. $=205$ in;
Perimeter $=(4\times 205)$ in. $=820$ in.

10. Area of one field $=(945\times 1344)$ sq. yd.;
Shorter side of other $=\dfrac{945\times 1344}{1134}$ yd. $=1120$ yd.

EXERCISE CCLXIII. PAGE 168.

10. $\dfrac{350+440+750}{2}=770$;
Area of triangle $=\sqrt{(770\times 420\times 330\times 20)}$ sq. yd.
$=46200$ sq. yd. $=\frac{105}{11}$a.
Now, rent of $\frac{105}{11}$ a. $=\$31.50$;
$\therefore$ " " 1 a. $=\frac{11}{105}$ of $\$31.50=\$3.30$.

EXERCISE CCLXIV. PAGE 169.

2. The quantity of water passing through the pipes is in proportion to the areas of the pipes.

The areas are proportional to the squares of the measures of their like dimensions.

Hence, the areas of the pipes are proportional to 9, 9, 16.

The two whose areas are proportional to 9 and 9 are equivalent to one whose area is equivalent to 18.

Hence, the two pipes would carry more than the single one.

4. Radius of pond and walk=30 ft.;

Area of " " $=(\frac{22}{7}\times 30^2)$ sq. ft.;

Area of pond$=(\frac{22}{7}\times 26^2)$ sq. ft.;

$\therefore$ " " walk$=(\frac{22}{7}\times 56\times 4)$ sq. ft.=704 sq. ft.

10. Length of circumference$=\frac{22}{7}\times$diameter;

" " diameter $=\frac{7}{7}\times$ "

$\therefore$ difference between cir. and diam.$=\frac{15}{7}\times$ "

$\therefore$ $\frac{15}{7}\times$diameter=45 in.;

$\therefore$ diameter$=\frac{7}{15}$ of 45 in.=21 in.

Area of circle$=[\frac{22}{7}\times(\frac{21}{2})^2]$ sq. in.

$=346\frac{1}{2}$ sq. in.

EXERCISE CCLXV. PAGE 169.

1. Draw the figure and it will be at once seen that B E C and A E D are similar triangles and have the sides about the angles E B C and E A D proportional.

Hence, 40 : 16+A E :: 32 : A E;

$\therefore$ 40×A E=16×32+32×A E;

$\therefore$ 8×A E=16×32;

$\therefore$ A E=64;

Length of B E=(64+16) ft.=80 ft.

Again, 40 : 18+D E :: 32 : D E.

$\therefore$ 40×D E=18×32+32×D E;

$\therefore$ 8×D E=18×32;

$\therefore$ D E=72;

Length of E C=(72+18) ft.=90 ft.

7. Draw the figure.

The following will be apparent:

Distance from end of shorter side to perpendicular=25 ft.;

" " " longer " " " =32 ft.;

$$(\text{Shorter side}+3)^2-32^2=(\text{shorter side})^2-25^2;$$
$$\therefore 6\times \text{shorter side}=32^2-25^2-3^2;$$
$$\therefore \text{shorter side}=\frac{1024-625-9}{6}=65.$$
$$\text{Longer side}=65+3 \qquad =68;$$
$$\text{Perpendicular}=\sqrt{(65^2-25^2)}=60.$$

EXERCISE CCLXVI. PAGE 170.

1. Since $\frac{22}{7}\times(\text{radius})^2=246400$

$$\therefore \text{radius}=\sqrt{(\tfrac{7}{22}\text{ of }246400)} \quad =280\text{ ft.};$$

Length of standing part $=\sqrt{(287^2-280^2)}$ ft. $=63$ ft.

6. Area of field $=(5\times160)$ sq. rd. $=800$ sq. rd.;

Sum of length and breadth $=60$ rd.;

Resolve 800 into factors whose sum shall be 60. These are evidently 40 and 20.

Length $=40$ rd.; breadth $=20$ rd.

8. $6\frac{1}{4}$ gal. $=1$ c. ft.;

$$\therefore 4500\text{ gal.}=\frac{4500}{6\frac{1}{4}}\text{ of 1 c. ft.}=720\text{ c. ft.};$$

Area of square tank $=\frac{720}{5}$ sq. ft. $\quad =144$ sq. ft.;

Length of side $=(\sqrt{144})$ ft. $\;=12$ ft.

10. Area to be painted $=\frac{120\times60}{2}$ sq. ft. $=\frac{120\times60}{9\times2}$ sq. yd.;

$$\text{Cost of painting}=\frac{120\times60}{9\times2}\times8\text{c.} \quad =\$32.$$

EXERCISE CCLXVIII. PAGE 171.

7. Cubical content of plate $=(2\times\frac{22}{7}\times4\times4)$ c. in.

Cubical content of each shot $=(\frac{4}{3}\times\frac{22}{7}\times\frac{5}{100}\times\frac{5}{100}\times\frac{5}{100})$ c. in.

$$\text{No. of shot}=\frac{2\times\frac{22}{7}\times4\times4}{\frac{4}{3}\times\frac{22}{7}\times\frac{5}{100}\times\frac{5}{100}\times\frac{5}{100}}=192000.$$

9. Cubical content of sphere $=(\frac{4}{3}\times\frac{22}{7}\times6^3)$ c. in.;

Area of iron in the end of cylinder $=(\frac{22}{7}\times7^2-\frac{22}{7}\times5^2)$ sq. in.;

$$=(\tfrac{22}{7}\times12\times2)\text{ sq. in.};$$

$$\text{Length of cylinder}=\frac{\frac{4}{3}\times\frac{22}{7}\times6\times6\times6}{\frac{22}{7}\times12\times2}\text{ in.}=12\text{ in.}$$

10. The cubical content of spheres vary as the cubes of their like dimensions.

Hence, the weight of the smaller sphere would be $\frac{64}{125}$ of 75 oz., if they were composed of the same material.

Actual weight of smaller sphere $=\frac{125}{100}$ of $\frac{64}{125}$ of 75 oz. $=48$ oz.

EXERCISE CCLXXVII. PAGE 179.

5. No vulgar fraction in its lowest terms can be expressed as an exact decimal unless it can be transformed to one which has 10 or some power of 10, for its denominator.

Now, no number can by multiplication be made a power of 10 unless it is composed of factors each of which is 2 or 5.

Thus, 4 can be made into a power of 10 by multiplying it by 5×5.

125 can be made into a power of 10 by multiplying it by $2\times2\times2$.

40 can be made into a power of 10 by multiplying it by 5×5.

$$\tfrac{3}{4}=\frac{3}{2\times2}=\frac{3\times5\times5}{2\times2\times5\times5}=\frac{75}{100}=.75;$$

$$\tfrac{5}{8}=\frac{5}{2\times2\times2}=\frac{5\times5\times5\times5}{2\times2\times2\times5\times5\times5}=\frac{625}{1000}=.625;$$

$$\tfrac{5}{6}=\frac{5}{2\times3}=\frac{5\times5}{2\times3\times5}=\frac{25}{10\times3}=\frac{2.5}{3}=.8\dot{3}.$$

$$\tfrac{7}{12}=\frac{7}{2\times2\times3}=\frac{7\times5\times5}{2\times2\times3\times5\times5}=\frac{175}{100\times3}=\frac{1.75}{3}=.58\dot{3}.$$

Such numbers as 6, 11, 12, 9, 44, 7 cannot be made into powers of 10 by multiplication, and hence $\frac{5}{6}$, $\frac{7}{11}$, $\frac{7}{12}$, $\frac{7}{9}$, $\frac{5}{44}$, $\frac{3}{7}$ cannot be reduced to exact decimals and hence will form repeating decimals.

EXERCISE CCLXXVIII. PAGE 179.

1. See Exercise CCLXXVII.

2. " " "

3. When a vulgar fraction in its lowest terms is reduced to an exact decimal, the number of figures in the decimal part is expressed by the greatest number of times that either the factors 2 or 5 occurs in the denominator.

Thus,

$10=2\times5$,	hence there will	be	1 figure	in the	decimal;
$8=2\times2\times2$,	"	"	"	3 figures	"	"
$32=2\times2\times2\times2\times2$,	"	"	"	5	"	"	"
$200=2\times2\times2\times5\times5$,	"	"	"	3	"	"	"

4. See Exercise CCLXXVII.

5. $6=2\times3$, here there will be 1 figure in the non-repeating part;
$18=2\times3\times3$, here there will be 1 figure in the non-repeating part;

$30=2\times5\times3$, here there will be 1 figure in the non-repeating part;

$24=2\times2\times2\times3$, here there will be 3 figures in the non-repeating part;

$96=2\times2\times2\times2\times2\times3$, here there will be 5 figures in the non-repeating part.

The number of figures in the non-repeating part is determined in the same way as the number of figures in an exact decimal, viz., by the number of times the factors 2 or 5 occurs in the denominator.

6. $7.45648-7.456=.00048$;

$7.457-7.45648=.00052$;

Hence, 7.45648 is more nearly represented by 7.456.

9. Since .0015 is represented by 1;

$\therefore$ 1 " " " $\frac{1}{.0015}$;

$\therefore$ .0002 " " " $\frac{.0002\times1}{.0015}$, or by $.1\dot{3}$.

EXERCISE CCLXXIX. PAGE 179.

1. Elder son's share $=\frac{518}{999}$ of property $=\frac{14}{27}$ of property;

Sum remaining $=\frac{13}{27}$ of property;

$\therefore$ younger son's share $=\frac{14}{27}$ of $\frac{13}{27}$ of property;

$\therefore$ difference between the shares $=(\frac{14}{27}-\frac{14}{27}$ of $\frac{13}{27})$ of property;

$=(\frac{14}{27}\times\frac{14}{27})$ of property

$=\frac{196}{729}$ of property;

$\therefore \frac{196}{729}$ of property $=\$1960$;

$\therefore$ property $=\frac{729}{196}$ of $\$1960=\$7290$;

$\therefore$ elder son's share $=\frac{14}{27}$ of $\$7290=\$3780$.

2. Area of surface of water $=(63\frac{1}{3}\times48\frac{1}{2})$ sq. ft.;

Cubical content of 3 ft. in depth $=(3\times63\frac{1}{3}\times48\frac{1}{2})$ c. ft. $=9215$ c. ft.

3. $1\frac{1}{3}+2\frac{1}{2}+2\frac{7}{9}=6\frac{11}{18}$;

Share of A $=\frac{1\frac{1}{3}}{6\frac{11}{18}}$ of $\$654.50=\$132$;

" B $=\frac{2\frac{1}{2}}{6\frac{11}{18}}$ of $\$654.50=\$247.50$;

" C $=\frac{2\frac{7}{9}}{6\frac{11}{18}}$ of $\$654.50=\$275$.

4. $\frac{6}{9}$ of $\frac{6}{10}$ of money $=\$40$;

$\therefore$ " $=\frac{9}{6}$ of $\frac{10}{6}$ of $\$40=\$100$.

5. $1\frac{1}{2}+\frac{6}{9}+\frac{75}{100}=2\frac{11}{12}$;
$\frac{24}{90}-\frac{15}{100}=\frac{7}{60}$;
$2\frac{11}{12}\times\frac{7}{60}\div43\frac{5}{9}=\frac{35}{12}\times\frac{7}{60}\times\frac{9}{392}=.0078125.$

6. $.\dot{6}+.\dot{1}\dot{8}=\frac{28}{33}$.
Remainder of journey $=\frac{5}{33}$ of distance;
$\therefore\ \frac{5}{33}$ of distance $=(1\frac{2}{3}\times3)$ mi.;
$\therefore$ " $=(\frac{33}{5}$ of $\frac{5}{3}$ of $3)$ mi. $=33$ mi.

7. Area of field $=(72.9\times160)$ sq. rd.;
Length of side $=\sqrt{(72.9\times160)}$ rd. $=108$ rd.;
Distance to be walked $=4\times108$ rd.;
Time to walk $(3\frac{1}{3}\times320)$ rd. $=1$ hr.;
$\therefore$ " " (4×108) rd. $=\dfrac{4\times108}{3\frac{1}{3}\times320}$ hr. $=.405$ hr.

8. Let him buy 1 lb., 2 lb., and 3 lb., respectively.
Cost price $=(1\times30+2\times33\frac{1}{3}+3\times36\frac{2}{3})$c. $=206\frac{2}{3}$c.;
Selling price $=[(1+2+3)\times40]$c. $=240$c.;
Gain on $206\frac{2}{3}$c. $=(240-206\frac{2}{3})$c. $=33\frac{1}{3}$c.;
$\therefore$ " 100c. $=\dfrac{100}{206\frac{2}{3}}$ of $33\frac{1}{3}$c. $=16\frac{4}{31}$c.;
$\therefore$ gain $=16\frac{4}{31}\%$.

9. $.\dot{0}\dot{8}-.08=\frac{8}{99}-\frac{8}{100}=\frac{8}{9900}$;
$\frac{8}{9900}\div.625=\frac{8}{9900}\times\frac{1000}{625}=.001\dot{2}\dot{9}$.

EXERCISE CCLXXX. PAGE 180.

1. Part done by A and B in 1 da. $=\frac{4}{3}$ work;
" " B " $=\frac{2}{5}$ "
$\therefore$ " " A " $=\frac{4}{3}-\frac{2}{5}$ work $=\frac{14}{15}$ work;
Part A does more than B $=(\frac{14}{15}-\frac{2}{5})$ work $=\frac{8}{15}$ work.

2. A does 2 works in 1 da.;
B " 3 " " 1 da.;
C " 4 " " 1 da.;
A, B, and C do 9 " " 1 da.;
$\therefore$ A, B, and C do 1 " " $\frac{1}{9}$ da.

3. Part A, B, and C do in 1 da. $=\frac{4}{5}$ work;
" A and B " " $=(\frac{2}{9}+\frac{5}{18})$ work $=\frac{1}{2}$ work;
Part C does " $=(\frac{4}{5}-\frac{1}{2})$ " $=\frac{3}{10}$ "
Time for C to do $\frac{3}{10}$ work $=1$ da.;
$\therefore$ " " " 1 work $=\frac{10}{3}$ of 1 da. $=3\frac{1}{3}$ da.

Or, L.C.M. of $4\frac{1}{2}$, $3\frac{3}{5}$ and $1\frac{1}{4}$ is 90.

A, B, and C do the work 72 times in 90 da.;

A does " 20 " 90 da.;

B " " 25 " 90 da.;

∴ C " " 27 " 90 da.;

∴ C " " 1 " $3\frac{1}{3}$ da.

4. Since A's working power is $\frac{5}{6}$ of B's;

∴ A will require $\frac{6}{5}$ of B's time to do the same work

Time for B$=12\frac{1}{4}$ da.;

∴ " " A$=\frac{6}{5}$ of $12\frac{1}{4}$ da.$=14.7$ da.

5. If B takes 7 days to do the work,

then A " 8 " " " ;

∴ B does $\frac{1}{7}$ of the work in 1 day

and A " $\frac{1}{8}$ " " 1 day;

∴ B does $\frac{8}{56}$ " " 1 day;

and A " $\frac{7}{56}$ " " 1 day;

∴ A's working power is $\frac{\frac{7}{56}}{\frac{8}{56}}$ of B's working power,

or A's " " is $\frac{7}{8}$ of B's " "

6. Part which A and B do in 1 day$=\frac{1}{18}$ of work;

B's part of this is 1 share;

A's " " " 2 shares;

∴ 3 shares of work$=\frac{1}{18}$ of work;

∴ 1 " " $=\frac{1}{3}$ of $\frac{1}{18}$ of work$=\frac{1}{54}$ of work$=$B's part;

2 " " $=\frac{2}{54}$ of work$=$A's part.

B does $\frac{1}{54}$ of work in 1 day, ∴ the whole work in 54 days;

A " $\frac{2}{54}$ " " 1 day, ∴ " " 27 "

7\. Time for A, B and C together to do the work$=\frac{8}{5}$ of $3\frac{1}{2}$ da.

$=\frac{28}{5}$ da.;

Part of work which A, B and C do in 1 day$=\frac{5}{28}$ work;

If C does 1 share of this;

A " 2 " "

B " 4 " "

Hence, 7 times C's work in 1 day$=\frac{5}{28}$ work;

∴ C's " " 1 " $=\frac{1}{7}$ of $\frac{5}{28}$ work

$=\frac{5}{196}$ work;

Hence, C would do the whole work in $\frac{196}{5}$ da., or $39\frac{1}{5}$ da.

As A does twice as much each day as C;
A will require $\frac{1}{2}$ of $39\frac{1}{5}$ days, or $19\frac{3}{5}$ da.
As B does four times as much each day as C;
B will require $\frac{1}{4}$ of $39\frac{1}{5}$ days, or $9\frac{4}{5}$ da.

8. Time for 10 men to finish the work $=3$ da.;
$\therefore$ “ “ 4 “ “ “ $=\frac{10}{4}$ of 3 da.;
$=7\frac{1}{2}$ da.

Time to begin work $=(12-7\frac{1}{2})$ days, or $4\frac{1}{2}$ days from the commencement of the work.

9. Part which A does in 12 days $=\frac{12}{27}$ work;
“ “ B “ 5 “ $=\frac{5}{15}$ “
“ “ C “ 4 “ $=(1-\frac{12}{27}-\frac{5}{15})$ work $=\frac{2}{9}$ work;
“ “ C “ 1 day $=\frac{1}{4}$ of $\frac{2}{9}$ work $=\frac{1}{18}$) work;
$\therefore$ time for C to do all the work $=18$ days.

10. Time for 1 man to do the work $=75$ hr.;
“ “ 1 boy “ “ $=240$ hr.
Part which 1 man does in 1 hr. $=\frac{1}{75}$ work;
“ “ 1 boy “ “ $=\frac{1}{240}$ “
$\therefore$ part which 3 men and 48 boys do in 1 hr. $=(\frac{3}{75}+\frac{48}{240})$ work
$=\frac{6}{25}$ work;
$\therefore$ time for 3 men and 48 boys to do all the work $=\frac{25}{6}$ hr. $=4\frac{1}{6}$ hr.

11. Part of the work done by A, B and C in 1 day $=\frac{1}{20}$ work.
The parts done by A, B and C are as 1, $\frac{1}{2}$ and $\frac{1}{3}$, or as 6, 3 and 2.
Hence, part done by A in 1 day $=\frac{6}{11}$ of $\frac{1}{20}$ work $=\frac{3}{110}$ of work;
$\therefore$ time for A to complete the work $=\frac{110}{3}$ da. $=36\frac{2}{3}$ da.;
Time for B “ “ “ “ $=(2\times36\frac{2}{3})$ da. $=73\frac{1}{3}$ da;
“ “ C “ “ “ “ $=(3\times36\frac{2}{3})$ da. $=110$ da.

12. Time for A to do the whole work $=\frac{5}{4}$ of 12 da. $=15$ da.;
Part of work done by B in $1\frac{1}{2}$ da. $=\left(1-\frac{13\frac{1}{2}}{15}\right)$ of work
$=\frac{1}{10}$ of work;
$\therefore$ “ “ “ B in 1 da. $=\frac{2}{3}$ of $\frac{1}{10}$ of work
$=\frac{1}{15}$ of work;
$\therefore$ time for B to do all the work $=15$ da.

13. Part which A and B together do in 1 hr. $=\frac{15}{224}$ of work;
$\therefore$ part which B alone does in 1 hr. $=(\frac{15}{224}-\frac{1}{28})$ of work
$=\frac{7}{224}$ of work;
$\therefore$ time for B to do all the work $=\frac{224}{7}$ hr. $=32$ hr.

Or, A and B can do the work 15 times in 224 hr.;
A " " " 8 " " 224 hr.;
∴ B " " " 7 " " 224 hr.;
∴ B " " " 1 " " 32 hr.

14. A, B and C do, respectively, $\frac{4}{15}$, $\frac{2}{5}$, $\frac{8}{25}$ of work in 1 hr.;
Work done by A in $\frac{1}{2}$ hr $=\frac{1}{2}$ of $\frac{4}{15}$ of work$=\frac{2}{15}$ of work;
Work done by A and B in $22\frac{1}{2}$ min.$=\frac{3}{8}$ of $(\frac{4}{15}+\frac{2}{5})$ of work
$=\frac{1}{4}$ of work;
Part of work to be done by the three$=(1-\frac{2}{15}-\frac{1}{4})$of work
$=\frac{37}{60}$ of work;
Time for A, B and C to do this$=\dfrac{\frac{37}{60}}{\frac{4}{15}+\frac{2}{5}+\frac{8}{25}}$ hr.$=\frac{5}{8}$ hr.$=37\frac{1}{2}$ min.
Total time to do the work$=(30+22\frac{1}{2}+37\frac{1}{2})$ min.$=90$ min.

15. Since 2 horses=3 mules;
∴ 6 horses=9 mules and
∴ 6 horses and 5 mules=(9+5) mules;
Now 7 tons are drawn by 14 mules;
∴ 1 t. is " by 2 "
and $9\frac{1}{2}$ t. is " by 19 "
Hence, 12 mules must work along with 7 mules to do the work;
But 12 mules=($\frac{2}{3}$ of 12) horses=8 horses.

16. A, B and C do, respectively, $\frac{1}{12}$, $\frac{1}{15}$ and $\frac{1}{10}$ of work in 1 day.
If each one had worked as long as C, they would do $(1+\frac{2}{12}+\frac{3}{2}$ of $\frac{1}{15})$ of work, or $(1+\frac{1}{6}+\frac{1}{10})$ of work.
Work done by A, B and C in 1 day$=(\frac{1}{12}+\frac{1}{15}+\frac{1}{10})$ of work;
Time for A, B and C to do $(1+\frac{1}{6}+\frac{1}{10})$ of work$=\dfrac{1+\frac{1}{6}+\frac{1}{10}}{\frac{1}{12}+\frac{1}{15}+\frac{1}{10}}$ da.
$=5\frac{1}{15}$ da.

17. A does $\frac{2}{15}$ and B $\frac{4}{33}$ of work in 1 hr.;
Time to do the whole work$=\dfrac{1}{\frac{2}{15}+\frac{4}{33}}$ hr.$=\frac{55}{14}$ hr.;
Part of work done by A$=\frac{55}{14}$ of $\frac{2}{15}$ of work$=\frac{11}{21}$ of work;
" " " B$=\frac{55}{14}$ of $\frac{4}{33}$ " $=\frac{10}{21}$ "
Sum received by A$=\frac{11}{21}$ of \$2.10 =\$1.10;
" " " B$=\frac{10}{21}$ of \$2.10 =\$1.00.

Or, A earns $\dfrac{210}{7\frac{1}{2}}$c., or 28c. per hr.;

B " $\dfrac{210}{8\frac{1}{4}}$c., or $25\frac{5}{11}$c. per hr.;

Time each works$=[1\div(\frac{2}{15}+\frac{4}{33})]$ hr.$=\frac{165}{42}$ hr.;

$\therefore$ sum earned by A$=(\frac{165}{42}\times 28)$c. $=\$1.10$;

" " " B$=(\frac{165}{42}\times 25\frac{5}{11})$c. $=\$1.00$.

EXERCISE CCLXXXI. PAGE 181.

NOTE.—In teaching clock problems the dial of a clock should be studied until it becomes clear that the minute hand gains

55 minute-spaces in 60 minutes of time,

or, 11 minute-spaces are gained in 12 minutes of time,

or, 1 minute-space is " " $1\frac{1}{11}$ " "

Much oral drill should be given to accustom the pupil to recognize the position of the hands, *i.e.*, to tell how many minute-spaces separate them.

2. 11 minute-spaces are gained in 12 min.;

$\therefore$ $16\frac{1}{2}$ " " " " $\frac{16\frac{1}{2}\times 12}{11}$ min., or 18 min.

3. At 3 the minute-hand is 15 minute-spaces behind the hour-hand.

Time to gain 11 minute-spaces$=$12 min.;

$\therefore$ " " 15 " " $=\frac{15\times 12}{11}$ min.$=16\frac{4}{11}$ min.

Hence, the time is $16\frac{4}{11}$ min. past 3.

4. To be at right-angles the hands must be 15 minute-spaces apart. At 6 o'clock they are 30 minute-spaces apart. Hence, 15 minute-spaces must be caught up by the minute hand.

Time to gain 11 minute-spaces$=$12 min.;

$\therefore$ " " 15 " " $=\frac{15\times 12}{11}$ min.$=16\frac{4}{11}$ min.

Hence, the time is $16\frac{4}{11}$ min. past 6.

5. At 6 o'clock the hands are 30 minute-spaces apart.

Time to gain 30 minute-spaces$=\frac{30\times 12}{11}$ min.$=32\frac{8}{11}$ min.

Hence, the time is $32\frac{8}{11}$ min. past 6.

6. At 5 o'clock the hands are 25 minute-spaces apart.

To be 3 minute-spaces apart 22 minute-spaces must be gained.

Time to gain 22 minute-spaces$=\frac{22\times 12}{11}$ min.$=$24 min.

Hence, the time is 24 min. past 5.

7. At 6 o'clock the hands are 30 minute-spaces apart.

To be 8 minute-spaces apart either 22 or 38 minute-spaces must be gained.

Time to gain 22 minute-spaces$=\frac{22\times 12}{11}$ min.$=24$ min.;

" " 38 " " $=\frac{38\times 12}{11}$ min.$=41\frac{5}{11}$ min.

Hence, the time is either 24 min. or $41\frac{5}{11}$ min. past 6.

8. In (*a*) 40 minute-spaces have to be gained.

Time to gain 40 minute-spaces$=\frac{40\times 12}{11}$ min.$=43\frac{7}{11}$ min.

In (*b*) (40—15) minute-spaces have to be gained.

Time to gain 25 minute-spaces$=\frac{25\times 12}{11}$ min.$=27\frac{3}{11}$ min.

In (*c*) (40—30) minute-spaces have to be gained.

Time to gain 10 minute-spaces$=\frac{10\times 12}{11}$ min.$=10\frac{10}{11}$ min.

9. At 9 the hands are 45 minute-spaces apart.

Time to gain 45 minute-spaces$=\frac{45\times 12}{11}$ min.$=49\frac{1}{11}$ min.

At 10 the hands are 50 minute-spaces apart.

Time to gain 50 minute-spaces$=\frac{50\times 12}{11}$ min.$=54\frac{6}{11}$ min.

At 11 the hands are 55 minute-spaces apart.

Time to gain 55 minute-spaces$=\frac{55\times 12}{11}$ min.$=60$ min.

10. If the hour-hand moves a certain distance, the minute-hand moves 12 times as far;

Hence, 1 distance $+12$ distances$=25$ minute-spaces;

$\therefore$ 1 distance $=\frac{25}{13}$ "

$=1\frac{12}{13}$ "

Hence, the time is $(25-1\frac{12}{13})$ min. past 5, or $23\frac{1}{13}$ min. past 5.

EXERCISE CCLXXXII. PAGE 182.

1. $48^\circ=\frac{48}{360}$ of 60 minute-spaces$=8$ minute-spaces;

At 4 o'clock the hands are 20 minute-spaces apart;

Hence, (20—8) minute-spaces must be gained;

Time to gain 12 minute-spaces$=\frac{12\times 12}{11}$ min.$=13\frac{1}{11}$ min.

2. $18° = \frac{18}{360}$ of 60 minute-spaces $=3$ minute-spaces;

To be 3 minute-spaces apart for the 2nd time after 3, $(15+3)$ minute-spaces must be gained.

Time to gain 18 minute-spaces $=\frac{18\times12}{11}$ min. $=19\frac{7}{11}$ min.

3. If the hour-hand moves forward 1 distance from the figure 6, the minute-hand moves forward 12 distances from the figure 12.

Hence, 12 distances $-\frac{1}{2}$ distance $=30$ minute-spaces;

$$\therefore 1 \text{ distance} = \frac{30}{11\frac{1}{2}} \quad \text{"}$$
$$= 2\frac{14}{23} \quad \text{"}$$

Hence, the minute-hand is $1\frac{7}{23}$ minute-spaces past the figure 6,

The time will be $(30+1\frac{7}{23})$ or $31\frac{7}{23}$ min. past 6.

4. In 24 hours the clocks are 15 sec. apart;

Time to be $\frac{1}{4}$ min. apart $=24$ hr.;

$\therefore$ " " 30 min. " $=30\times4\times24$ hr. $=120$ da.;

Time lost in 120 da. $=120\times8$ sec. $=16$ min.

Hence, the time is 44 min. past 11.

Time gained in 120 da. $=120\times7$ sec. $=14$ min.

Hence, the time is 14 min. past 12.

5. Time in which the clocks are 2 sec. apart $=12$ hr.;

$\therefore$ " " " " 300 " " $=\frac{300\times12}{2}$ hr. $=75$ da.

6 Time to lose 5 sec. $=24$ min.;

$\therefore$ " " 17 min $=\frac{17\times60}{5}$ of 24 min. $=3$ da. 9 hr. 36 min.;

3 da. 9 hr. 36 min. from 9 p.m. Monday is 36 min. past 6 a.m. on Friday.

7. From 12 o'clock to 5 o'clock are 300 min.;

295 min. on watch $=5$ hr. of true time:

$\therefore$ 300 " " " $=\frac{300}{295}$ of 5 hr. of true time $=5$ hr. $5\frac{5}{59}$ min.

8. Time in which the clocks are 3 min. apart $=12$ hr.;

$\therefore$ " " " " $8\frac{1}{4}$ " " $=8\frac{1}{4}$ min. of 12 hr.

$=33$ hr.

33 hr. from noon on Friday is 9 p. m. on Saturday.

Time gained is 33 hr. $=\frac{33}{12}\times2$ min $=5\frac{1}{2}$ min.;

Hence, the time is $5\frac{1}{2}$ min. past 9 p. m. Saturday.

Time lost in 33 hr. $=\frac{33}{12}\times1$ min. $=2\frac{3}{4}$ min.;

Hence, the time is $57\frac{1}{4}$ min. past 8 on Saturday.

9. From a quarter to 11 p. m. on May 2 to 9 a. m. on May 7 are $106\frac{1}{4}$ hr.

Time to lose $9\frac{4}{9}$ min. $=106\frac{1}{4}$ hr.;

$\therefore$ " " $1\frac{4}{9}$ min. $=\dfrac{1\frac{4}{9}}{9\frac{4}{9}}$ of $106\frac{1}{4}$ hr. $=16\frac{1}{4}$ hr.;

$16\frac{1}{4}$ hr. from a quarter to 11 p. m. on May 2 is 3 p. m. May 3.

10. From noon on Tuesday till 10 p.m. Saturday are 106 hr.

Now, $1443\frac{2}{3}$ min. on the watch $=1440$ min. true time;

$\therefore$ 106 hr. " " $=\dfrac{106\times 60}{1443\frac{2}{3}}$ of 1440 min. true time

$=105$ hr. $43\frac{3667}{4331}$ min.

Hence, the time is 9 hr. $43\frac{3667}{4331}$ min.

EXERCISE CCLXXXIII. PAGE 182.

1. Distance gone in (60×60) sec. $=(30\times 5280)$ ft.;

$\therefore$ " " " 1 sec. $=\dfrac{30\times 5280}{60\times 60}$ ft. $=44$ ft.

2. Distance gone in 1 sec. $=50$ ft.;

Distance gone in (60×60) sec. $=(60\times 60\times 50)$ ft. $=34\frac{1}{11}$ mi.

3. Distance gone in 11 sec. $=22$ rd.;

Distance gone in (60×60) sec. $=\dfrac{60\times 60}{11}$ of 22 rd. $=22\frac{1}{2}$ mi.

4. Time to move (30×1760) yd. $=(60\times 60)$ sec.;

$\therefore$ " " 110 yd. $=\dfrac{110}{30\times 1760}$ of 60×60 sec.

$=7\frac{1}{2}$ sec.

5. Time for coach to go 2 mi. $=\frac{7}{49}$ of 1 hr.;

$\therefore$ " " " 3.36 mi. $=\dfrac{3.36}{2}$ of $\frac{7}{49}$ of 1 hr. $=14\frac{2}{5}$ min.

6. Time for A to gain $\frac{1}{2}$ mi. $=1$ hr.;

$\therefore$ " " " 4 mi. $=\dfrac{4}{\frac{1}{2}}$ of 1 hr. $=8$ hr.

7. A and B approach each other at the rate of $(4\frac{1}{2}+3\frac{1}{2})$ mi. per hr.

Time to approach 8 mi. $=1$ hr.;

$\therefore$ " " 60 mi. $=\frac{60}{8}$ of 1 hr. $=7\frac{1}{2}$ hr.;

Distance A goes in $7\frac{1}{2}$ hr. $=(7\frac{1}{2}\times 4\frac{1}{2})$ mi. $=33\frac{3}{4}$ mi.

8. A runs 1760 yd. while B runs 1680 yd. and C, 1673;
$\therefore$ B runs 1680 yd. while C runs 1673 yd.;
$\therefore$ B " 1760 yd. " C " $\frac{1760}{1680}$ of 1673 yd., or $1752\frac{2}{3}$ yd.;
Hence, B can beat C by $(1760-1752\frac{2}{3})$ yd., or $7\frac{1}{3}$ yd. in a mile.

9. A can run 1760 yd. while B can run 1738 yd.;
B " 1760 yd. " C " 1749 yd.;
$\therefore$ B " 1738 yd. " C " $\frac{1738}{1760}$ of 1749 yd.
or, $1727\frac{11}{80}$ yd.;
Hence, A can give C $(1760-1727\frac{11}{80})$yd. or, $32\frac{69}{80}$ yd. start in a mile.

10. When the hands of a clock move correctly, they coincide every $65\frac{5}{11}$ min. To coincide every 65 min. $\frac{5}{11}$ min. must be gained.
Gain in 65 min. $=\frac{5}{11}$ min.;
$\therefore$ " 1440 min. $=\frac{1440}{65}$ of $\frac{5}{11}$ min. $=10\frac{10}{143}$ min.

EXERCISE CCLXXXIV. PAGE 183.

1. Greater No. + Less No. = 365;
Greater No. − Less No. = 83;
$\therefore$ twice greater No. = 365 + 83 = 448;
$\therefore$ greater No. $=\frac{448}{2}=224$;
and twice less No. = 365 − 83 = 282;
$\therefore$ less No. $=\frac{282}{2}=141$.

2. Rate down stream = 7 mi. per hr.;
$\therefore$ " in still water $=(7-1\frac{1}{2})$ mi. per hr. $=5\frac{1}{2}$ mi. per hr.;
$\therefore$ " up stream $=(5\frac{1}{2}-1\frac{1}{2})$ mi. " $=4$ mi. per hr.

3. Rate down stream per hr. = 4 mi.;
" up " " $=2\frac{2}{3}$ mi.
Rate in still water + rate of stream = 4 mi. per hr.
" " " − " " $=2\frac{2}{3}$ "
$\therefore$ twice rate of stream $=1\frac{1}{3}$ "
rate of stream $=\frac{1\frac{1}{3}}{2}$ " $=\frac{2}{3}$ mi. per hr.

4. Rate up stream per hr. $=\frac{10}{2\frac{1}{2}}$ mi. $=4$ mi.;
" in still water " $=(4+1\frac{1}{2})$ mi. $=5\frac{1}{2}$ mi.
" down stream " $=(5\frac{1}{2}+1\frac{1}{2})$ mi. $=7$ mi.
Time to row 7 mi. = 1 hr.
" " $3\frac{1}{2}$ mi. $=\frac{3\frac{1}{2}}{7}$ of 1 hr. $=\frac{1}{2}$ hr.

5. Rate down stream per hr. $=\frac{15}{2\frac{1}{2}}$ mi. $=6$ mi.;

" up " " $=\frac{15}{4}$ mi. $=3\frac{3}{4}$ mi.;

∴ rate in still water + rate of stream $=6$ mi. per hr.;

" " " − " " $=3\frac{3}{4}$ " "

∴ twice rate in still water $=9\frac{3}{4}$ " "

∴ " " " $=\frac{9\frac{3}{4}}{2}$ " " $=4\frac{7}{8}$ mi. per hr.

6. Rate down stream per hr. $=4$ distances;

" up " " $=3$ "

∴ rate in still water + rate of stream $=4$ times the distance;

and " " " − " " $=3$ "

∴ twice rate of stream $=1$ "

" " $=\frac{1}{2}$ "

Hence $\frac{1}{2}$ the distance $=1$ mi.;

∴ the distance $=2$ mi.

7. Rate in still water $=4$ mi. per hr.;

" down stream $=(4\times1\frac{1}{3})$ mi. per hr. $=5\frac{1}{3}$ mi. per hr.;

∴ rate of stream $=(5\frac{1}{3}-4)$ mi. per hr. $=1\frac{1}{3}$ mi. per hr.;

∴ " up " $=(4-1\frac{1}{3})$ mi. per hr. $=2\frac{2}{3}$ mi. per hr.

Time to return $1\frac{1}{3}$ mi. up stream $=\frac{1\frac{1}{3}}{2\frac{2}{3}}$ hr. $=\frac{1}{2}$ hr.

" " 1 mi. on the canal $=\frac{1}{4}$ hr.;

∴ " " to starting place $=(\frac{1}{2}+\frac{1}{4})$ hr. $=\frac{3}{4}$ hr.

8. Rate per hr. up stream $=2$ mi.;

" " down " $=6$ mi.;

Time to go 1 mi. up and back $=(\frac{1}{2}+\frac{1}{6})$ hr. $=\frac{2}{3}$ hr.

Distance the man goes and returns in $\frac{2}{3}$ hr. $=1$ mi.;

∴ " " " " " 2 hr. $=\frac{2}{\frac{2}{3}}$ of 1 mi. $=3$ mi.

9. Rate down per hr. $=\frac{\text{the distance}}{2}$;

" up " $=\frac{\text{the distance}}{3}$;

∴ rate in still water + rate of stream $=\frac{\text{the distance}}{2}$;

and rate in still water − rate of stream $=\frac{\text{the distance}}{3}$;

$\therefore$ twice rate in still water $=\frac{5 \text{ times the distance}}{6}$;

$\therefore$ " " $=\frac{5 \text{ times the distance}}{12}$.

Again, twice rate of stream $=\frac{\text{the distance}}{6}$;

$\therefore$ " " $=\frac{\text{the distance}}{12}$.

Hence, the rate in still water is to rate of stream as $\frac{5 \text{ times the distance}}{12}$ is to $\frac{\text{the distance}}{12}$, or as 5 to 1.

10. Rate per hr. up stream $=\frac{6\frac{2}{3}}{2\frac{1}{2}}$ mi. $=2\frac{2}{3}$ mi.;

" " in still water $=(2\frac{2}{3}+1\frac{1}{3})$ mi. $=4$ mi.;

" " down stream $=(4+1\frac{1}{3})$ mi. $=5\frac{1}{3}$ mi.

Time to row $5\frac{1}{3}$ mi. down stream $=1$ hr.;

$\therefore$ " " 12 mi. " " $=\frac{12}{5\frac{1}{3}}$ of 1 hr. $=2\frac{1}{4}$ hr.

EXERCISE CCLXXXV. PAGE 184.

1. Distance skated against the wind in 50 min. $=6$ mi.;

$\therefore$ " " " " " 60 " $=\frac{60}{50}$ of 6 mi.

$=7\frac{1}{5}$ mi.

Rate of skating in calm + rate of wind $=10$ mi. per hr.;

" " " " − " " $=7\frac{1}{5}$ mi. per hr.;

$\therefore$ twice rate of wind $=(10-7\frac{1}{5})$ mi. per hr.;

$\therefore$ " " $=\frac{10-7\frac{1}{5}}{2}$ mi. per hr.

$=1\frac{2}{5}$ mi. per hr.

2. Sum of the rates in 5 sec. $=(110+88)$ yd.;

" " " per hr. $=\frac{60\times 60}{5}$ of 198 yd. $=81$ mi.

Difference of the rates in 45 sec. $=(110+88)$ yd.;

" " " per hr. $=\frac{60\times 60}{45}$ of 198 yd. $=9$ mi.

Rate of faster + rate of slower $=81$ mi.;

" " − " " $=9$ mi.;

$\therefore$ twice rate of faster $=90$ mi.;

$\therefore$ " " $=\frac{90}{2}$ mi. $=45$ mi.;

Again, twice rate of slower $=(81-9)$ mi.;

$\therefore$ " " $=\frac{72}{2}$ mi. $=36$ mi.

3. Rate of first train per sec. $=\frac{1}{60}$ of $\frac{1}{2}$ of 5280 ft. $=44$ ft.;
Sum of rates of the trains per sec. $=(44+40)$ ft. $=28$ yd.;

Distance gone in 1 sec. $=28$ yd.

$\therefore$ " " 8 sec. $=(8\times 28)$ yd. $=224$ yd.

Length of second train $=(224-110)$ yd. $=114$ yd.

4. Rate per 4 sec. $=88$ yd.

Rate per hr. $=\frac{60\times 60}{4}$ of 88 yd. $=45$ mi.

5. Distance the freight is ahead $=\frac{5\frac{1}{2}}{60}$ of 38 mi.

Distance the express gains per hr. $=19$ mi.

Time to gain 19 mi. $=1$ hr.;

$\therefore$ time to gain $\frac{11}{120}$ of 38 mi. $=\frac{11}{120}$ of 38 of $\frac{1}{19}$ hr. $=\frac{11}{60}$ hr.

6. Distance the train goes in 10 sec. $=\frac{10\times 30\times 5280}{60\times 60}$ ft. $=440$ ft.

" " man " " $=\frac{10\times 3\times 5280}{60\times 60}$ ft. $=44$ ft.

440 ft. is the sum of the length of the train and 44 ft.;

$\therefore$ length of the train $=(440-44)$ ft. $=396$ ft.

Or, distance gained by the train in 1 hr. $=27$ mi. $=(27\times 5280)$ ft.;

$\therefore$ " " " " 10 sec. $=\frac{10\times 27\times 5280}{60\times 60}$ ft. $=396$ ft.

7. Distance the train goes in 20 sec. $=\frac{20\times 25\times 1760}{60\times 60}$ yd.

$=\frac{2200}{9}$ yd.

$\frac{2200}{9}$ yd. is the sum of the length of the bridge and the length of the train;

$\therefore$ length of the bridge $=(\frac{2200}{9}-150)$ yd. $=94\frac{4}{9}$ yd.

8. Distance the man goes in 10 sec. $=\frac{10\times 4\times 1760}{60\times 60}$ yd. $=\frac{176}{9}$ yd.

Distance the train goes in 10 sec. $=(88+\frac{176}{9})$ yd.;

$\therefore$ " " " " 1 hr. $=\frac{60\times 60}{10}$ of $(88+\frac{176}{9})$ yd.

$=22$ mi.

9. Distance the man goes in 8 sec. $=\frac{8\times 5\times 1760}{60\times 60}$ yd. $=\frac{176}{9}$ yd.;

Distance the train goes in 8 sec. $=(110-\frac{176}{9})$ yd.;

" " " " 1 hr. $=\frac{60\times 60}{8}$ $(110-\frac{176}{9})$ yd.

$=23\frac{1}{5}$ mi.

10. Distance the man goes in 15 sec. $=\frac{15\times4\times5280}{60\times60}$ ft. $=88$ ft.;

" " train " 15 sec. $=(352+88)$ ft. $=440$ ft.;

" " " " 9 sec. $=\frac{9}{15}$ of 440 ft. $=264$ ft.;

∴ " " man " 9 sec. $=(352-264)$ ft. $=88$ ft.;

∴ " " " " 1 hr. $=\frac{60\times60}{9}$ of 88 ft. $=6\frac{2}{3}$ mi.

11. Distance the train goes in 10 sec. $=(88+44)$ yd. $=132$ yd.;

" " " " 1 hr. $=\frac{60\times60\times132}{10}$ yd. $=27$ mi.;

Time for train to go 180 mi. $=\frac{180}{27}$ hr. $=6\frac{2}{3}$ hr.

EXERCISE CCLXXXVI. PAGE 185.

1. Length of the first piece $=\frac{5364}{149}$ yd. $=36$ yd.;
Cost of 1 yd. of the second piece $=\frac{6912}{36}$c. $=\$1.92$.

2. No. in first bag $=\left(\frac{625-25}{2}+25\right)=325$;

No. in third bag $=\frac{300-50}{2}$ $=125$;

No. in second bag $=125+50$ $=175$.

3. Sum spent in 10 yr. $=\$(6\times700+4\times500)=\$6200$;
∴ income for 1 yr. $=\$\frac{6200}{10}$ $=\$620$.

4. £288 4s. 11d. $=276716$ far.;
£3 17s. $10\frac{1}{4}$d. $=3737$ far.:
276716 far. $\div$ 3737 far. $=74$ and 178 far. remaining;
178 far. $=3$s. $8\frac{1}{2}$d.

5. 5c. + 10c. + 25c. = 40c.;

No. of each $=\frac{680}{40}$c. $=17$.

6. No. is the G. C. M. of $(365-20)$, $(546-17)$ and $(1140-13)$;
G. C. M. of 345, 529 and 1127 is 23.

7. Share of B and C = £40 2s. 8d.
" A and C = £29 10s.;
∴ shares of A, B and twice C's = £69 12s. 8d.;
But, " A, B and C = £53 6s. 8d.;
∴ C's share = £16 6s.

8. Perimeter of field $=2\times(120+30)$ rd. $=4950$ ft.;
No. of boards to go once round $=\frac{4950}{15}=330$;
∴ " " " 6 times " $=6\times330 = 1980$.

9. No. of furrows in 16 ft. $=\frac{16\times12}{8}=24$;
Time to plough 1 furrow $=6$ min.;
∴ " " " 24 " $=(24\times6)$ min. $=2$ hr. 24 min.

10. Value of ducks $=(8\times75)$c. $=600$c.;
Value of chickens $=(13\times50)$c. $=650$c.;
1250c.;
No. of pounds bought for \$1 $=16$;
" " " \$$12\frac{1}{2}=12\frac{1}{2}\times16=200$.

EXERCISE CCLXXXVII. PAGE 185.

1. $6\frac{1}{4}$ gal. $=1$ c. ft.;
∴ 500 gal. $=\frac{500}{6\frac{1}{4}}$ c. ft. $=80$ c. ft.;
No. of c. ft. $=(8\times6\times4)$ c. ft. $=192$ c. ft.
Time for 80 c. ft. to enter cistern $=60$ min.;
∴ " 192 c. ft. " " $=\frac{192\times60}{80}$ min. $=144$ min.

2. Share of boy $=1$ share;
" woman $=1$ " $+25$c.;
" man $=1$ " $+50$c.;
Shares of 7 boys $=7$ shares;
" 8 women $=8$ " $+200$c.;
" 10 men $=10$ " $+500$c.;
25 " $+700$c.;
∴ 25 shares $+700$c. $=2575$c.;
∴ 25 shares $=(2575-700)$c. $=1875$c.;
∴ 1 share $=\frac{1875}{25}$c. $=75$c. $=$ boy's share;
1 share $+25$c. $=\$1.00$ $=$ woman's share;
1 " $+50$c. $=\$1.25$ $=$ man's share;
Money received by boys $=7\times75$c. $=\$5.25$;
" " " women $=8\times\$1.00$ $=\$8.00$;
" " " men $=10\times\$1.25$ $=\$12.50$.

3. L.C.M. of 250c. and 175c. $=1750$c.;
No. of barrels to cost 1750c. $=\frac{1750}{250}$c. $=7$.

4. Dividend $=7469\times728+19=5437451$;
$(5437451-411)\div5320=1022$.

5. Distance A runs further than B=25 yd.;
but distance A runs 1 yd. " " B=3 yd.;
∴ " A " 25 yd. " " B=(25×3) yd.=75 yd.

6. No. of hours for 30 men to do the work=240;
∴ " " " 1 man " " =30×240;
∴ " " " 25 men " " $=\frac{30\times240}{25}=288$;
No. of working days in 288 hr.$=\frac{288}{9}=32$.

7. No. of miles$=\frac{7480\times12}{5280}=17$.

8. Price paid for 1 a.$=\$\frac{7800}{120}=\$65$;
Sum received for 30 a.=30×\$75=\$2250;
" " " 45 a.=45×\$50=\$2250;
\$4500;

Sum for which 45 a. must be sold=\$(7800+300−4500)=3600;
Price for which 1 a. must be sold$=\$\frac{3600}{45}=\$80$.

9. If all were sheep he would have received \$(250×8), or \$2000;
Every time a pig is sold this sum is reduced by \$2.
\$(2000−1720)=\$280.
To reduce \$2000 to \$1720 there must be $\frac{280}{2}$ pigs sold, or 140 pigs.
Hence, there were (250−140) sheep, or 110 sheep.

10. Let cost of 1 vest =1 vest;
Then " 1 coat =1 vest+\$1.25;
∴ " 4 vests=4 vests;
and " 5 coats=5 vests+\$6.25;
∴ cost of 4 vests and 5 coats=9 vests+\$6.25=\$33.25;
∴ cost of 9 vests=\$27;
∴ " 1 vest =\$3;
∴ " 1 coat =\$4.25;
∴ " 12 coats=\$51.

EXERCISE CCLXXXVIII. PAGE 186.

1. Each time he paid \$4.50 he owed \$8.
No. of times he paid \$4.50$=\frac{\$7200}{\$4.50}=1600$;
∴ sum owed=\$(1600×8)=\$12800.

2. Cost of 128 gal.=\$(128×1.70) =\$217.60;
Sum to be realized=\$(217.60+33.40)=\$251;
No. of gals. sold=251;
Quantity of water added=(251−128) gal.=123 gal.

3. Cost of 9 horses and 7 cows=\$1200;
" 6 " " 13 " =\$1200;
∴ " 18 " " 14 " =\$2400;
and " 18 " " 39 " =\$3600;
∴ cost of 25 " =\$1200;
∴ " 1 cow =\$48.

The cost of 9 horses=\$(1200−7×48)=\$864;
∴ " " 1 horse =\$96;
Cost of 3 horses and 8 cows=\$(3×96+8×48)=\$672.

4. Cost of 475 apples=(95×6)c. =570c.;
∴ cost of second lot=(1020−570)c.=450c.;

No. of times he spent 5c.$=\frac{450c.}{5c.}$ =90;

No. of apples in second lot=(90×8) =720.

5. Area of field=$(40\times16\frac{1}{2}\times24\times16\frac{1}{2})$ sq. ft.;
Area of 1 lot=(165×66) sq. ft.;

No. of lots$=\frac{40\times16\frac{1}{2}\times24\times16\frac{1}{2}}{165\times66}=24$;

Price of 1 lot$=\$\frac{1800}{24}=\$75$.

6. Sum received for the tea=60×40×3s. 6d.=£420;
Gain=£(420−280)=£140.

7. No. of cords in the pile$=\frac{60\times8\times8}{128}$;

Value of the wood$=\$\left(\frac{60\times8\times8}{128}\times3.25\right)=\$97.50$.

8. 23 hr. 2 min. 36 sec.=82956 sec.;
46 hr. 8 min. 55 sec.=166135 sec.;
The G. C. M. of 82956 sec. and 166135 sec. is 223 sec.

9 Let Henry get 1 share;
then Thomas gets 8 shares;
and John gets 40 shares.
49 shares.

Value of 49 shares=\$1078;
∴ " " 1 share $=\$\frac{1078}{49}=\$22$;
Henry gets \$22;
Thomas " \$(8×22), or \$176;
John " \$(40×22), or \$880.

10. No. of quarts raised$=\frac{60\times240}{5}=2880$;

" bushels " $=\frac{2880}{32}=90$.

Sum received for 90 bu.=5040c.;

∴ " " " 1 bu.$=\frac{5040}{90}$c.=56c.

EXERCISE CCLXXXIX. PAGE 187.

1. Area of room=(18×12×162) sq. in.;

Length of Carpet$=\frac{(18\times12\times162)}{27}$ in.$=\frac{18\times12\times162}{36\times27}$ yd.=36 yd.

Cost of carpet=(36×75)c.=\$27.

2. Length of rope=2×4 ft.+2×(3 ft. 2 in.)+4×(2 ft. 8 in.) +1 ft. 6 in.;

=8 ft. +6 ft. 4 in.+10 ft. 8 in.+1 ft. 6 in.

=26 ft. 6 in.

3. Aggregate height of 6 boys=6×(4 ft. 5 in.)=26 ft. 6 in.

" " " 4 boys=4×(4 ft. 7 in.)=18 ft. 4 in.;

8 2

∴ " " " 2 boys=8 ft. 2 in.;

∴ average of a boy=4 ft. 1 in.

4. 12 apples=10c.;

∴ 3 apples or 2 oranges=¼ of 10c., or 2½c.;

∴ 4 oranges or 5 lemons=2×2½c., or 5c.;

∴ 15 lemons=3×5c., or 15c.

5. G.C.M. of 1089 ft. and 1375 ft.=11 ft.;

Perimeter of field=(2×1089+2×1375) ft.=4928 ft.

No. of rails to go 1 time round$=\frac{4928}{11}=448$.;

" " " 6 times " =6×448=2688.

6. 212520=2×2×2×3×5×7×11×23.;

These arranged to make 4 consecutive numbers become,

2×2×5, 3×7, 2×11, 23.

7. L.C.M. of 3, 4 and 5=60:

∴ least number possible=60+2=62.;

62 is not a multiple of 7;

2×60+2=122. This is not a multiple of 7;

3×60+2=182. This is a multiple of 7;

∴ least number of marbles=182.

8. Perimeter of lot $=(2\times120+2\times60)$ ft. $=360$ ft.;
Area of surface $=(6\times360)$ sq. ft. $=240$ sq. yd.;
Cost of painting $=(240\times25)$c. $=\$60$.

9. Area of roof $=(35\times12\times25\times12)$ sq. in.;
Area of 1 shingle $=(4\times4)$ sq. in.;

$$\therefore \text{No. of shingles} = \frac{35\times12\times25\times12}{4\times4}=7875.$$

10. Cubic content of car $=(33\times8\times6)$ c. ft.;
" " bale $=(3\times2\times1\frac{1}{2})$ c. ft.;

$$\text{No. of bales in car} = \frac{33\times8\times6}{3\times2\times1\frac{1}{2}}=176.$$

Weight of 176 bales $=26400$ lb.;
$\therefore$ " " 1 bale $=\frac{26400}{176}$ lb. $=150$ lb.

EXERCISE CCXC. PAGE 187.

1. As the farms are of equal size one farm must cost $\frac{2}{5}$ of \$19500 and the other $\frac{3}{5}$ of \$19500.

$\frac{2}{5}$ of \$19500 $=$ \$7800, and $\frac{3}{5}$ of \$19500 $=$ \$11700.

Cost of 130 a. $=$ \$7800;
$\therefore$ " 1 a. $=\$\frac{7800}{130}=\$60$;
and " 130 a. $=$ \$11700;
$\therefore$ " 1 a. $=\$\frac{11700}{130}=\$90$.

2. The L.C.M. of 10 ft. and 12 ft. $=60$ ft.

In going 60 ft. the front wheel turns 1 time more than the hind one.

Distance in which the front wheel turns 1 time more $=60$ ft.;
$\therefore$ " " " " " 264 " " $=(264\times60)$ ft.
$=3$ mi.

3. Wages for 100 days $=(100\times95)$c. $=\$95$.;
Wages lost $=\$(95-76.80)=\$18.20$;
Wages lost in 1 day $=(95+45)$c. $=\$1.40$;

$$\text{No. of days idle to lose \$18.20} = \frac{\$18.20}{\$1.40} = 13.$$

No. of days he worked $=100-13=87$.

4. Perimeter of lot $=(2\times120+2\times90)$ ft. $=420$ ft.;
Cost of fencing $=(420\times15)$c. $=\$63$.
Area of lot $=(120\times90)$ sq. ft. $=10800$ sq. ft.;
Area of walk and lot $=[(120+9)\times(90+9)]$ sq. ft. $=12771$ sq. ft.;

Area of walk $=(12771-10800)$ sq. ft. $=1971$ sq. ft.
$=219$ sq. yd.;
Cost of walk $=(219\times29)$c. $=\$63.51$.;
Difference in cost $=\$(63.51-63)$ $=51$c.

5. Area of the sides $=(2\times16\frac{1}{2}\times7)$ sq. ft. $=231$ sq. ft ;
" " ends $=(2\times7\frac{1}{2}\times7)$ sq. ft. $=105$ sq. ft.;
" " bottom $=(16\frac{1}{2}\times7\frac{1}{2})$ sq. ft. $=123\frac{3}{4}$ sq. ft.
$459\frac{3}{4}$ sq. ft.

No. of sq. yd. $=\frac{1839}{9\times4}=\frac{613}{12}$;
Weight $=(\frac{613}{12}\times8)$ lb.;
Cost $=\$\left(\frac{613\times8}{12}\times\frac{4.90}{100}\right)$ $=\$20.024\dot{6}$.

6. Injured goods $=\frac{7}{8}$ of $\frac{1}{5}$ of goods $=\frac{7}{40}$ of goods;
$\therefore$ value of $\frac{7}{40}$ of uninjured goods $=\$1680$;
$\therefore$ value of goods $=\frac{40}{7}$ of \$1680 $=\$9600$.
Sum received $=\$(840+300)$ $=\$1140$;
Loss $=\$(9600-1140)=\$8460$.

7. Interest on the principal for 4 yr. $=\$(1339.80-1131)=\$208.80$;
$\therefore$ " " " " 5 yr. $=\frac{5}{4}$ of \$208.80 $=\$261$;
Hence, the principal $=\$(1131-261)$ $=\$870$.
The interest on \$870 for 5 yr. $=\$261$;
$\therefore$ " " 100 " 1 yr. $=\$\frac{100\times261}{870\times5}$ $=\$6$;
Hence, the rate $=6\%$.

8. No. of rings $=\frac{2\text{ lb. }3\text{ dwt. }8\text{ gr.}}{9\text{ dwt. }16\text{ gr.}}=50$;
Value of the rings $=\$(50\times8.75)$ $=\$437.50$.
2 lb. 3 dwt. 8 gr. $=24\frac{1}{6}$ oz.;
Value of the gold $=\$(24\frac{1}{6}\times18)$ $=\$435$;
Gain $=\$2.50$.

9. 60 men are to do the work of 150 men for $(40-34)$ da.
Time for 150 men $=6$ da.;
$\therefore$ " " 60 " $=\frac{150\times6}{60}$ da. $=15$ da.;
Hence, the 60 men must begin work $(34-15)$ da. after the commencement, or 19 da.

EXERCISE CCXCI. PAGE 188.

1. Sum remaining after paying legacy duty $=\frac{9}{10}$ of \$4500 = \$4050;

$2+3+4=9$;

Share of first $=\frac{2}{9}$ of \$4050 = \$900;

" second $=\frac{3}{9}$ of \$4050 = \$1350;

" third $=\frac{4}{9}$ of \$4050 = \$1800.

2. No. of horses $=\frac{5}{12}$ No. of cattle;

" cattle $=\frac{12}{12}$ " "

$\frac{17}{12}$ of " " $=85$;

$\therefore$ " " $=\frac{12}{17}$ of $85=60$;

and " horses $=\frac{5}{12}$ of $60=25$.

If all had been horses the cost would have been \$$(60\times 8)$ more than \$2580, or \$3060.

Hence, value of 85 horses = \$3060;

$\therefore$ " 1 " $=\$\frac{3060}{85}=\$36$.

3. No. of bushels of barley $=\frac{3024}{48}$ $=63$;

Value of 63 bu. $=(63\times 46)$c. $=\$28.98$.

No. of bushels of rye $=\frac{3024}{56}$ $=54$;

Value of 54 bu. $=(54\times 55)$c. $=\$29.70$.

Farmer's gain $=\$(29.70-28.98)=72$c.

4. No. of acres represented by 4 sq. in. $=640$;

$\therefore$ " " " (48×30) sq. in. $=\dfrac{48\times 30\times 640}{4}$

$=23040$.

5. 10 a. 268 sq. rd. = 1868 sq. rd.;

Area of field, 4 rd. $\times$ 3 rd. = 12 sq. rd.;

The first field is $\frac{1868}{12}$ times as large as the second.

Hence, the length of the first field is $\sqrt{\frac{1868}{12}}$ times 4 rods;

and the width " " " $\sqrt{\frac{1868}{12}}$ " 3 rods;

$(\sqrt{\frac{1868}{12}}\times 4)$ rd. $=(12.476\times 4)$ rd. $=49.90\ldots$ rd.;

$(\sqrt{\frac{1868}{12}}\times 3)$ rd. $=(12.476\times 3)$ rd. $=37.428\ldots$ rd.

6. Selling price $=\frac{84}{100}$ of cost price;

$\therefore$ $\frac{84}{100}$ of cost price + 50c. $=\frac{104}{100}$ of cost price;

$\therefore$ $\frac{20}{100}$ of cost price = 50c.;

$\therefore$ cost price $=\frac{100}{20}$ of 50c. = \$2.50.

7. Length of wall $=\dfrac{8550}{10\times 2\frac{1}{2}}$ ft. = 342 ft.;

By drawing the figure, it will be seen that

$2\times$ length $+2($length -24 ft. -5 ft.$)=342$ ft.;

$\therefore$ length $+$length-29 ft.$=171$ ft.;

$\therefore$ $2\times$length$=(171+29)$ ft.$=200$ ft.

$\therefore$ length$=\frac{200}{2}$ ft. $=100$ ft.

and breadth$=(100-24)$ ft.$=76$ ft.

8. $\frac{66\frac{2}{3}}{100}$ (of father's money$-\$128.50)=\$128.50$;

$\therefore$ $\frac{2}{3}$ of father's money$-\$85\frac{2}{3}=\$128.50$;

$\therefore$ $\frac{1}{3}$ of father's money$=\$214.16\frac{2}{3}$;

$\therefore$ father's money$=\frac{3}{2}$ of $\$214.16\frac{2}{3}$ $=\$321.25$;

or, $\frac{2}{3}$ of remainder$=\$128.50$;

$\therefore$ remainder$=\$192.75$;

$\therefore$ father's money$=\$(192.75+128.50)=\$321.25$.

9. Interest for 1 yr. 9 mo.@$6\%=\frac{7}{4}$ of $\frac{6}{100}$ of note$=\frac{21}{200}$ of note;

$\therefore$ amount of note$=\frac{221}{200}$ of note $=\$391.17$;

$\therefore$ note$=\frac{200}{221}$ of $\$391.17$ $=\$354$.

10. Wages for 12 mo.$=8$ sheep$+\$180$;

" " 7 mo.$=8$ sheep$+\$80$;

$\therefore$ " " 5 mo.$=\$100$;

$\therefore$ " " 1 yr. $=\frac{12}{5}$ of $\$100=\$240$;

Value of 8 sheep$=\$(240-180)$;

$\therefore$ " of 1 sheep$=\$\frac{60}{8}=\$7\frac{1}{2}$.

EXERCISE CCXCII. PAGE 189.

1. In 1 day A and B earn $\frac{462}{7}$c. or 66c.;

" " A and C " $\frac{700}{10}$c., or 70c.;

" " B and C " $\frac{836}{11}$c., or 76c.;

$\therefore$ " " A, B and C " $\frac{66+70+76}{2}$c., or 106c.;

$\therefore$ " " C earns $(106-66)$c., or 40c.;

" " B " $(106-70)$c., or 36c.;

" " A " $(106-76)$c., or 30c.

2. 5 of hare's leaps$=2$ of hounds;

but the hare takes 4 leaps while the hound takes 2;

$\therefore$ 1 hare's leap is gained every 2 leaps of the hound;

$\therefore$ 80 hare's leaps is gained every (80×2) leaps, or 160 leaps of the hound.

3. A's selling price$=\frac{112}{100}$ of cost to A;

B's " " $=\frac{115}{100}$ of $\frac{112}{100}$ of cost to A;

$\therefore$ $\frac{115}{100}$ of $\frac{112}{100}$ of cost to A$=\$3155.60$;

$\therefore$ cost to A$=\frac{100}{115}$ of $\frac{100}{112}$ of $\$3155.60=\$2450$.

4. The faster train gains $(21-16)$ mi., or 5 mi. per hr.

Time to gain (5×1760) yd. $=(60\times60)$ sec.;

$\therefore$ " " $(184+135)$ yd. $=\dfrac{319\times60\times60}{5\times1760}$ sec. $=130\frac{1}{2}$ sec.

5. Value of house $=8\frac{1}{2}\times$ value of lot;

$\therefore$ $8\frac{1}{2}$ value of lot $+$ value of lot $=\$4750$;

$\therefore$ " " $=\$\dfrac{4750}{9\frac{1}{2}}=\$500$.

6. Since $\frac{90}{100}$ of cost $=\$277\frac{1}{3}$;

$\therefore$ $\frac{108}{100}$ of cost $=\frac{108}{90}$ of $\$277\frac{1}{3}=\$332.80$;

$\therefore$ cost of 1 bu. $=\frac{33280}{416}$c. $=80$c.

7. Distance travelled $=(\frac{72}{60}$ of $4\times5280)$ ft.;

Area cut $=(2\frac{2}{11}\times4840\times9)$ sq. ft.;

$\therefore$ width cut $=\dfrac{2\frac{2}{11}\times4840\times9}{\frac{72}{60}\text{ of }4\times5280}$ ft. $=3\frac{3}{4}$ ft.

8. Cost of sugar $=\$(160+13)=\$173$;

Cost at 7c. $=\$140$;

Difference from actual cost $=\$(173-140)=\$33$;

Each pound bought at 10c. increases this price by $(10-7)$c., or 3c.

Hence, No. of pounds at 10c. $=\frac{3300}{3}=1100$;

" " " " 7c. $=2000-1100=900$.

9. Divide the field in four square fields by dividing the length into 4 equal parts and drawing lines parallel to the ends through the points of section.

Then, area of each square field $=\frac{15376}{4}$ sq. yd. $=3844$ sq. yd.;

$\therefore$ width of the large field $=\sqrt{(3844)}$ yd. $=62$ yd.;

$\therefore$ length " " " $=(4\times62)$ yd. $=248$ yd.

10. L.C.M. of $\frac{3}{4}$ yd., 1 yd., $1\frac{1}{4}$ yd. and $1\frac{1}{2}$ yd. $=15$ yd.;

$\therefore$ width of hall $=15$ yd.;

No. of strips $\frac{3}{4}$ yd. wide $=\dfrac{15}{\frac{3}{4}}=20$;

Length of each strip $=\frac{60}{3}$ yd. $=20$ yd.;

No. of yd. needed $=(20\times20)$ yd. $=400$ yd.;

Cost $=\$(400\times1.10)=\$440$.

EXERCISE CCXCIII. PAGE 189.

1. Distance the train gains in 1 hr. $=27$ mi.;

" " " " 10 sec. $=\dfrac{10\times27\times5280}{60\times60}$ ft. $=396$ ft.;

Hence, length of train $=396$ ft.

2. $\frac{6}{100}$ of \$450=\$27;

Sum to be divided according to capital=\$(450−27)=\$423;

Capital=\$(4900+1400)=\$6300;

A's share=$\frac{49}{63}$ of \$423 =\$329.

3. The successive multiples of the divisor are 1641, 1094, 1641 and 1641.

The G.C.M. of 1641, 1094, 1641 and 1641 is 547.

Hence, the divisor is 547 and the quotient 3233.

4. The difference between a gain of 7% and a loss of 17% is $\frac{24}{100}$ of cost;

$\therefore$ $\frac{24}{100}$ of cost=\$1;

$\therefore$ cost=\$$\frac{100\times 1}{24}$=\$$4\frac{1}{6}$.

5. Cost of (25 cows+25 calves)=\$(427.75+303.50)=\$731.25;

Cost of 25 calves=\$(25×5.75) =\$143.75;

" " 25 cows =\$731.25−143.75) =\$587.50;

$\therefore$ " " 1 cow =\$$\frac{587.50}{25}$ =\$23.50.

6. When B was supposed to win he ran 1760 yd. while A ran 1738 yd.;

Hence, A's rate is to B's as 1738 is to 1760;

But 1760 represents $\frac{4}{3}$ of B's actual rate;

$\therefore$ $\frac{4}{3}$ of B's rate=1760;

$\therefore$ B's rate=1320.

Hence, A's rate is to B's as 1738 is to 1320, or as 79 to 60.

7. Number A and B can dig=68;

" A and C " =66;

" B and C " 62;

" all together " =98;

L.C.M. of 36, 32, 30, 68, 66, 62 and 98 =409036320.

8. Gain=$\frac{40}{100}$ of \$4860=\$1944;

Let what B gets=1 share;

Then A gets=2 shares;

and C gets=6 shares;

$\therefore$ 9 shares=\$1944;

$\therefore$ 1 share =\$ 216=B's share;

and 2 shares =\$ 432 = A's "

and 6 shares =\$1296 = C's "

9. My taxes $=\$(1150-1063.75)=\$86.25$;

Rate on the dollar $=\dfrac{86.25}{1150}$c. $=7\frac{1}{2}$c. $=7\frac{1}{2}\%$.

After paying the tax there remained $92\frac{1}{2}\%$ of income;

$\therefore \dfrac{92\frac{1}{2}}{100}$ of income $=\$1554$;

$\therefore$ income $=\dfrac{100}{92\frac{1}{2}}$ of $\$1554$ $=\$1680$.

10. Selling price of 1st lot $=\frac{120}{100}$ of cost;

$\therefore$ cost of 1st lot $=\frac{100}{120}$ of selling price

$=\frac{5}{6}$ of selling price.

Selling price of 2nd lot $=\frac{80}{100}$ of cost;

$\therefore$ cost of 2nd lot $=\frac{100}{80}$ of selling price $=\frac{5}{4}$ selling price.

Hence, cost of both lots $=(\frac{5}{6}+\frac{5}{4})$ of selling price

$=\frac{50}{24}$ of selling price

$=2\frac{1}{12}$ times selling price;

$\therefore$ loss $=\frac{1}{12}$ selling price.

$\therefore \frac{1}{12}$ of selling price $=\$30$;

$\therefore$ " " $=\$360$;

Cost of 1st lot $=\frac{100}{120}$ of $\$360=\$300$;

" 2nd lot $=\frac{100}{80}$ of $\$360=\$450$.

EXERCISE CCXCIV. PAGE 190.

1. $2016=2\times2\times2\times2\times2\times3\times3\times7=2^3+2^2\times3^2\times7$;

$\therefore$ to become a perfect cube it must be multiplied by

$2\times3\times7^2$, or by 294.

The cube root of $2^3\times2^3\times3^3\times7^3=2\times2\times3\times7=84$.

2. Interest for 3 yr. @ $4\frac{1}{2}\%=\frac{27}{200}$ of principal;

" " $2\frac{1}{4}$ yr. @ $5\%=\frac{45}{400}$ "

$\therefore \frac{27}{200}$ of first sum $=\frac{45}{400}$ of second sum.

Hence, first sum is to second as 54 is to 45, or as 6 is to 5.

Hence, first sum $=\frac{6}{11}$ of $\$1650=\$900$;

and second sum $=\frac{5}{11}$ of $\$1650=\$750$.

3. Selling price $=\frac{90}{100}$ of cost;

Supposed " " $=\dfrac{112\frac{1}{2}}{100}$ "

$\therefore \left(\dfrac{112\frac{1}{2}}{100}-\dfrac{90}{100}\right)$ of cost $=\$45$;

$\therefore$ cost $=\dfrac{100}{22\frac{1}{2}}$ of $\$45=\$200$.

4. $\frac{(.201-.102)^2}{(.201)^2-(.102)^2}=\frac{(.201-.102)(.201-.102)}{(.201+.102)(.201-.102)}=\frac{.201-.102}{.201+.102}$
$=\frac{.099}{.303}=\frac{33}{101}$.

5. Fair gain $=\frac{40}{100}$ of cost;
Unfair gain $=2$ oz. for every 14 oz. sold
$=\frac{1}{7}$ of goods;
Sum realized for $\frac{1}{7}$ of goods $=\frac{1}{7}$ of $\frac{140}{100}$ of cost $=\frac{20}{100}$ of cost;
Total gain $=\frac{60}{100}$ of cost $=\$240$;
$\therefore$ cost $=\frac{100}{60}$ of \$240 $=\$400$.

6. Interest at $\frac{1}{2}$% for the given time $=\$(336-330)=\$6$;
$\therefore$ " " 8% " " " $=16\times\$6 = \$96$;
$\therefore$ principal " " " $=\$(336-96) = \$240$;
Interest on \$240 for 1 yr. at 8% $=\$19.20$;
Time for which \$19.20 is interest $=1$ yr.;
$\therefore$ " " " \$96 " $=\frac{96}{19.20}$ of 1 yr. $=5$ yr.

7. Cubic content of sphere $=\frac{4\pi\times6^3}{3}$ c. in.;
Area of iron in end of cylinder $=\pi\,[(6\frac{1}{2})^2-r^2]$;
Cubic content of cylinder $=8\pi\,[(6\frac{1}{2})^2-r^2]$;
$\therefore 8\pi\,[(6\frac{1}{2})^2-r^2]=\frac{4\pi\times6\times6\times6}{3}$;
$\therefore (6\frac{1}{2})^2-r^2=6\times6$;
$\therefore r^2=6\frac{1}{4}$.
Hence, radius of opening $=\frac{5}{2}$ in.;
Thickness of iron $=(6\frac{1}{2}-2\frac{1}{2})$ in. $=4$ in.

8. 2 hound's leaps $=3$ hare's leaps;
$\therefore$ 3 " " $=\frac{3}{2}$ of 3 hare's leaps $=4\frac{1}{2}$ hare's leaps;
Now while the hound takes 3 leaps the hare takes 4;
Hence, $\frac{1}{2}$ leap is gained each time the hound takes 3 leaps;
$\therefore$ 1 " " " " " 6 "
$\therefore$ 50 leaps are " " " " 300 "

9. $8\frac{1}{2}+5\frac{1}{10}=13\frac{6}{10}$;
Time the fire is burning $=\frac{5\frac{1}{10}}{13\frac{6}{10}}$ of 24 hr. $=9$ hr.

10. Since the time is inversely proportioned to the rate and the supposed time is $\frac{3}{4}$ of the actual time;

$\therefore$ if the actual rate is 3 the supposed rate is 4, 1 more than the actual rate;

Now, 1 mi. per hr. slower would make the rate 2.
L.C.M. of 3 and 2 is 6;
1 hr. is the difference in time to walk 6 mi.;
4 " " " " " 24 mi.

EXERCISE CCXCV. PAGE 191.

1. Selling price per gal. $=\frac{120}{100}$ of \$3.20 = \$3.84.
After 20% has leaked away $\frac{80}{100}$ of a gal. remains;
∴ selling price of $\frac{80}{100}$ of gal. = \$3.84;
∴ " " " 1 gal. $=\frac{100}{80}$ of \$3.84 = \$4.80.

2. 1% of \$2700 = \$27;
Interest on \$(2275+2700) at lower rate = \$(425−27);

$$\therefore \text{ interest on \$100 at lower rate} = \$\frac{100\times398}{4975} = \$8;$$

∴ lower rate = 8%;
and higher rate = 9%.

3. $\frac{125}{100}$ of cost of 1st kind = 45c.;
∴ " " " $=\frac{100}{125}$ of 45c. = 36c.
$\frac{140}{100}$ " " 2nd " = 42c.;
∴ " " " " $=\frac{100}{140}$ of 42c. = 30c.

$$\text{Average cost} = \frac{36+30}{2}\text{c.} = 33\text{c.}$$

Gain on 33c. = $10\frac{1}{2}$c.;

$$\therefore \text{ gain per cent.} = \frac{100\times10\frac{1}{2}}{33} = 31\tfrac{9}{11}.$$

4. $\frac{12}{500}$ of $\frac{3}{4}$ of value of wheat = \$44.07;

$$\therefore \text{ " " } = \$\frac{500\times4\times44.07}{12\times3};$$

$$\text{No. of bu. at 80c.} = \$\frac{500\times4\times4407}{80\times12\times3} = 3060\tfrac{5}{12}.$$

5. A received $\frac{23}{48}$ of the votes; B received $\frac{25}{48}$;
∴ $\frac{2}{48}$ of number of votes = 124;
∴ " " $=24\times124=2976$.
No. who did not vote = 3200−2976 = 224.

6. L.C.M. of 4 and 7 = 28;
In $28\times\frac{22}{7}$ ft., one wheel turns 3 times more than the other.
Distance to turn 3 times more = 88 ft.;
∴ " " 180 " " $=60\times88$ ft.;
∴ rate per minute $=60\times88$ ft.;
∴ " " hour $=60\times60\times88$ ft. = 60 mi.

7. Time for 1 man to do the work = 10 times the time;
" " 1 " " " " = 7 times the time and 49 da.;
∴ 10 times the time " " " = 7 times the time and 49 da.;
∴ 3 " " " " " " = 49 da.;
∴ 1 time " " " " " = $\frac{49}{3}$ da. = $16\frac{1}{3}$ da.

Or, 7 men + 3 men do 1 work in 1 time;
∴ 7 men do $\frac{7}{10}$ work in 1 time;
∴ 7 men do $\frac{3}{10}$ work in 7 days;
∴ 10 men do $\frac{10}{10}$ work in $\frac{7\times10\times7}{10\times3}$ da., or $16\frac{1}{3}$ da.

8. Area of part within the circumference of path
$= (\frac{22}{7}\times149^2)$ sq. yd.;
" " " " path $= (\frac{22}{7}\times145^2)$ sq. yd.;
Area of path $= \frac{22}{7}\times(149^2-145^2)$ sq. yd.
$= (\frac{22}{7}\times294\times4)$ sq. yd.;
Quantity of gravel $= (\frac{8}{36}\times\frac{22}{7}\times294\times4)$ c. yd.
$= 821\frac{1}{3}$ c. yd.

9. In 1 da. A does $\frac{1}{10}$ of the work; B, $\frac{1}{12}$; and C, $\frac{1}{15}$.
Work done by B in $1\frac{1}{2}$ da. $= (1\frac{1}{2}\times\frac{1}{12})$ of work $= \frac{1}{8}$ of work;
" " C " 2 da. $= (2\times\frac{1}{15})$ of work $= \frac{2}{15}$ of work;
Hence, if B and C worked as long as A, they would do
$(1+\frac{1}{8}+\frac{2}{15})$ of work;
Time to do $(\frac{1}{10}+\frac{1}{12}+\frac{1}{15})$ of work = 1 da.;
" $(1+\frac{1}{8}+\frac{2}{15})$ " $= \dfrac{1+\frac{1}{8}+\frac{2}{15}}{\frac{1}{10}+\frac{1}{12}+\frac{1}{15}}$ da. $= 5\frac{1}{30}$ da.

10. Let D's share = 1 share;
C's " $= \frac{105}{100}$ share;
B's " $= \frac{110}{100}$ of $\frac{105}{100}$ share;
A's " $= \frac{112}{100}$ of $\frac{110}{100}$ of $\frac{105}{100}$ share;
Hence, $(1+\frac{105}{100}+\frac{110}{100}$ of $\frac{105}{100}+\frac{112}{100}$ of $\frac{110}{100}$ of $\frac{105}{100})$ share = \$2699.16;
or, $\frac{44986}{10000}$ share = \$2699.16;
∴ 1 share $= \frac{10000}{44986}$ of \$2699.16 = \$600 = D's share;
$\frac{105}{100}$ of \$600 = \$630 = C's "
$\frac{110}{100}$ of \$630 = \$693 = B's "
$\frac{112}{100}$ of \$693 = \$776.16 = A's "

EXERCISE CCXCVI. PAGE 192.

1. Let quantity raised the second yr. = 1 share;
then " " " first yr. $= \frac{13}{10}$ "
∴ " " both yr. $= \frac{23}{10}$ " = 3105 bu.;
∴ 1 share $= \frac{10}{23}$ of 3105 bu. = 1350 bu.;
Quantity raised more the first yr. $= \frac{30}{100}$ of 1350 bu. = 405 bu.

2. Area of walls $=\frac{3360}{30}$ sq. yd. $=112$ sq. yd. $=1008$ sq. ft.;

Let height $=1$ height;

breadth $=1\frac{1}{2}$ "

length $=2$ "

perimeter $=(4+3)$ heights $=7$ heights;

Area of walls $=[7\times(\text{No. of units of height})^2]$ sq. ft.

$=1008$ sq. ft.;

$\therefore$ (No. of units of height)$^2=144$;

No. of units of height $=12$.

Hence, height $=12$ ft.; breadth $=18$ ft.; length $=24$ ft.

3. $62\frac{1}{2}$ lb. $=1$ cu. ft. of water;

$7.112\times62\frac{1}{2}$ lb. $=1$ cu. ft. of iron;

$\therefore$ 1 lb. $=\dfrac{1}{7.112\times62\frac{1}{2}}$ cu. ft. of iron;

$\therefore$ 2000 lb. $=\dfrac{2000}{7.112\times62\frac{1}{2}}$ cu. ft of iron $=4.5$ cu. ft. nearly.

4. Sum received from \$5000 stock $=\frac{5000}{100}$ of \$110 $=$ \$5500;

" " " \$7600 " $=\frac{7600}{100}$ of \$115 $=$ \$8740;

Income from \$(5500+8740) $=\$\dfrac{14240\times4\frac{1}{2}}{89}=\$720$;

Original income from \$5000 stock $=$ \$250;

" " " \$7600 " $=$ \$456;

Gain in income $=\$720-\$(250+456)=\$14$.

5. Buying price per orange $=\frac{3}{5}$c.;

Selling " " " $=\frac{3}{4}$c.;

Gain " " $=(\frac{3}{4}-\frac{3}{5})$c. $=\frac{3}{20}$c.;

No. of oranges to gain $\frac{3}{20}$c. $=1$;

$\therefore$ " " " 1200c $=\dfrac{1200\times20\times1}{3}=8000$.

6. Value of 1 lb. gold $=$ £46 14s. 6d.;

Value of 18 lb. gold $+$ 18 lb. silver $=$ £896 8s.;

Value of 18 lb. gold $=$ £841 1s.;

$\therefore$ value of 18 lb. silver $=$ £55 7s.;

$\therefore$ " " 1 oz. " $=5\frac{1}{8}$s.

Value of 18 lb. of gold $=$ £841 1s.;

Value of 18 lb. of mixture $=$ £637 7s.;

£203 14s.

Each pound of silver reduces the value by £46 14s. 6d. — £3 1s. 6d.

No. of pounds of silver $=\dfrac{\text{£203 14s.}}{\text{£43 13s.}}=4\frac{2}{3}$;

No. of pounds of gold $=18-4\frac{2}{3}=13\frac{1}{3}$.

7. $\frac{120}{100}$ of $\frac{3}{8}$ cost$=\frac{360}{800}$ cost;
$\frac{115}{100}$ of $\frac{3}{10}$ cost$=\frac{345}{1000}$ cost;
$\frac{90}{100}$ of $(1-\frac{3}{8}-\frac{3}{10})$ cost$=\frac{1170}{4000}$ cost;
Selling price$=\frac{4350}{4000}$ cost;
Gain$=\frac{350}{4000}$ cost$=\$217$;
$\therefore$ cost$=\$\frac{4000\times 217}{350}=\$2480$.

8. Time to row 30 mi. up=time to row 50 mi. down;
$\therefore$ time to row 30 mi. up and back=time to row 80 mi. down;
$\therefore$ rate of rowing down$=\frac{80}{12}$ mi. per hr.
$=6\frac{2}{3}$ mi. per hr.;
Time to row 30 mi. down=time to row 18 mi. up;
$\therefore$ time to row 30 mi. down and back=time to row 48 mi. up;
$\therefore$ rate of rowing up$=\frac{48}{12}$ mi. per hr.
$=4$ mi. per hr.;
Rate in still water+rate of stream$=6\frac{2}{3}$ mi. per hr.;
" " " $-$ " " $=4$ mi. per hr.;
$\therefore$ $2\times$ rate of stream$=2\frac{2}{3}$ mi. per hr.;
$\therefore$ " " $=1\frac{1}{3}$ mi. per hr.

9. If the time past 3 p.m. is represented by 4;
then " " " 1 a. m. will be represented by 1;
$\therefore$ the time between 3 p.m. and 1 a.m. will be represented by 3;
The time between 3 p.m. and 1 a.m. is 10 hr.
If 3 represents 10 hr.;
$\therefore$ 1 will represent 3 hr. 20 min.
Hence, the time is 3 hr. 20 min. past 1 a.m., or 4 hr. 20 min. a.m.

10. Since the width is $\frac{5}{6}$ of the length;
$\therefore$ $\frac{1}{6}$ of area$=\frac{1}{6}$ of $\frac{3}{4}$ a. $=\frac{1}{8}$ a.;
$\frac{3}{4}$ a.$-\frac{1}{8}$ a.$=100$ sq. rd.;
Width$=(\sqrt{100})$ rd. $=10$ rd.;
Length$=\frac{6}{5}$ of 10 rd. $=12$ rd.;
Area to be painted$=[(20+24)\times 5\frac{1}{2}\times 2]$ sq. yd.$=484$ sq. yd.;
Cost$=(484\times 45)$c. $=\$217.80$.

EXERCISE CCXCVII. PAGE 193.

1. $2\times$(No. of units of side)$^2=48^2=2304$;
$\therefore$ (" " ")$^2=\frac{2304}{2}=1152$;
$\therefore$ area$=1152$ sq. ft.

2. Selling price of the hay $=\$(1000\times 20) = \$20000$;

$\therefore$ commission $=\$[20000-(18325+875)]=\$800$;

Commission on \$20000 = \$800;

" " \$100 = \$4;

$\therefore$ rate $=4\%$.

3. Wine remaining after the 1st drawing off $=\frac{3}{4}$ of cask;

" " " 2nd " " $=\frac{3}{4}$ of $\frac{3}{4}$ of cask;

" " " 3rd " " $=\frac{3}{4}$ of $\frac{3}{4}$ of $\frac{3}{4}$ of cask;

" " " 4th " " $=\frac{3}{4}$ of $\frac{3}{4}$ of $\frac{3}{4}$ of $\frac{3}{4}$ of cask

$=\frac{81}{256}$ of cask.

4. Let the number of units of thickness = thickness;

then " " " height $=5\times$ thickness;

and " " " length $=40\times$ thickness;

$\therefore$ cubic content of wall $=200\times(\text{thickness})^3$

$=625630\frac{5}{64}$ c. in.;

$\therefore (\text{thickness})^3=625630\frac{5}{64}\div 200$

$=3128.150390625$;

$\therefore$ thickness $=\sqrt[3]{(3128.150390625)}$ in.

$=14.625$ in.

5. $25.\dot{1}3\dot{5}\times 13\frac{7}{31}=25\frac{5}{37}\times 13\frac{7}{31}=\frac{30\times 410}{37}$

$61.375\times 5\frac{15}{37}=61\frac{3}{8}\times 5\frac{15}{37} =\frac{491\times 25}{37}$

$\frac{30\times 410}{37}-\frac{491\times 25}{37}=\frac{25}{37} \qquad =.\dot{6}7\dot{5}$

6. 2 lb. of standard gold $=1869$s.;

$\therefore$ 1 " " " $=934.5$s.;

$\therefore$ 1 " pure " $=\frac{12}{11}$ of 934.5s. $=\frac{11214}{11}$s.

1 lb. of standard silver $=66$s.;

$\therefore$ 1 " pure " $=\frac{40}{37}$ of 66s. $=\frac{2640}{37}$s.;

$\therefore$ 1 lb. of gold : 1 lb. of silver :: $\frac{11214}{11} : \frac{2640}{37}$, or as $14.288 : 1$.

7. $\frac{112\frac{1}{2}}{100}$ of cost price $=45$c.;

$\therefore$ " " $=40$c.;

Cost of 1500 lb. $=(1500\times 40)$c. $=\$600$;

Selling price $=\frac{120}{100}$ of \$600 $=\$720$;

Sum now realized $=(300\times 45)$c. $=\$135$;

" to be " $=\$(720-135)=\$585$;

Selling price per lb. $=\frac{58500}{1200}$c. $=48\frac{3}{4}$c.

8. $\frac{125}{100}$ of $\frac{1}{4}$ cost $=\frac{125}{400}$ of cost;

$\frac{120}{100}$ of $\frac{1}{3}$ cost $=\frac{120}{300}$ of cost;

$\frac{85}{100}$ of $\frac{5}{12}$ cost$=\frac{425}{1200}$ of cost;

$\therefore$ $(\frac{125}{400}+\frac{120}{300}+\frac{425}{1200})$ of cost$=\$800$;

$\therefore$ cost of 240 yds.$=\frac{1200}{1280}$ of \$800;

$\therefore$ cost of 1 yd. $=\frac{1}{240}$ of $\frac{1200}{1280}$ of \$800$=\$3\frac{1}{8}$.

9. Length of side of square field$=\sqrt{(40\times160)}$ rd.$=80$ rd.;

Perimeter of square field$=(4\times80)$ rd. $=320$ rd.

Breadth of rectangular field $=\dfrac{40\times160}{90}$ rd. $=71\frac{1}{9}$ rd.;

Perimeter of " " $=322\frac{2}{9}$ rd.;

Difference in length of fences$=2\frac{2}{9}$ rd.:

" " cost " " $=(2\frac{2}{9}\times81)$c.$=\$1.80$.

10. Income from \$1000 in 3% stock$=\$30$;

" " new investment $=\$40$;

$\therefore$ $\frac{8}{100}$ of new stock$=\$40$;

$\therefore$ " " $=\$500$.

Sum invested in \$500 " $=\frac{95}{100}$ of \$1000$=\$950$;

$\therefore$ " " " \$100 " $=\$\frac{950}{5}=\$190$;

$\therefore$ price of new stock$=190$.

EXERCISE CCXCVIII. PAGE 193.

1. 27 a. 48 sq. rd.$=4368$ sq. rd.;

Taking $\frac{8}{21}$ of 4368 sq rd. away will leave a field equal to a square;

$\frac{13}{26}$ of 4368 sq rd.$=2704$ sq. rd.;

Side of square field$=\sqrt{2704}$ rd. $=52$ rd.;

Width of field$=52$ rd.;

Length " $=\frac{21}{13}$ of 52 rd. $=84$ rd.;

Perimeter " $=(2\times84+2\times52)$ rd.$=272$ rd.

2. Cost of \$18300 stock @ 75$=\$13725$;

Interest on \$13725 @ 8%$=\$1098$;

Total outlay$=\$14823$;

Income from \$18300 stock @ 3%$=\$549$;

Net outlay$=\$14274$;

Sum received for \$18300 stock$=\$(14274-122)$;

$\therefore$ selling price of stock$=\frac{14152}{183}=77\frac{1}{3}$.

3. It is evident that 10c., the difference between the two prices, must be divided into two parts proportional to 20c. and 30c.

Smaller part$=\frac{2}{5}$ of 10c. $=4$c.;

Selling price$=(20+4)$c.$=24$c.

What cost $(20+30)$c, is sold for 48c.;

$\therefore$ loss on 50c $=2$c.;

$\therefore$ " " 100c. $=4$c.;

$\therefore$ loss per cent. $=4$.

4. Let height = number of units of height;

Number of units of area in one side $=408$;

" " " " " end $=312$;

$$\text{Number of units of length}=\frac{408}{\text{height}};$$

$$\text{" " " of breadth}=\frac{312}{\text{height}};$$

$$\frac{408}{\text{height}}\times\frac{312}{\text{height}}=884;$$

$$\therefore (\text{height})^2=\frac{408\times 312}{884};$$

$$\therefore \text{height}=12;$$

$$\text{Length}=\tfrac{408}{12}=34;$$

$$\text{Breadth}=\tfrac{312}{12}=26.$$

5. Time for the first to go 3 mi. $=\frac{3}{4}$ hr.

$$\text{" " " " 12 mi.}=\frac{12}{5\frac{1}{4}} \qquad =2\tfrac{2}{7}\text{ hr.};$$

Time for him to complete the journey $=(2\frac{2}{7}+\frac{3}{4})$ hr. $=3\frac{1}{28}$ hr.;

Distance the second goes in $3\frac{1}{28}$ hr. $=(3\frac{1}{28}\times 3\frac{1}{2})$ mi. $=10\frac{5}{8}$ mi.;

" " " has yet to go $=(15-10\frac{5}{8})$ mi. $=4\frac{3}{8}$ mi.

6. 6 a. $=(6\times 43560)$ sq. ft.;

Quantity of ice $=(6\times 43560\times\frac{1}{2})$ cu. ft.;

Weight of 1 cu. ft. of water $=1000$ oz.;

Weight of $(6\times 43560\times\frac{1}{2})$ cu. ft. of ice $=\frac{9}{10}$ of $6\times 43560\times\frac{1}{2}\times 1000$ oz.

$=3675\frac{3}{8}$ t.

7. $\frac{115}{100}$ of $\frac{4}{9}$ cost $=\frac{460}{900}$ cost;

$\frac{150}{100}$ of $\frac{1}{7}$ " $=\frac{150}{700}$ "

$\frac{75}{100}$ of $\frac{26}{63}$ " $=\frac{1950}{6300}$ "

Sum received $=(\frac{46}{90}+\frac{15}{70}+\frac{195}{630})$ cost $=\frac{652}{630}$ cost;

Net gain $=\frac{22}{630}$ cost $=\$15.40$;

$$\therefore \text{cost}=\$\frac{630\times 15.40}{22} \qquad =\$441.$$

8. A's age : B's age :: 9 : 5;

$\therefore$ A's age $=\frac{9}{5}$ of B's age;

But A's age -23 : B's age -23 :: 10 : 3;

$\therefore$ A's age $-23=\frac{10}{3}$ of (B's age -23);

$\therefore$ A's age $-23+\frac{10}{3}$ of (B's age-23);

$\therefore$ $23+\frac{10}{3}$ of B's age$-\frac{230}{3}=\frac{9}{5}$ of B's age;

$\therefore$ $(\frac{10}{3}-\frac{9}{5})$ of B's age$=76\frac{2}{3}-23$;

$\therefore$ B's age$=\frac{15}{23}$ of $53\frac{2}{3}=35$;

and A's age$=\frac{9}{5}$ of $35=63$.

9. Since the weight of 1728 cu. in.$=(1215\times5760)$ gr.;

$\therefore$ " " " " 1 " $=4050$ gr.;

1 gr.$=56$ sq. in.;

$\therefore$ 4050 gr.$=(4050\times56)$ sq in.;

$\therefore$ number of leaves$=\dfrac{4050\times56}{400}=567$.

10. 5 men$=$ 15 boys and 9 men$=27$ boys;

4 women$=8$ boys and 15 women$=30$ boys;

$\therefore$ 5 men, 4 women and 3 boys $=26$ boys;

and 9 " 15 " " 18 " $=75$ ";

Time for 26 boys to do 1 work $=150$ da.;

$\therefore$ " 75 " " 1 " $=\dfrac{26\times150}{75}$ da.;

$\therefore$ " 75 " " 2 " $=\dfrac{2\times26\times150}{75}$ da.

$=104$ da.

EXERCISE CCXCIX. PAGE 194.

1. Since $\frac{1}{2}$ cir.$+$diam.$=60$ ft. and the cir.$=\frac{22}{7}\times$diam.;

$\therefore$ $\frac{1}{2}$ of $\frac{22}{7}$ diam.$+$diam.$=60$ ft.;

$\therefore$ diam.$=\frac{7}{18}$ of 60 ft. $=\frac{70}{3}$ ft.;

Area of semicircle$=(\frac{1}{2}$ of $\frac{22}{7}\times\frac{35}{3}\times\frac{35}{3})$ sq. ft.

$=213\frac{8}{9}$ sq. ft.;

$\therefore$ No. of plants$=213$.

2. A's selling price$=\frac{85}{100}$ of cost;

B's " " $=\frac{90}{100}$ of $\frac{85}{100}$ of cost;

C's " " $=\frac{110}{100}$ of $\frac{90}{100}$ of $\frac{85}{100}$ of cost;

$\therefore$ $\frac{110}{100}$ of $\frac{90}{100}$ of $\frac{85}{100}$ of cost$=\$3786.75$;

$\therefore$ cost$=\frac{100}{110}$ of $\frac{100}{90}$ of $\frac{100}{85}$ of $\$3786.75=\$4500$.

3. Rate down stream per hr.$=4\frac{1}{2}$ mi.$+$rate of stream;

" up " " $=4\frac{1}{2}$ mi.$-$rate of stream;

$\therefore$ $4\frac{1}{2}$ mi.$+$rate of stream$=3$ ($4\frac{1}{2}$ mi.$-$rate of stream);

$\therefore$ $4\times$rate of stream$=9$ mi.;

$\therefore$ rate of stream$=2\frac{1}{4}$ mi.

4. The quantity of each kind must be inversely proportional to the prices, *i.e.*, to 48c. and 60c., or as 4 to 5;

Quantity at 48c.$=\frac{5}{9}$ of 180 lb.$=$100 lb.;

" " 60c.$=\frac{4}{9}$ of 180 lb.$=$ 80 lb.

5. Cost of coffee per lb.$=\frac{3400}{100}$c. $=$34c.;

" chicory " $=\frac{1000}{100}$c. $=$10c.;

5 lb. chicory @ 10c.$=$50c.;

7 lb. coffee @ 34c.$=$238c.;

Cost of 12 lb. of mixture$=$288c.;

$\therefore$ of 1 lb. " $=\frac{288}{12}$c. $=$24c.;

Selling price$=\frac{7}{6}$ of 24c.$=$28c.

6. $\frac{96}{100}$ of sum levied$=$\$2520;

$$\therefore \text{ " " } = \$\frac{100\times2520}{96}=\$2625.$$

7. Sum received by creditors$=\dfrac{62\frac{1}{2}}{100}$ of \$5000$=$\$3125;

" " on $\frac{1}{2}$ goods$=\dfrac{68\frac{3}{4}}{100}$ of $\frac{1}{2}$ cost$=\frac{275}{800}$ cost;

" " on $\frac{1}{2}$ goods$=\dfrac{87\frac{1}{2}}{100}$ of $\frac{1}{2}$ cost$=\frac{175}{400}$ cost;

$\therefore$ $(\frac{275}{800}+\frac{175}{400})$ cost$=$\$3125;

$$\therefore \text{ cost}=\$\frac{800\times3125}{625}=\$4000.$$

8. Weight of 1 cu. ft of iron$=(7.492\times62.5)$ lb.;

Weight of 1 cu. ft. of cedar$=$20 lb.;

$\therefore$ iron weighs $\dfrac{20}{7.492\times62.5}$ of an equal bulk of cedar;

Thickness of cedar of a certain wt.$=$9 in.;

$\therefore$ " " iron of same wt. $=\dfrac{20}{7.492\times62.5}$ of 9 in.$=$.384 . . in.

9. Edge of cube$=\sqrt[3]{(11390625)}$ in.$=$225 in.;

Length of diagonal$=$225 $(\sqrt{3})$ in.;

Difference in lengths$=$225 $(\sqrt{3}-1)$ in. $=$164.7114 . . in.

10. A's loss$=\frac{10}{26}$ of 2080 $=$\$800;

A's capital$=$\$(10000$-$800) $=$\$9200;

B's loss$=\frac{16}{26}$ of \$2080 $=$\$1280;

B's capital$=$\$(16000$-$1280) $=$\$14720;

\$(9200$+$14720$\times$18000)$=$\$41920;

B's capital$=\$(16000-1280)=\$14720$;

$\$(9200+14720+18000)=\$41920$;

A's share$=\frac{9200}{41920}$ of $\$4716=\$1035$;

B's " $=\frac{14720}{41920}$ of $\$4716=\$1656$;

C's " $=\frac{18000}{41920}$ of $\$4716=\$2025$.

EXERCISE CCC. PAGE 195.

1. $\frac{126}{100}$ of cost$=\$4.32$;

$\therefore$ cost$=\frac{100}{126}$ of $\$4.32=\$3\frac{3}{7}$;

Wine at \$3 sold for $\$3\frac{3}{7}$ gains $\$\frac{3}{7}$, or $\$\frac{15}{35}$;

" $\$3\frac{3}{5}$ " $\$3\frac{3}{7}$ loses $\$\frac{6}{35}$;

To make the gain and loss equal 6 doz. at \$3 must be mixed with 15 doz. at $\$3\frac{3}{5}$, or 2 doz. must be mixed with 5 doz.

2. By drawing the triangle it will be seen that the angle between the hands is 45°.

$45°=\frac{45}{360}$ of 60 minute-spaces$=7\frac{1}{2}$ minute-spaces.

At 4 o'clock the hands are 20 minute-spaces apart.

To be $7\frac{1}{2}$ minute-spaces apart $(20-7\frac{1}{2})$ or $(20+7\frac{1}{2})$ minute-spaces must be gained.

Time to gain $12\frac{1}{2}$ minute-spaces$=(12\frac{1}{2}\times\frac{12}{11})$ min.$=13\frac{7}{11}$ min.;

" " $27\frac{1}{2}$ " " $=(27\frac{1}{2}\times\frac{12}{11})$ min.$=30$ min.

3. Selling price$=\frac{80}{100}$ of cost;

Supposed cost$=\frac{90}{100}$ "

Supposed selling price$=\frac{80}{100}$ " $+\$13$;

$\therefore$ $\frac{125}{100}$ of $\frac{90}{100}$ of cost$=\frac{80}{100}$ " $+\$13$;

$\therefore$ $(\frac{9}{8}-\frac{80}{100})$ of cost$=\$13$;

$$\therefore \text{cost}=\$\frac{40\times 13}{13}=\$40.$$

4. A cost of \$9 gives a width of 1 yd.;

$\therefore$ " \$45 " " $\frac{45\times 1}{9}$ yd., or 5 yd.

5. Cost of 11 lb. @ 88c.$=968$c.;

Increased cost due to 4 lb. of better kind$=$ 88c.;

$\therefore$ cost of 11 lb. of poorer " $=880$c.;

$\therefore$ " 1 lb. of " " $=$ 80c.;

$\therefore$ " 1 lb. of better " $=(80+22)$c.$=102$c.

6. $\frac{8}{100}$ of cost of 1st$=\frac{10}{100}$ of cost of 2nd;

$\therefore$ cost of 1st$=\frac{5}{4}$ of cost of 2nd;

$\therefore \frac{5}{4}$ of cost of 2nd+cost of 2nd=\$5850;

$\therefore$ “ “ $=\frac{4}{9}$ of \$5850=\$2600;

and “ 1st$=\frac{5}{4}$ of \$2600=\$3250.

7. Sum of areas$=(232\frac{3}{4})^2$ sq. ft.$+(23\frac{1}{3})^2$ sq. ft.;

$=\frac{7879249}{144}$ sq. ft.;

Length of side$=\sqrt{\frac{7879249}{144}}$ ft.$=\frac{2807}{12}$ ft.$=233\frac{11}{12}$ ft.

8. The L.C.M. of 1330 and 350=6650;

Time for 175 men and 240 boys=1330;

$\therefore$ “ 35 “ “ 48 “ =6650;

“ 603 “ “ 1005 “ = 350;

$\therefore$ “ $\frac{603}{19}$ “ “ $\frac{1005}{19}$ “ =6650.

Hence, 35 men and 48 boys$=\frac{603}{19}$ men and $\frac{1005}{19}$ boys;

$\therefore$ 665 men and 912 boys=603 men and 1005 boys;

$\therefore$ 62 men =93 boys;

or 2 men=3 boys.

Hence a man's work : boy's work : : 3 : 2.

9. Area of room=49 sq. yd.;

Cost of 49 sq. yd. of carpet$=\$(49\times2.40)$ =\$117.60;

Actual cost of carpet and oil-cloth=\$90.60;

Each yard of oil-cloth reduces the price by (240—96)c., or by 144c.;

Area of oil-cloth$=\frac{2700}{144}$ sq. yd. $=18\frac{3}{4}$ sq. yd.;

Area of carpet$=(49-18\frac{3}{4})$ sq. yd.$=30\frac{1}{4}$ sq. yd.;

Length of side of carpet$=(\sqrt{30\frac{1}{4}})$ yd. $=5\frac{1}{2}$ yd.;

Width of oil-cloth$=\dfrac{21-16\frac{1}{2}}{2}$ ft. $=2\frac{1}{4}$ ft.

10. A's profit for 6 mo.=\$(1200—960)=\$240;

$\therefore$ “ “ 12 mo.=\$480;

$\therefore$ A's stock and profit at end of 12 mo.=\$(960+480) =\$1440.

Stock to produce \$1440 in 12 mo.=\$960;

$\therefore$ “ “ “ \$2440 in 12 mo.$=\$\dfrac{2400\times960}{1440}$

=\$1600 =B's stock.

C's profit=\$(1040—640)=\$400.

Time for \$1600 to gain \$800=12 mo.;

$\therefore$ “ “ \$640 “ \$400$=\dfrac{1600\times400\times12}{640\times800}$ mo.

=15 mo.

EXERCISE CCCI. PAGE 196.

1. If the train passes 1 pole per minute the true speed per hr.

$$=\frac{60\times 60}{1760}\text{ mi.}=2\tfrac{1}{22}\text{ mi.};$$

Approximate speed$=2$ mi.:

$\therefore$ error$=\frac{1}{22}$ mi.

Now, $\frac{1}{22}$ mi. is $\frac{1}{45}$ of $\frac{45}{22}$ mi.

2. $50\times\frac{1}{2}$c.$=25$c.; $\$8-25$c.$=\$7.75$;

Cost of 1 lb. of B's$=\frac{775}{80}$c. $=9\frac{11}{16}$c.;

" " " A's$=(9\frac{11}{16}+\frac{1}{2})$c. $=10\frac{3}{16}$c.;

A's share$=(50\times 10\frac{3}{16})$c.$=\$5.09\frac{3}{8}$;

B's " $=(30\times 9\frac{11}{16})$c. $=\$2.90\frac{5}{8}$.

3. Interest on $1300 for 1 yr. @ $3\frac{1}{2}\%=\$45.50$;

" " $1790 " " $5\%=\$89.50$;

Total interest for 12 mo.$=\$135$;

Time for which $135 is interest$=12$ mo.;

$\therefore$ " " " $112.50 " $=\frac{112.50\times 12}{135}$ mo.$=10$ mo.

4. Diameter of plate$=\sqrt{(6^2+8^2)}$ in.$=10$ in.

5. $\frac{110}{100}$ of $\frac{1}{4}$ cost$=\frac{110}{400}$ cost;

$\frac{105}{100}$ of $\frac{3}{4}$ cost$=\frac{315}{400}$ cost;

$\therefore$ $\frac{425}{400}$ cost$=\frac{108}{100}$ cost$-\$21$;

$\therefore$ $\frac{7}{400}$ cost$=\$21$;

$\therefore$ cost$=\frac{400}{7}$ of $21;

and cost of 1 yd.$=\frac{1}{400}$ of $\frac{400}{7}$ of $\$21=\$3$.

6. Net income$=\frac{95}{100}$ of $\frac{3}{100}$ of stock$=\$1140$;

Stock$=\frac{100}{95}$ of $\frac{100}{3}$ of $\$1140=\$40000$;

Cost of $40000 stock$=\$36000$;

$\therefore$ " $100 " $=\$\frac{36000}{400}$ $=\$90$.

7. Part done by James in 1 hr.$=\frac{1}{8}$ of $\frac{2}{3}$ $=\frac{1}{12}$;

" " John " 1 hr.$=\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{1}{3}=\frac{1}{8}$;

" " Charles " 1 hr.$=\frac{60}{40}$ of $\frac{1}{12}$ $=\frac{1}{8}$;

" " all " 1 hr.$=\frac{8}{24}$.

Time to do $\frac{8}{24}$ of work$=1$ hr.;

$\therefore$ " " all the " $=3$ hr.

8.
$$\begin{array}{r} 5\times 7=35 \\ 3\times 11=33 \\ 2\times 13=\underline{26} \\ 94 \end{array}$$

Weight of 1st metal $=\frac{35}{94}$ of 1410 lb. $=525$ lb.;
" " 2nd " $=\frac{33}{94}$ of 1410 lb. $=495$ lb.;
" " 3rd " $=\frac{26}{94}$ of 1410 lb. $=390$ lb.

9. $\frac{1}{120}$ of 1st sum $=\frac{1}{100}$ of 2nd $=\frac{1}{60}$ of 3rd $=\frac{1}{40}$ of 4th;
∴ 1st sum $=\frac{6}{5}$ of 2nd $=2\times$3rd $=3\times$4th;
∴ 1st sum $+\frac{5}{6}$ of 1st, $\times\frac{1}{2}$ of 1st, $+\frac{1}{3}$ of 1st $=\$5600$;
∴ 1st sum $=\frac{3}{8}$ of $\$5600=\$2100$;
2nd " $=\frac{5}{6}$ of $\$2100=\$1750$;
3rd " $=\frac{1}{2}$ of $\$2100=\$1050$;
4th " $=\frac{1}{3}$ of $\$2100=\$700$.

10. Time A runs 1 mi. $=\dfrac{5280\times12}{270\times48}$ min. $=4\frac{8}{9}$ min.;
Distance B runs after 4 min. $=(63360-4\times300\times44)$ in.
$=10560$ in.;
Time to run 10560 in. $=\dfrac{10560}{320\times44}$ min. $=\frac{3}{4}$ min.;
Time for B to run a mile $=4\frac{3}{4}$ min.;
∴ B wins by $[(4\frac{8}{9}-4\frac{3}{4})\times270\times48]$ in. $=50$ yd.

EXERCISE CCCII. PAGE 197.

1. Area of triangle $=\sqrt{(48\times25\times19\times4)}$ sq. ft.
$=301.993376$ sq. ft.
Area of equilateral triangle $=(16\times16\times\sqrt{3})$ sq. ft.;
$=443.404\,.\,.$ sq. ft.;
Difference of area $=141.4\,.\,.$ sq. ft.

2. A's share $=\frac{3}{10}$ of D's;
C's " $=\frac{3}{10}$ of A's $=\frac{3}{10}$ of $\frac{3}{10}$ of D's $=\frac{9}{100}$ of D's;
B's " $=(\frac{3}{10}+\frac{9}{100})$ of D's $=\frac{39}{100}$ of D's;
∴ $(\frac{3}{10}+\frac{39}{100}+\frac{9}{100}+\frac{100}{100})$ of D's share $=\$3560$;
∴ D's share $=\frac{100}{178}$ of $\$3560=\$2000$;
A's " $=\frac{3}{10}$ of $\$2000=\$600$;
B's " $=\frac{39}{100}$ of $\$2000=\$780$;
C's " $=\frac{9}{100}$ of $\$2000=\$180$.

3. Selling price of first lot $=\$1875+\dfrac{6\frac{1}{4}}{100}$ of selling price;
∴ $\dfrac{93\frac{3}{4}}{100}$ of " " " $=\$1875$;
∴ " " " $=\frac{16}{15}$ of $\$1875=\$2000$;
∴ price of second lot $=\frac{108}{100}$ of $\$1200=\$1296$.

4. Interest on \$34000 for 1 yr. at $\frac{1}{2}\%$=\$170;
∴ the interest on \$(34000+14000) for 1 yr. at lower rate=\$1920;
∴ " " " \$100 " 1 yr. " " " $=\$\frac{1920}{480}=\$4$;
∴ rate=4% and $4\frac{1}{2}\%$.

5. Cost price of the mixture $=\frac{100}{120}$ of 66c. =55c.;
Price of 2 lb. of tea $=(2\times55)$c. =110c.;
Price of 1 lb. of better kind $=(110-50)$ c.=60c.

6. No. of days' work for 1 man $=24\times$No. of men;
" " " " 1 " $=16\times$(No. of men+16);
∴ $24\times$No. of men $=16\times$No. of men+256;
∴ $8\times$No. of men=256;
∴ No. of men=32.

7. When B is at the town, A is $\frac{1}{4}$ of the way there.
Distance A goes in 70 min. $=(\frac{70}{60}\times3)$ mi. $=3\frac{1}{2}$ mi.
But B meets A $\frac{2}{3}$ of 12 mi. from the town;
∴ the distance $=\frac{1}{4}$ of the distance $+(3\frac{1}{2}+8)$ mi.;
∴ $\frac{3}{4}$ of the distance $=11\frac{1}{2}$ mi.;
∴ the distance $=\frac{4}{3}$ of $11\frac{1}{2}$ mi. $=15\frac{1}{3}$ mi.

8. Capital at end of 4th yr. $=\frac{100}{4}$ of \$3660.25=\$91506.25;
Hence, $\frac{11}{10}$ of $\frac{11}{10}$ of $\frac{11}{10}$ of $\frac{11}{10}$ of original capital=\$91506.25;
∴ original capital $=\frac{10000}{14641}$ of \$91506.25=\$62500.

9. Circumference of dial $=(2\times9\times\frac{22}{7})$ ft.;
∴ No. of min.-spaces in 11 ft. $=\dfrac{11}{2\times9\times\frac{22}{7}}$ of 60 min.-spaces
$=11\frac{2}{3}$ min.-spaces;
Hence, the time is $11\frac{2}{3}$ min. past 12.

10. (1) Interest on \$120 for 1 time =\$5;
∴ " " \$120 " 2 times=\$10;
∴ discount on \$130 " 2 " =\$10;
∴ " " \$125 " 2 " $=\$\dfrac{125\times10}{130}=9\frac{8}{13}$.

(2) Interest for 1 time $=\frac{1}{24}$ of principal;
∴ amount for 1 " $=\frac{25}{24}$ " "
∴ " " 2 times $=(\frac{25}{24})^2$ of principal
$=\frac{625}{576}$ of principal;
∴ interest for 2 times $=\frac{49}{576}$ " "
∴ discount for 2 times on $\frac{625}{576}$ of principal $=\frac{49}{576}$ of principal;
∴ discount for 2 times on \$125 $=\$(125\times\frac{49}{625})=\$9\frac{4}{5}$.

PART III.

Suggestions for Teaching Arithmetic.

I. NUMBER WORK.

I. Counting.

Let the pupils use splints, pebbles, measure with rules, etc.

II. Numbers from 1-10.

(*a*) Teach each number as a whole by means of (1) objects, (2) definite measures, and (3) number-pictures, and have the pupils learn the symbol (figure) for each.

The following is suggested as blackboard summary:

● = 1=one.	● ● ● ● ● ● = 6=six.
● ● = 2=two.	● ● ● ● ● ● ● = 7=seven.
● ● ● = 3=three.	● ● ● ● ● ● ● ● = 8=eight.
● ● ● ● = 4=four.	● ● ● ● ● ● ● ● ● = 9=nine.
● ● ● ● ● = 5=five.	● ● ● ● ● ● ● ● ● ● =10=ten.

NOTE.—Only one form of number-picture should be presented so that the child may get a definite picture of it.

(*b*) Teach the facts of addition and subtraction in connection with each number and introduce the symbols of operation + and —.

Thus, by objects as splints, definite measures as 1 in., 1 ft., etc., and by number-pictures teach that

5+1=6.	6—5=1.
1+5=6.	6—1=5.
4+2=6.	6—4=2.
2+4=6.	6—2=4.
3+3=6.	6—3=3.

If thought preferable begin with 3+3=6, etc.

No fact should be symbolized until the pupils have performed the operations by counting, measuring, etc.

These facts will be fixed by brief, repeated drills (*a*) with objects, (*b*) without objects, (*c*) having pupils form problems, (*d*) having pupils work problems.

NOTE.—(1) In practice it is better to postpone the facts of multiplication and division until the pupil has a general knowledge of numbers to 20.

NOTE.—(2) The pupils may be exercised in making and solving such exercises as

$4+2=?$	$4+?=6.$	$?+2=6.$
$3-1=?$	$3-?=2.$	$?-1=2.$
$3+3=?$	$3+?=6.$	$?+3=6.$
$5-2=?$	$5-?=3.$	$?-2=3.$

III. The Numbers from 11-20.

(*a*) Analyze these numbers as in step II., the work being confined to the facts of addition and subtraction and one of the parts being always 10. Thus,

$$17=10+7=10+4+3=14+3=10+3+4=13+4, \text{ etc.}$$

(*b*) The numbers from 1-20 should now be measured by such units as 2, 3, 4, etc., and such questions answered, as how many times 2 inches make 8 inches? How many twos make 8? etc.

IV. The Numbers from 21-100.

By using number-pictures, definite measures, etc., these numbers should be analyzed to discover the chief facts of addition, subtraction, multiplication and division, the tens being mainly dealt with, thus,

$$37=30+7=20+17=10+10+10+7, \text{ etc.}$$

NOTE.—No attempt should be made to break the number up into such parts as

$$37=19+18=16+21=13+24, \text{ etc.}$$

II. ARABIC NOTATION AND NUMERATION.

I. The Units.

As the numbers from 1 to 9 are developed their symbols are given and pupils learn to associate each number with its symbol.

II. The Tens.

Have bundles of ten and single splints.

Have pupils represent 1 splint, thus, 1.

Have pupils represent 1 bundle of ten, thus, 1.

How is the 1 representing 1 splint to be distinguished from the 1 representing 1 bundle?

The teacher explains that this is done by the position of the 1's. The 1 representing 1 bundle of ten is always placed to the left of the 1 representing 1 splint.

Hold up 2 bundles of ten and 1 splint and have the number represented by 21; 3 bundles of ten and 1 splint and have it represented by 31, etc.

Now, put down the 1 splint and holding the bundle, give the question, How can the one bundle of ten be represented to show that is a bundle of ten and not a single splint?

The teacher will thus be able to show the use of 0 is to give position to the significant figures.

Continue to 100, as 30, 40, 50, 60, 70, 80, etc.

III. Combine the Tens and the Units.

Thus, 25, 27, 34, 39, 47, etc.

IV. Hundreds.

(*a*) Using bundles of ten-tens, show, in a similar way, that 1, 2, 3, 4, etc., in the 3rd place represent hundreds.

(*b*) Combine hundreds, tens and units together and give such drill as will enable pupils to write and read with readiness any number expressed by three figures.

V. Thousands.

(*a*) In a similar way teach the thousands' place and have much practice in reading and writing numbers expressed by four figures.

(*b*) Introduce the fifth place, give its name, and, as before, give much practice in reading and writing numbers of five figures.

(*c*) Introduce the complete period and give practice in reading and writing it as before.

(*d*) Teach the meaning of period and how many places constitute a period. Show that in the two periods that have been taught that units, tens, and hundreds occupy the same relative positions.

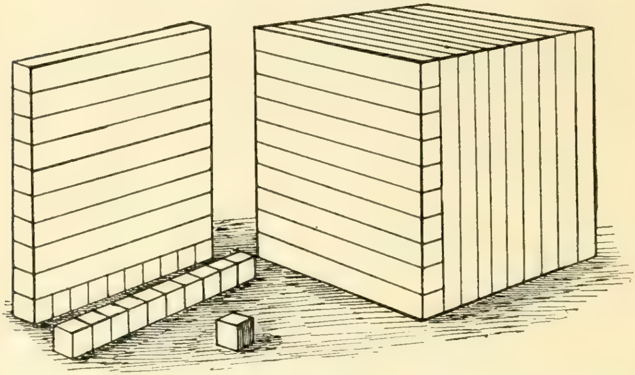
NOTE.—The first period is the *units'* period; the second one, the thousands', &c. The first period is never to be called the hundreds' period as some teachers do.

VI. Large Numbers.

(*a*) Teach the millions' period by giving its name and drilling in reading and writing numbers of 7, 8 and 9 figures.

(*b*) Continue thus giving the names and periods in order and having much drill.

The following device of Sonnenschein and Nesbitt is suggested as a better means of impressing the idea that each digit has two values, an *absolute* one, and a *relative* one depending upon its position in a number:



The small cube represents 1 unit; the stave is 1 ten; the plate is 1 hundred; and the large cube, 1 thousand.

A handy teacher can make these for himself. There should be at least 19 each of the small cubes, staves, and plates. It is to be used in a similar manner to the splints, bundles of ten, etc.

NOTE.—In our system of notation 3 figures make a period. This is the French system of notation. In the English 6 figures make a period. One million is expressed in the same way in each case, viz., 1,000000. One billion in the English system is expressed thus, 1,000000,000000, and is hence 1000 times as great as our billion.

III. ROMAN NOTATION.

I. Numbers from 1-10.

(*a*) By means of the reader, let the pupils observe how the lessons from 1 to 10 are numbered.

Tabulate the results as 1=I; 2=II; 3=III; 4=IV; 5=V; etc.

(*b*) From observation of these symbols teach the following:—

1. The effect of repeating a symbol.

2. The effect of writing a symbol of lower value to the left of one of greater value.

3. The effect of writing a symbol of less value to the right of one of greater value.

4. How often a symbol is repeated.

II. Numbers from 11-100.

(*a*) Teach how to read and write the tens as

20, 30, 50, 40, 60, 70, 80, 100, 90.

(*b*) Combine the tens and units as 17, 24, 43, etc.

III. Numbers from 100-2000.

(*a*) Teach the hundreds alone as 100, 200, 300, 500, 400, etc.

(*b*) Combine the hundreds and tens as 110, 130, 250, 340, etc.

(*c*) Combine the hundreds, tens and units as 275, 456, 945, etc.

NOTE.—Pupils will thus see that one place in Arabic Notation must always be represented by its Roman equivalent before going to the next place. Thus,

1897=1000+800+90+7=MDCCCXCVII.

1901=1000+900+0+1 =MCMI.

IV. ADDITION.

I. Oral Addition, or the Addition Table.

Pupils should not be given written addition until they can combine numbers without counting by ones. Facility in oral addition can be secured by (1) constant drill and (2) correct practice.

Drill Exercises.

(1) Count by twos beginning with (*a*) 2, and (*b*) 1, up to and just beyond 100.

Count by threes beginning with (*a*) 3, (*b*) 2, and (*c*) 1, up to and just beyond 100.

Treat each of the other numbers up to 10 in a similar way.

(2) Teach the sum of two numbers, as 7+8; 8+7.

Pay attention to the ending, as 17+8; 18+7; 27+8; 28+7, etc. When a second combination is known as 5+5, use these in giving drill in rapid addition as (add from the left)

7+8+5+8+7+5+8=

8+7+5+7+8+5+7+8=

When a third combination, as 4+3, 3+4 is known, other exercises are given as,

4+3+8+5+3+4+8+5+7=

3+4+8+5+7+8+5+4+3=

As each new combination is learned, as 9+8, the pupil should use it as shown above, thus,

9+8+8+5+9+8+8+5+4+3=

Thus, the pupil is never called on to add combinations which he has not learned and thus compelled to resort to counting on his fingers, or making marks which he counts.

Pupils should be taught from the beginning to give the *sum* at once; thus, in the last line: 17, 25, 30, 39, 47, etc., and not 9 and 8 is 17; 17 and 8 is 25, etc.

(3) Place a number in the centre of a circle with other numbers round the circumference. Have the pupils add the number in the centre to those on the circumference as they are rapidly pointed to.

Vary the tens in the numbers on the circumference.

(4) Write the digits on the blackboard, as

3, 7, 5, 8, 9, 6, 4, 7, 5, 2, 8, 3, 7, 9, etc.,

and as the digits are pointed to have the pupils add. At first take the answer as soon as the sum is sufficiently large; but after a time have a number of such exercises before there is any interruption in the concentrated effort necessary, the pupils placing the answer to each exercise as soon as the teacher stops pointing and being ready to begin adding as soon as he begins to point to the digits again.

Gradually increase the speed of pointing.

(5) Write a column of digits on the blackboard and as soon as the last figure is on erase the whole. The pupils must add as the digits are placed on the board.

The answers may be taken as in (4) above.

(6) Train pupils to look for easy combinations, especially those that make ten.

NOTE.—1. In all the exercises used to secure accuracy and rapidity in addition, experience shows that *rapidity and accuracy are closely associated together*.

2. When a pupil fails, ascertain where and why and give special exercises on the combinations causing the difficulty.

II. Written Addition.

(a) Addition where there is no carrying.

(1) Show how the addends must be placed.

(2) Give examples for practice.

(3) Give problems to be solved.

NOTE.—In an example the pupil is told what he is to do; in a problem he is required to find the process for himself.

(b) Addition where there is carrying.

(1) Show the necessity of carrying.

(2) Show the convenience of beginning at the right hand side.

(3) Give examples for practice.

(4) Give problems to be solved.

To save Ts time.
1. Own old method of t.
2. Cover card. Young p. 234.

In introducing carrying, let the pupils do a few examples, thus,

4	3	7
6	5	9
8	7	8
18	15	24

Then their knowledge of notation should be used to show that 24 units are the same as 2 tens and 4 units, etc. Make use of the plates, staves and cubes, or bundles of splints.

The exercises on pages 15 and 16 of the Public School Arithmetic are so arranged that they may be added without transferring the figures to the slate or note book and any number of addends may be given up to 29 on page 16 or 43 on page 15.

III. Methods of Proof.

Pupils should be shown how to prove their work.

(1) Add upwards and downwards. If the sums agree the work is likely correct.

(2) Divide the addends into a number of groups. Find the sum of each group. Then find the sum of these sums. If the result agrees with the sum of all the addends the work is likely correct.

(3) By adding across, or casting out nines,

(4) Add again the sum, result must be twice [illegible]

V. SUBTRACTION.

I. Oral Subtraction, or the Subtraction Table.

This is merely the converse of the addition table.

It should be impressed by much skilful drill and practice.

Drill Exercises.

(1) Count backwards by twos from 100.

"	"	"	"	"	99.
"	"	"	threes	"	99.
"	"	"	"	"	98.
"	"	"	"	"	97.
"	"	"	fours	"	100.
"	"	"	"	"	99, etc.

(2) Describe a circle; place the subtrahend at the centre and round the circumference write numbers. Let the pupil subtract as the teacher points to a number on the circumference.

(3) Place the minuend at the centre of a circle and the digits in any order round the circumference. Let the pupil subtract as the teacher points to a subtrahend.

(4) Place the digits in any order upon the blackboard, as

7, 4, 9, 5, 3, 8, 6, 2, 1,

and require the pupils to subtract each from a given number, as 70, when the digit is pointed to.

(5) Place the digits as in (4) and let the numbers indicated by the digits be successively subtracted from a given number, as 65. Thus, 65, 58, 54, 45, 40, 37, 29, 23, 21, 20.

II. Written Subtraction.

(a) Subtraction where every figure in the minuend is of a greater value than the corresponding figure in the subtrahend.

(1) Give examples and show how the subtrahend is placed with reference to the minuend.

(2) Give problems.

(b) Subtraction where every figure of the minuend is not greater than the corresponding figure of the subtrahend.

There are several modes of procedure here.

(I) The method of Decomposition often improperly called the "borrowing" method.

Thus, from 32 splints take 14 splints.

$$\begin{array}{lllllll} & \text{tens} & & \text{units} & & \text{tens} & & \text{units} \\ 32 = & 3 & + & 2 & = & 2 & + & 12 \\ 14 = & & & & & 1 & + & 4 \\ \hline & & & & & 1 & + & 8 = 18. \end{array}$$

Illustrate by using bundles of ten splints, or by taking staves and changing one into its equivalent number of cubes.

(II) The method of Equal Additions often improperly called the "borrowing and carrying" method.

Borrowing is incorrect because in borrowing several conditions are to be fulfilled. (*a*) There must be a lender. (*b*) The lender has less after than before the lending. (*c*) The lender must be repaid. (*d*) He will then possess the original amount. These conditions are not fulfilled in subtraction.

The method of Equal Additions depends upon the principle that adding the same number to both minuend and subtrahend does not alter the remainder.

Illustrate this principle by many examples, as

From	5	7	9	6	16	26
Take	3	5	7	2	12	22
	2	2	2	4	4	4

NOTE.—While it is important for the teacher to understand the rationale of processes, the important thing for the pupil is facility in performing operations; hence, at this stage the pupil may be shown there is a reason for what is done, but he should not be expected to reproduce the reason.

(III) Computers' Subtraction.

This method is based on the fact that the sum of the remainder and the subtrahend is equal to the minuend.

From 49721 take 15837.

49721	7+**4**=11; 4+**8**=12; 9+**8**=17; 6+**3**=9; 1+**3**=4.
15837	This is the most direct way of proceeding from the known
33884	addition table to the answer required in subtraction.

NOTE.—The *arithmetical complement* of a number is the difference between it and the unit of the next higher order. Thus, 4, 8, 143, respectively, are the arithmetical complements of 6, 92, 857, being the differences, respectively, of 6, 92, and 857 and 10, 100, and 1000.

The computers' method is sometimes called the method of *complementary addition* for the reason that the complement of the figure in the subtrahend which is greater than its corresponding figure in the minuend is added to the latter figure.

Thus, in the examples given above, the complement of 7 is 3; 3+1=4; the complement of 4 is 6; 6+2=8; etc.

The advantages of this method are the following:

(*1*) The sum of a number of addends can be subtracted at once from a given number.

Thus, from 8073 take the sum of 247, 968, and 3714.

8073	
247	4+8+7+(4)=23
968	2+1+6+4+(4)=17
3714	1+7+9+2+(1)=20
3144	2+3+(3)= 8

(*2*) Subtraction and addition can be taught together.

Thus, addition, 2+3=5, or 2 and 3 is 5;
subtraction, 2+(3)=5, or 2 from 5 leaves 3.

(*3*) It may be used to shorten long division.

Thus, divide 183410 by 379;

379)183410(483

3181	4×9+(8)=44; 4×7+4+(1)=33; 4×3+3+(3)=18.
1490	8×9+(9)=81; 8×7+8+(4)=68; 8×3+6+(1)=31.
353	Etc.

(*4*) In business, it is the method followed in making change.

(*5*) No special device is needed to obviate the difficulty when a digit in the subtrahend is of greater value than the one above it; hence, the labor required to teach subtraction is greatly lessened.

(6) They prove as they subtract.

III. Methods of Proof.

(1) By Addition. The sum of the remainder and the subtrahend is equal to the minuend, if the work is correct.

(2) By Subtraction. When the remainder is taken from the minuend, the result is the same as the subtrahend, if the work is correct.

VI. MULTIPLICATION.

I. Oral Multiplication, or the Multiplication Table.

Using splints, definite units in measuring, the numeral frame, etc., build up the table.

Thus, 1 in.+1 in.=2×1 in.=2 in., or 2 times 1 in. is 2 in.
2 in.+2 in.=2×2 in.=4 in., or 2 times 2 in. are 4 in.
3 in.+3 in.=2×3 in.=6 in., or 2 times 3 in. are 6 in.; etc.

Then, 2×1=2;
2×2=4; proceeding from the concrete to the abstract.

1 in.+1 in.+1 in.+1 in.=4×1 in.=4 in., or 4 times 1 in. is 4 in.
2 in.+2 in.+2 in.+2 in.=4×2 in.=8 in., or 4 times 2 in. are 8 in.
3 in.+3 in.+3 in.+3 in.=4×3 in.=12 in., or 4 times 3 in. are 12 in.

Then, 4×1=4;
4×2=8, etc.

Thus it will be seen that multiplication is a short method of doing addition when the addends are all the same.

It will also be understood why *the multiplier is always an abstract number*.

Fix the table by much drill and as soon as a few of the facts are learned begin written multiplication.

Drill Exercises.

(1) Describe a circle; place the multiplier in the centre and round the circumference range the figures, including 0, in any order. Have the pupils tell the product as a figure is pointed to.

(2) Write the figures in any order in a row upon the blackboard, and placing the multiplier above, have the product given as a figure is pointed to.

(3) Write the figures as in (2) and have a pupil begin at one end to state the successive products to the other.

NOTE.—When a pupil fails, he should discover the correct product by using addition.

II. Written Multiplication.

(a) Short Multiplication.

(1) Where there is no carrying.

Show how to place the multiplicand and multiplier and how to proceed.

Thus, John and James have each 243 marbles. How many have they together?

Marbles	Marbles
243	243
243	2
486	486

(2) Where there is carrying.

Show by easy examples the connection between carrying in addition and in multiplication.

Thus,		
	756	756
	756	2
	1512	1512

Give (1) examples; (2) problems.

In problems the pupil should learn (1) why the multiplicand may be any kind of a quantity; (2) why the multiplier must always be an abstract number (so many times); and (3) why the product must be of the same denomination as the multiplicand.

This will save both teacher and pupil from blundering later on by talking of such impossible operations as multiplying length by width and producing something different from either multiplicand or multiplier.

(b) Multiplication by Factors.

This must now be introduced to lead to Long Multiplication.

(1) Teach the term, Factor.

Use the pupil's knowledge of the multiplication table and place on the blackboard such products as

$3\times7=21$	3 and 7 are the factors of 21;
$4\times9=36$	4 " 9 " " " " 36;
$7\times5=35$	7 " 5 " " " " 35; etc.

(2) By using numbers not greater that 12 show

(i) That multiplying by a number and by the factors of the number produce the same results.

Thus,				
	17	17	17	17
	12	3	4	6
	204	51	68	102
		4	3	2
		204	204	204

(ii) That it does not matter in which order the factors are used.

(iii) That when a number can be resolved into several sets of factors it does not matter which set is used.

In connection with this give much practice in multiplying by 20, 30, 40, etc., 200, 300, 400, etc., 2000, 3000, 4000, etc.

Show that the convenient factors are 10 and 2; 10 and 3; 10 and 4; etc.; 100 and 2; 100 and 3; 100 and 4; etc.; 1000 and 2, etc.

(c) Long Multiplication.

Illustrate the principle of long multiplication, viz.: Multiplying the several parts of the multiplicand by the several parts of the multiplier, multiplies the aggregate multiplicand by the multiplier.

Thus the pupil will see there is a reason for what he is doing. He should not be expected to fully reproduce it.

$$\begin{array}{rl} 8 &= 5+3 \\ \underline{9} &= \underline{2+7} \\ 72 &= 2\times5+2\times3+7\times5+7\times3. \end{array}$$

Multiply 362 by 23.

$$23=20+3.$$

The pupil can multiply by 3 and by 20 and, hence, by 23.

$$\begin{array}{rlrl} 362 & & & \\ \underline{23} & & & \\ 1086 &= & 3 & \text{times } 362. \\ \underline{7240} &= & \underline{20} & \text{“} \quad 362. \\ 8326 &= & 23 & \text{“} \quad 362. \end{array}$$

Multiply 457 by 205.

$$205=200+5.$$

$$\begin{array}{rlrl} 457 & & & \\ \underline{205} & & & \\ 2285 &= & 5 & \text{times } 457. \\ \underline{91400} &= & \underline{200} & \text{“} \quad 457. \\ 93685 &= & 205 & \text{“} \quad 457. \end{array}$$

The uselessness of putting down the 0's will soon be observed and it will be noticed that the correct result is found by placing the first figure of each partial product under that figure of the multiplier by which it is obtained.

NOTE.—As accuracy and rapidity are closely associated, much drill should be given in doing a certain amount of multiplication in a given time. A number, as 768594, may be taken and the pupil required to multiply successively by a given multiplier (as multiply by 2, the resulting product by 2, the new product by 2, etc.) as often as possible in the specified time, as one minute; or a given number may be multiplied successively by a given multiplier a fixed number of times and the time required to do this observed.

In connection with problems, care must be taken to see that the correct multiplier is used.

Thus, find the cost of 7 hats @ \$3 each.

The pupil must understand (1) that 7 is the multiplier and (2) why 7 hats should cost 7 times as much as 1 hat. This step is often neglected.

Sometimes the multiplier is large and the multiplicand small when the above solution is used, hence another is suggested.

Find the cost of 7896 articles at \$5 each.

Cost of 7896 articles at \$1 each=\$7896;

∴ " 7896 " " \$5 " =5 times \$7896=\$39480.

III. Proof of Multiplication.

(1) Until division is reached the only proof possible is to multiply the multiplier by the multiplicand. If the two products are the same the work is likely correct.

(2) After division is known two other methods of proof may be used:

(*a*) Divide the product by the multiplicand. If the quotient is the same as the multiplier the work is likely correct.

This method has the advantage of locating the error, should there be one.

(*b*) *Casting out the nines.*

Cast the nines out of the multiplicand and out of the multiplier, multiply the remainders together and cast the nines out of the product. The remainder thus obtained should be the same as that from casting the nines out of the product of the multiplicand and multiplier. Observe this method fails:

(i) If the order of figures in the product is misplaced, as 375 for 735.

(ii) If errors are made which counterbalance each other as 259 for 619, the remainder in each case being 7.

(iii) If 9 is written for 0, or 0 for 9, or if either is omitted or inserted too often.

IV. Mechanical Work in Multiplication. F. more [illegible]

In giving mechanical work from the blackboard the teacher should make the finding of the answer easy for himself. Short "cuts" are good for the teacher, not for the pupil.

A few of these are suggested here.

1. (*a*) $53 \times 53 = 50 \times 56 + 3^2 = 2809$.

   (*b*) $78 \times 78 = 80 \times 76 + 2^2 = 6084$.

   (*c*) $793 \times 793 = 800 \times 786 + 7^2 = 628849$.

2. (*a*) $46\times44=50\times40+6\times4=2024$.
   (*b*) $123\times127=130\times120+3\times7=15621$.
   (*c*) $152\times158=160\times150+2\times8=24016$.

Notice the number of tens is the same in both multiplier and multiplicand and that the sum of the units of the multiplier and multiplicand is ten.

3. Use the Arithmetical complement.

The arithmetical complement of a number is the difference between the number and the unit of the next superior order.

Thus, the A.C. of 6 is 10—6, or 4; of 78 is 100—78, or 22; of 584 is 1000—584, or 416.

Multiply 9984 by 9996.

9984; A.C. is 16	
9996; A.C. is 4	
99800064	0064

The A.C. of the multiplicand and multiplier are multiplied together and as many figures are written in the product by placing naughts to the left as will make the number of places equal to the number of figures in the multiplicand or multiplier. Set down this product. Then the A.C. of the multiplier is subtracted from the multiplicand or vice versa and the remainder is written to the left of the figures already placed.

NOTE.—(1) The multiplier and multiplicand should have the same number of figures.

(2) For convenience numbers with small A.C.'s should be used.

V. Contractions in Multiplication.

It is recommended that practice in such contractions as the following be given *advanced* students.

Such exercises are valuable in that:—

(1) They facilitate results.

(2) They give pupils a clearer conception of the nature of the multiplication process.

(3) They furnish excellent practice to the pupils in determining the relations between numbers.

1. To multiply by such numbers as 999, 798, 9998, etc.

Ex. 1. Multiply 734564 by 999.

734564000 =	1000	times	the	multiplicand.
734564 =	1	"	"	"
733829436 =	999	"	"	"

Ex. 2. Multiply 587469 by 798.

469975200 =	800	times	the	multiplicand.
1174938 =	2	"	"	"
468800262 =	798	"	"	"

II. To multiply in a given number of lines of partial products.

Ex. 3. Multiply 785649 by 369 in two lines of partial products.

```
   785649
    (36)9
  7070841  =  9 times the multiplicand.
 28283364  = 40  "  9 times, or 360 times the multiplicand.
289904481  =369  "  the multiplicand.
```

Ex. 4. Multiply 81234567 by 64432 in three lines of products.

```
   81234567
   (64)(4)(32)
  324938268     =   400 times the multiplicand.
   2599506144   =     8  "  4 times, or 32 times the multiplicand.
5199012288      =  2000  "  32, or 64000 times the multiplicand.
5234105620944   = 64432  "  the multiplicand.
```

III. To multiply by 5, 25, 125.

(1) To multiply by 5, multiply by 10 and divide the product by 2. $5=10\div2$.

(2) To multiply by 25, multiply by 100 and divide the product by 4. $25=100\div4$.

(3) To multiply by 125, multiply by 1000 and divide the product by 8. $125=1000\div8$.

VII. DIVISION.

I. Oral Division, or the Division Table.

The Division Table is the converse of the Multiplication Table and can be learned from it.

The pupil should discover similar answers to the following questions in connection with each product in the multiplication table:

(1) How often can 2 be taken from 6?

(2) How many 2's are there in 6?

(3) How many times 2 make 6?

(4) How often is 2 contained in 6?

(5) One of the factors of 6 is 2, what is the other?

(6) Divide 6 into 2 equal parts.

(7) What is the half of 6?

Suggestions for Drill.

(*a*) Describe a circle. Place the divisor in the centre, and round the circumference place numbers up to 12 times the divisor. Point to these and let the pupil tell instantly the quotient and

remainder. If he cannot tell, he should discover the required answer by using objects, or number-pictures.

(*b*) Place the digits in any order upon the blackboard with the divisor above, as

2

6, 8, 7, 3, 5, 4, 8, 9, 0, 2, 7, 6, 7, 9, 8.

Point to a digit and have a pupil divide. If there is a remainder place it to the left of the next digit pointed to and divide the resulting number, etc.

NOTE.—During the learning of the division table the pupils should gradually discover (1) that division is the process by which having given a product and one factor the other may be found; and (2) that Division is a short process of performing several successive subtractions of the same number.

Let the pupils discover the latter by working with objects such problems as, Among how many boys may 30 marbles be divided, giving each boy 5 marbles?

30	marbles	
5	"	Taking 5 marbles and giving them to the first boy,
25	"	there are 25 remaining.
5	"	Taking 5 of these and giving them to the second boy,
20	"	there are 20 marbles remaining.
5	"	Taking 5 of these and giving them to the third boy,
15	"	there are 15 marbles remaining.
5	"	Taking 5 of these and giving them to the fourth boy,
10	"	there are 10 marbles remaining.
5	"	Taking 5 of these and giving them to the fifth boy,
5	"	there are 5 marbles remaining.
5	"	Taking these and giving them to the sixth boy,
0	"	there are none remaining.

Hence, 30 marbles may be divided among 6 boys, giving to each boy 5 marbles.

If 30 marbles are divided equally among 6 boys, how many does each boy receive?

30	marbles	
6	"	Taking 6 marbles and giving each boy 1,
24	"	there are 24 remaining.
6	"	Taking 6 of these and giving each boy 1,
18	"	there are 18 remaining and each boy has 2.
6	"	Taking 6 of these and giving each boy 1,
12	"	there are 12 remaining and each boy has 3.
6	"	Taking 6 of these and giving each boy 1,
6	"	there are 6 remaining and each boy has 4.
6	"	Taking these 6 and giving each boy 1,
0	"	there are none remaining and each boy has 5.

II. Short Division by the Long Division Method.

Divide 68 splints equally between 2 boys.

Let the pupils discover how many each boy will get by dividing 6 bundles of ten and 8 splints.

Then show the figuring on the blackboard, thus,

```
      Splints.  Splints.
2)      68       (34
        6
        -
         8
         8
         -
```

Divide 72 splints equally between 2 boys.

As before, the pupils have 7 bundles and 2 splints. The half of 6 bundles is readily found and after a time they find they must untie the bundle and then they have 12 splints, which they can easily divide into 2 equal parts.

Show the figuring:

```
      Splints.  Splints.
2)      72       (36
        6
        --
        12
        12
        --
```

III. Short Division by the Usual Method.

NOTE.—In working problems pupils should learn that there are two kinds of exercises, viz.: (1) those in which the divisor is an abstract number and (2) those in which the divisor is of the same name as the dividend.

In each of these cases they should be carefully trained to know the nature of the quotient and the remainder.

Thus, when the divisor is an abstract number, the quotient and remainder are always like the dividend. When the divisor is of the same name as the dividend, the quotient is always so many *times* and as in (1) the remainder is of the same name as the dividend.

IV. Long Division.

Pupils should be trained in the following steps until they are automatic:—

(1) See how many times the divisor is contained in the part of the dividend to be considered.

(2) Place the figure in the quotient.

(3) Multiply the divisor by this number and place the product under the part of the dividend under consideration.

(4) Subtract the product from this part of the dividend.

(5) Bring down the next figure in the dividend.

After a pupil knows these steps there still remain two difficulties to be overcome:—

(*a*) To tell the quotient figure.

(*b*) To supply a 0 in the quotient when the divisor is not contained in the part of the dividend to be divided.

To overcome (*a*) let the pupils form the multiplication table for the given divisor. Thus, suppose the divisor is 19, let them put down 19 times.

$19\times1=19$
$19\times2=38$
$19\times3=57$
$19\times4=76$
$19\times5=95$
$19\times6=114$
$19\times7=133$
$19\times8=152$
$19\times9=171$

It will be an easy matter to divide correctly by 19.

After a time this help should be withdrawn and examples with divisors like 51, 61, 401 given for working, gradually increasing in difficulty as in Exercise XLIX.

To overcome (*b*) let the pupils go through the steps of Long Division just enumerated. This difficulty arises from attempting to crowd too much upon the pupil before he is ready for it.

Thus, divide 856905 by 21.

```
21)856905(40805
   84
   --
    16
    00
    --
    169
    168
    ---
      10
      00
      --
      105
      105
      ---
```

After a time the pupils will see the uselessness of setting down the product by 0 and will shorten the work in the usual way.

V. Division by Factors. For advantages see p.315.

(*a*) Division when there are two factors and no remainder.

(*b*) Division when there are two factors and a remainder with the first only.

(*c*) Division when there are two factors and remainders with each.

(*d*) Division when there are three or more factors and remainders with any or all the divisors.

Divide 132 by 12.

```
12)132      4)132      3)132      6)132
   ---        ---        ---        ---
    11       3)33       4)44       2)22
               --         --         --
               11         11         11
```

Working an example in this way shows

(1) A number is divided by another when it is divided by one factor of the divisor and the resulting quotient divided by the other;

(2) Either factor may be used as the first divisor;

(3) When a divisor can be resolved into several sets of factors, it matters not which set is used.

Ex. 1. How many packages of 15 lb. each can be made of 578 lb. of sugar, and how many pounds will remain?

The working of such by factors depends on the principle that the remainder is always of the same kind as the dividend.

Pounds.
3)578
5)192 bags of 3 lb. each and 2 lb. remaining;
38 packages of 15 lb. each and 2 bags of 3 lb. each remaining;
Hence, the remainder is $(2\times3+2)$ lb., or 8 lb.

As a first step put the sugar into bags of 3 lb. each. Now show that 5 of these must be put together to make a package of 15 lb.

The work should be shown to be correct by means of Long Division.

VI. Methods of Proof.

(*a*) Multiply the Divisor and Quotient together and to the product add the Remainder; the sum should equal the Dividend if the work is correct.

(*b*) Subtract the Remainder from the Dividend and divide the result by the Quotient; the result should equal the Divisor if the work is correct.

The following are suggested as TESTS of the THEORY of the Simple Rules:

Addition.

987
27684
384569
****** 785097
57694
1256031

2487
379
22383
17409
7461
942573
243
379)942816(2487
758
1848
1516
3321
3032
2896
2653
243

Subtraction.

******* 8436218
964847
7471371

or,

2876496
****** 485657
2390839

Division.

379)******(2487 or **)82031(976 84

243 over.

47 over.

3708)28157041(**** 7593
***** 25956
22010
***** 18540
34704
***** 33372
13321
***** 11124
2197

Multiplication.

75649
3×70
529543
******* 378245
226947
27006693

or ****** 780907
5687
******* 5466349
******* 6247256
******* 4685442
******* 3904535
4441018109

359,999,999,99
599,999
7999...
= 4 ...,199,999

6|00)416|70
ciphers on right of div.

VIII. PROBLEMS.

Experience shows that the chief difficulties in solving problems arise from the following causes:—

(i) Pupils do not understand the nature of the transaction involved in the problem.

(ii) They are unable to analyze the given conditions and so determine their relations to the quantity to be found.

(iii) They have an imperfect understanding of the nature of the four simple operations of Arithmetic.

Hence, the steps in teaching the solution of a problem are as follows:

(i) The pupil must acquire as clear an understanding of the nature of the transaction as possible. This should be illustrated by the teacher in the concrete. Failure to do this makes much of the commercial arithmetic incomprehensible, more especially to girls.

(ii) The problem must be analyzed to connect what is given with what is to be found.

When pupils understand the transaction involved, and by questions have been made to reason out the connection between the statement and the demand, they should be left to find a way of solution for themselves.

Many teachers fail at this point; for not satisfied with leading the pupil to the difficulty, they teach the solution as well. *In fact much of the questioning is often on the solution rather than on the reasoning.*

A pupil who understands the four fundamental operations and who is trained to be accurate in his mechanical work will have little difficulty in reaching a correct result.

Mechanical accuracy should be insisted upon. It should be felt to be a disgrace to make a mistake in addition, subtraction, etc. A pupil should be held responsible for every figure he sets down.

Suggestions.

(i) It is a good plan to train pupils to state the supposition or datum by itself and underneath it to write the demand.

(ii) In the case of "Rule of Three" problems the supposition should be written so as to bring the number of the same name as the answer required, at the end of the statement.

Then the pupil should reason from the number given to unity and from unity to the number required.

When he is able, he should reason at once from the number given to the number required.

(iii) When teaching, the problems should be classified; when testing, miscellaneous problems should be used.

(iv) In giving problems the teacher should have a definite end in view. This requires forethought on his part.

(v) All work should be done neatly and intelligibly.

(vi) Pupils should be taught to examine their answer to see if it is a reasonable one.

(viii) Questions occurring in practical life are often of great interest to pupils.

(ix) Pupils should be exercised in making problems; *i.e.* Problem-making should accompany problem-solving.

Ex. 1. Find the cost of 7 barrels of flour if \$30 pay for 5 barrels.

Solution:—Cost of 5 barrels=\$30; (*a*)
" " 1 barrel =\$30÷5 =\$6; (*b*)
" " 7 barrels=7 times \$6=\$42. (*c*)

The reasoning from (*a*) to (*b*) is 1 bbl. is $\frac{1}{5}$ of 5 bbl.; hence, the price of 1 bbl. will be $\frac{1}{5}$ of the price of 5 bbl.

Similarly, the reasoning from (*b*) to (*c*) is 7 bbl. is 7 times 1 bbl.; hence, the price of 7 bbl. will be 7 times the price of 1 bbl.

Ex. 2. A building-lot 30 ft. in frontage by 120 ft. deep is sold at \$100 per foot frontage. How much is that per acre?

Before such a problem is given, pupils should understand square measure and the finding of the area of a surface. It should not be necessary to teach these when the problem is given.

The new feature in this problem is the method of measuring city property. *The explanation of this should precede the solution*, a diagram being placed on the board to aid the explanation.

After this explanation the class should have an opportunity to work the problem without further aid.

For those unable to work it, such questions as the following will be found helpful:—

Show a foot frontage on the diagram. How deep is the lot?

What is the area of a foot frontage carried back to the depth of the lot? ANS. 120 sq. ft.

What is the value of this much land? ANS. \$100.

What is required to be found?

How many sq. ft. are there in an acre? ANS. (4840×9) sq. ft.

Solution:—Area of 1 ft. frontage=120 sq. ft.;

" " 1 a. =(4840×9) sq. ft.;

Cost of 120 sq. ft.=\$100;

$\therefore$ " " (4840×9) " $=\frac{4840\times 9}{120}$ of \$100=\$36300.

The reasoning in this case is (4840×9) sq. ft. is $\frac{4840\times 9}{120}$ times 120 sq. ft., hence the cost of (4840×9) sq. ft. is $\frac{4840\times 9}{120}$ times the cost of 120 sq. ft.

Mistakes.

(i) It is a mistake to conduct the work as if getting a solution and finding the answer were the main purpose in the solution of problems.

This accounts for pupils *being taught the solution of type* problems and being required to remember these as so many facts.

(ii) It is a mistake to divert attention to the merely mechanical operations involved.

(iii) It is a mistake to allow the "unitary method" to be mechanically used. Pupils must be required to reason at each step.

For further exemplification of solutions see Part II.

IX. REDUCTION AND THE COMPOUND RULES.

TABLES.

By using proper appliances, pupils should be led to discover most of the facts of the tables. Thus, they can discover that 2 pints make 1 quart; 4 quarts make 1 gallon; etc.; 12 inches make 1 foot; 3 feet make 1 yard; etc.; 144 square inches make 1 square foot; etc.; 1728 cubic inches make 1 cubic foot; etc.

During the learning of the tables many simple problems should be worked, nothing being said of Reduction, as

Ex. 1. How many pints are there in 5 qt.?

1 qt.=2 pt.;

But 5 qt.=5 times 1 qt.;

Hence, 5 qt.=5 times 2 pt., or 10 pt.

Similarly, How many pints in 2 qt.? in 7 qt.? in 12 qt.? etc.

Ex. 2. How many pints are there in 3 qt. 1 pt.?

1 qt.=2 pt.;

3 qt.=3 times 1 qt.;

∴ 3 qt.=3 times 2 pt., or 6 pt.;

∴ 3 qt. 1 pt.=6 pt.+1 pt., or 7 pt.

Similarly, how many pints in 4 qt. 1 pt.? in 7 qt. 1 pt.? in 10 qt. 1 pt.? etc.

Ex. 3. How many quarts are there in 12 pt.?

2 pt.=1 qt.;

2 pt.)12 pt.

6 times;

∴ 12 pt.=6 times 1 qt.=6 qt.

Similarly, how many quarts in 8 pt.? in 16 pt.? in 30 pt.?

Ex. 4. How many quarts are there in 7 pt.?

2 pt.=1 qt.;

2 pt.)7 pt.

3 times and 1 pt. over;

∴ 7 pt.=3 times 1 qt.+1 pt.=3 qt. 1 pt.

Similarly, how many quarts are there in 13 pt.? in 17 pt.? in 23 pt.?

SURFACE MEASURE.

1. Definition of Surface.

By having the pupils pass their hands over the outside of such objects as boxes, cylinders, spheres, sheets of paper, etc., their minds are prepared for the term *surface*. Have many *surfaces* now pointed out and touched.

2. Dimensions of Surface.

Show that each surface has length and breadth, or height and length, etc., but never length, breadth and thickness; also that the boundaries of a surface are lines.

3. Shapes of Surfaces.

Teach the terms rectangle, or oblong, and square. State that surfaces are measured in squares. Teach the terms, square inch, square foot, square yard.

4. The Table.

Have pupils draw a square foot and mark it off into square inches. When they have found the number of square inches in a square foot, give easy problems to utilize the information they have gained.

Treat the square yard similarly.

Have the pupils measure a square rod under your directions. Then have each draw a plan of the square rod, using an inch to represent a yard. Mark off the plan into square inches each of which will thus represent a square yard. It will be readily seen that there are 25 such squares and 10 oblongs, each representing one-half a square yard and 1 square half as large as one of the oblongs, making $30\frac{1}{4}$ square inches in all and representing $30\frac{1}{4}$ square yards in 1 square rod.

In a convenient place have the pupils measure 16 rd. by 10 rd.; tell them this is an acre; and have them draw a plan of this with 1 inch representing 1 rod. By marking the plan off into square inches, they will discover that 160 square rods make 1 acre.

CUBIC MEASURE.

1. Meaning of Solid.

Show solids and then withdraw them. Pupils will observe that objects take up room, or space. The term solid is given, and pupils give a definition. "A solid is anything that takes up, or occupies, space." Have pupils give examples.

2. Dimensions of Solids.

Show that solids are bounded by surfaces and that they have length, breadth, and thickness.

3. Volume and How it is Measured.

The amount of space an object occupies is called its volume. Teach the terms rectangular solid and cube. Tell pupils the volume of a solid is measured in cubes and teach the terms cubic inch and cubic foot. Let the pupils discover the number of inch-slices the cubic foot will make, the number of rows of cubic inches, and the number of cubic inches in each row of the slice. They will thus be able to find the number of cubic inches in the 12 slices.

Give easy problems to utilize the information acquired.

Teach the cubic yard in a similar manner.

DENOMINATE QUANTITIES; DENOMINATIONS.

\$6, \$9, \$2.75 are quantities that express value.

4 oz., 7 lb., 7 lb. 3 oz. are quantities that express weight.

3 hr., 7 min., 4 hr. 8 min. are quantities that express time.

All these are called Denominate Quantities.

Again,

In 6c. the unit is 1c.

In 4 oz. the unit is 1 oz.

In 5 mi. the unit is 1 mi.

These are Simple Denominate Quantities because the Denominate Quantity is expressed in one kind of unit.

The unit in which a *Simple Denominate Quantity* is expressed is called its DENOMINATION.

Again,

In $9.25 the units are $1 and 1c.

In 7 lb. 9 oz. the units are 1 lb. and 1 oz.

In 3 ft. 6 in. the units are 1 ft. and 1 in.

A denominate number expressed in more than one kind of unit is called a Compound Denominate Quantity.

The units in which a *Compound Denominate Quantity* is expressed are called its DENOMINATIONS.

Exercises should be given in determining whether a denominate quantity is simple or compound.

REDUCTION.

(*a*) By means of questions similar to the following, the terms, Reduction, Reduction Descending, Reduction Ascending, should be taught:

(1) In $9, how many cents are there?
In 4 lb., how many ounces are there?
In 3 lb. 7 oz., how many ounces are there?

(2) In 700 cts., how many dollars are there?
In 32 oz., how many pounds are there?
In 45 oz., how many pounds are there?

(*b*) When examples in written reduction are reached, the pupil should be required to write out the work in full for a time. When the process is understood he should be shown the abbreviated form in ordinary use and required to compare the new form, step by step, with the one used at first.

Ex. 1. Reduce 5 bu. 3 pk. 1 gal. 2 qt. to quarts.

1 bu.=4 pk.;
5 bu.=5 times 4 pk. =20 pk.;
20 pk.+3 pk.=23 pk.;
1 pk.=2 gal.;
23 pk.=23 times 2 gal.=46 gal.;
46 gal.+1 gal.=47 gal.;
1 gal.=4 qt.;
47 gal.=47 times 4 qt.=188 qt.;
188 qt.+2 qt.=190 qt.

5 bu. 3 pk. 1 gal. 2 qt.
4 pk.
23 pk.
2 gal.
47 gal.
4 qt.
190 qt.

In giving an oral explanation of this, the pupil should be required to give a full statement, thus, 1 bu. is 4 pk.; 5 bu. are 5 times 4 pk., or 20 pk.; 20 pk. plus 3 pk. are 23 pk.; etc.

It should be quite clear to the pupil that 4 pk., 2 gal., 4 qt. are the multiplicands and the respective multipliers are 5, 23, and 47, *abstract* numbers, not denominate ones.

If thought desirable, the "Law of Commutation" may be used. Thus,

4 times 5 pk. are the same as 5 times 4 pk.;
2 " 23 gal. " " " " 23 " 2 gal.;
4 " 47 qt. " " " " 47 " 4 qt.

(*c*) In Reduction Ascending similar steps should be followed.

Ex. 2. Reduce 317 qt. to bushels.

(i) 4 qt. = 1 gal.;
4 qt.)317 qt.
79 times and 1 qt. over.
∴ 317 qt. = 79 times 1 gal. + 1 qt. = 79 gal. 1 qt.
2 gal. = 1 pk.;
2 gal.)79 gal.
39 times and 1 gal. over.
∴ 79 gal. = 39 times 1 pk. + 1 gal. = 39 pk. 1 gal.
∴ 317 qt. = 39 pk. 1 gal. 1 qt., etc.

(ii) 317 qt. = $\frac{317}{4}$ gal. = 79 gal. 1 qt.;
79 gal. = $\frac{79}{2}$ pk. = 39 pk. 1 gal., etc.

(iii) 4 qt.)317 qt.
2 gal.)79 gal. 1 qt.
39 pk. 1 gal., etc.

In giving an oral explanation of this, the pupil should give a full statement, thus, 4 qt. make 1 gal., hence 317 qt. will make as many gallons as the number of times 317 qt. contains 4 qt., etc.

COMPOUND ADDITION.

I. Review Simple Addition.

H.	T.	U.
7	5	8
9	7	5
6	8	9
9	9	9
31	29	31

H.	T.	U.
7	5	8
9	7	5
6	8	9
9	9	9
34	2	1

By questions cause the pupil to discover why he sets down 1 in the units' place and carries 3 to the tens; etc.

II. Addition of Compound Quantities.

£.	S.	D.
7	5	8
9	7	5
6	8	9
9	9	9
31	29	31

£.	S.	D.
7	5	8
9	7	5
6	8	9
9	9	9
32	11	7

Make it clear that the only difference between Simple and Compound Addition is in the fact that the number of units making 1 of the next higher order is constant in Simple Addition, being always ten, while in Compound addition, this number varies. This can be well done by using the same numbers in Simple and Compound Addition.

COMPOUND SUBTRACTION.

The steps in Compound Subtraction are similar to those in Compound Addition.

Whatever method of subtraction is used in Simple Subtraction, the same method should be used in Compound Subtraction. In giving the first lesson, the same numbers should be used in Simple and Compound Subtraction.

COMPOUND MULTIPLICATION.

I. Review Simple Multiplication.

H.	T.	U.
7	5	9
		7
49	35	63

H.	T.	U.
7	5	9
		7
53	1	3

The pupil should be able to explain why he sets down 3 in the units' place and carries 6 to the tens; etc.

II. Multiplication of Compound Quantities.

£.	S.	D.
7	5	9
		7
49	35	63

£.	S.	D.
7	5	9
		7
51	0	3

In giving the first lesson, care should be taken to use the same numbers as multiplicand and multiplier in both Simple and Compound Multiplication.

COMPOUND DIVISION.

(A). WHEN THE DIVISOR IS AN ABSTRACT NUMBER.

I. Review Simple Division.

	H.	T.	U.
2)	9	7	6
	4	8	8

The pupil must be able to explain how he gets 4 hundreds, 8 tens and 8 units.

II. Division of a Compound Quantity.

	£.	S.	D.
2)	9	7	6
	4	13	9

(B). WHEN THE DIVISOR IS A DENOMINATE NUMBER.

Introduce this by problems such as the following:

1. How many yards of cloth at 2s. a yard can be bought for 20 s.?

2. How many yards of cloth at 2s. a yard can be bought for £1?

What step had to be taken in the latter case which was not taken in the former?

3. How often can you subtract 5s. from £2?

How often does £2 contain 5s.?

Explain the steps by which £2 is divided by 5s.

4. How often does £2 5s. contain 9s.?

Explain the steps in the solution.

Continue such examples until it becomes clear that to divide one denominate number by another, each must be reduced to the lowest denomination mentioned in either.

X. SIMPLE APPLICATIONS OF THE PREVIOUS RULES.

BILLS AND ACCOUNTS.

In teaching this subject, a bill neatly written and perfect in all its details should be on the blackboard.

The pupils should be questioned to cause them to discover the following:

(*a*) What is shown in the bill.

(*b*) Why each item of detail is shown.

(*c*) The arrangement of the various parts.

(*d*) The effect of omitting any detail.

After such a bill has been carefully studied, and the details tabulated, the pupils should be given practice in ruling bills, filling in details, etc.

The common mistake in teaching a bill is in attempting to build up a bill, *i.e.*, in teaching synthetically instead of analytically.

SIMPLE MEASUREMENTS.

THE RECTANGLE.

Case I. Given the length and width of a rectangle to find its area.

Pupils should draw rectangles of various dimensions. These should be subdivided into squares according to the unit of area selected. The pupils will thus learn that the unit of area can be placed along one side of the rectangle as often as there are corresponding linear units of measure in that side and that there are as many rows of such square units of area as there are corresponding linear units in the adjacent side of the rectangle.

Ex. 1. Find the area of a rectangle 1 ft. 4 in. long by 1 ft. wide. Let 1 sq. in. be the unit of area.

Length of rectangle = 16 in.;
Width " " = 12 in.;
Area of rectangle 1 in. long by 1 in. wide = 1 sq. in.;
" " " 16 in. " " 1 in. " $= 16 \times 1$ sq. in.;
" " " 16 in. " " 12 in. " $= 12 \times 16 \times 1$ sq. in.
$= 192$ sq. in.

A number of examples worked by the pupils by means of diagrams will show them that *the area of a rectangle is found by multiplying the number of units of length by the number of like units of width to obtain the number of corresponding units of area.*

Case II. Given the area of a rectangle and one of its dimensions to find the other.

As in Case I the pupil should be required to draw a rectangle and place along the side whose length is given as many square units of area as there are corresponding linear units of length. It will then be an easy matter to find how many such rows must be taken to form a rectangle as large as the given one. Thus the number of linear units of measurement in this dimension will be found.

Ex. 1. A rectangular floor contains 486 sq. ft. It is 27 ft. long, find its breadth.

Number of square units along the length = 27;
" " rows to make 486 sq. ft. $= 486 \div 27 = 18$.
Hence, the breadth is 18 ft.

Work a number of similar examples and the rule will follow:

Divide the number of units of area by the number of corresponding units of length or breadth and the quotient will be the number of like linear units in the side to be found.

Mistakes.

(*a*) In Case I the most common mistake is in teaching that *the length is multiplied by the width.*

One who talks of multiplying length and width together does not understand the nature of multiplication.

(*b*) In Case II the usual mistake is *to divide the area by the given dimension.* The foundation of this error lies in not understanding the nature of division. It is impossible to divide square units by linear units.

(*c*) Another mistake consists in giving a mechanical solution as follows:—The area is the product of two numbers. One of these is given. How is the other found? This is purely mechanical as it does not compel the pupil to analyze the problem and give an adequate reason for each step.

BOARD MEASURE.

To teach Board Measure, have several boards a foot square and of various thickness, as $\frac{1}{4}$ in., $\frac{1}{2}$ in., 1 in., 2 in., 3 in., etc.

Explain that the boards $\frac{1}{4}$ in. thick, $\frac{1}{2}$ in. thick, and 1 in. thick are all called *one board foot;* that the 2 in. one is 2 board feet; the 3 in. one is 3 board ft.; etc.

RECTANGULAR SOLIDS.

Case I. Given the length, width and thickness of a rectangular solid to find the cubic content.

The pupils should be required to draw rectangular solids of various dimensions. These should be subdivided into cubes according to the unit of cubic content selected.

Ex. 1. How many c. ft. of air are there in a rectangular room 18 ft. by 16 ft. and 10 ft. high?

No. of c. ft. which can be placed on length $=18$ c. ft.;
" " " " " on floor $=(16\times 18)$ c. ft.;
" " " " " in room $=(10\times 16\times 18)$ c. ft.
$=2880$ c. ft.

A number of examples in which the pupils are required to draw and subdivide the solid and set down a solution will show them that *the cubic content of a rectangular solid is found by multiplying*

the number of units of length by the number of like units of width and this result by the number of like units of thickness to obtain the number of corresponding units of solid content.

Case II. Given the cubic content of a rectangular solid and two of its dimensions to find the other.

As in Case I the pupils should be required to draw a rectangular solid and place a layer of cubic units corresponding to the measurements given along the side of the solid whose dimensions are given. It will then be an easy matter to find how many such layers are necessary to form the solid.

Ex. 1. There are 60 cords of wood in a pile 8 ft. wide and 12 ft. high. How long is the pile?

No. of c. ft. in the pile of wood$=(60\times128)$ c. ft.;

No. of c. ft. which can be placed on one end$=(8\times12)$ c. ft.;

No. of layers to make (60×128) c. ft.$=\frac{60\times128}{8\times12}=80$.

Hence, length of the pile is 80 ft.

Mistakes.

The following mistakes are often made:—

(*a*) Pupils are taught to multiply length, breadth and thickness together.

(*b*) The cubic content is divided by the area of one side and the quotient is called the third dimension.

(*c*) A solution is given which, although theoretically correct, becomes mechanical, *e.g.*,

The number of units of cubic content is the product of three factors. If this product is divided by the product of any two of them, the quotient is the third factor. A pupil reaches a correct result by this solution but soon loses sight of why it is necessarily correct.

SHARING.

In introducing Sharing, *objects* should be used to aid pupils in forming a conception of what is to be done and how it may be performed. These should not be continued too long. Pupils should be encouraged to "picture" to themselves what they are called on to do.

Case I. Sharing where provision has to be made for some receiving more than others.

Ex. 1. Divide 36 marbles between James and Frank so that Frank may have 10 more than James.

First give Frank 10.

This will leave 26 to be divided equally between Frank and James.

26 marbles÷2=13 marbles;

James' share=13 marbles;

Frank's " =(10+13) marbles=23 marbles.

Show that the shares 13 and 23 fulfil all the conditions of the problem.

It cannot be repeated too often that pupils should be trained to examine their results and find whether they fulfil the conditions required.

Ex. 2. Divide $112 among A, B and C, giving B $7 less than A who is to receive $16 more than C.

C receives the least.

Extra for C=$0;

" " A=$16;

" " B=$(16—7)=$9;

Extra for A and B=$25.

Setting this aside for A and B, the sum left to be divided equally among three is $(112—25), or $87.

$87÷3=$29.

A receives $(29+16), or $45.

B " $(29+9), or $38.

C " $(29+0), or $29.

The pupils should be required to show that these results are correct.

Case II. Sharing where the ratio of the shares is given.

Ex. 3. Divide $100 between James and Frank so that James may have 3 times as many as Frank.

When Frank has $1,

James has $3,

Both have $4.

It is evident that Frank will have as many times $1 and James as many times $3 as the number of times $4 is contained in $100.

$100=25×$4;

∴ Frank's share=$(25×1)=$25;

∴ James' " =$(25×3)=$75.

Ex. 4. Divide \$63 among A, B and C, giving B twice as much as A, and C three times as much as B.

When A gets \$1,
B gets \$2,
and C gets \$6,
out of every \$9.
\$63÷\$9=7 times;
∴ A will get 7 times \$1, or \$7;
B " " 7 " \$2, or \$14;
C " " 7 " \$6, or \$42.

Ex. 5. Divide \$45 between A and B so that B may receive \$3 as often as A receives \$2.

When A receives \$2
B " \$3
out of every \$5
\$45÷\$5=9 times;
A will receive 9 times \$2, or \$18,
and B " " 9 " \$3, or \$27.

Case III. Sharing where Cases I and II are combined.

Ex. 6. A, B, and C together have \$208. B has \$13 more than twice as much as A, and C has \$20 more than four times as much as A. How much has each?

Extra for B=\$13;
" " C=\$20;
Extra for B and C=\$33;
Remainder to be divided=\$(208—33)=\$175;
A has \$1,
B " \$2,
C " \$4,
out of every \$7.
\$175÷\$7=25 times.
A has 25 times \$1, or \$25.
B " 25 " \$2+\$13, or \$63.
C " 25 " \$4+\$20, or \$120.

Ex. 7. \$2.40 is made up of three times as many 5c. pieces and twice as many 10c. pieces as 25c. pieces. How many are there of each?

There are one 25c. piece,
two 10c. pieces,
and three 5c. pieces,
out of every 60c.

$2.40÷60c.=4 times.

Hence, there are 4 times 1 twenty-five cent piece, or 4 twenty-five cent pieces;

4 times 2 ten cent pieces, or 8 ten cent pieces;

and 4 times 3 five cent pieces, or 12 five cent pieces.

NOTE.—The "equational" method may with profit be introduced to more advanced pupils.

Ex. 8. Divide $407 among A, B, and C, so that B may have $20 less than 3 times A's share and C $17 more than twice B's.

Let A receive 1 share,
then B receives 3 shares—$20,
and C " 6 shares—$40+$17;
all receive 10 shares—$60+$17;
∴ 10 shares—$60+$17=$407;
∴ 10 shares=$407+$43;
1 share=$$\frac{450}{10}$=$45.
Sum B receives=3×$45—$20=$115.
" C " =6×$45—$23=$247.

XI. FACTORS, CANCELLATION, MEASURES AND MULTIPLES.

I. FACTORS.

I. Prime and Composite Numbers.

(*a*) Use the pupils' knowledge of factors (see Multiplication) to teach the distinction between Prime and Composite numbers.

Blackboard summary:—

No factors except one and the number;	Other factors than one and the number;
2	4
3	6
5	8
7	9
11	10
13	12
17	14
Prime Numbers.	Composite Numbers.

(*b*) In a similar manner teach the distinction between Odd and Even numbers.

II. To resolve a number into Prime Factors.

Ex. 1. Resolve 120 into prime factors.

2)120 2)60 2)30 3)15 5	Divide the given number by the smallest prime factor other than 1; treat the quotient in a similar manner, and so on until a quotient which is a prime number is obtained.

The divisors and last quotient will be the prime factors of the number.

The question for the pupil is, What is the smallest prime number that is a factor of 120? Then of 60? Then of 30? Then of 15?

(a) Gradually teach the common tests of divisibility.

(1) A number is exactly divisible by 2, if the right hand figure is 0 or indicates an even number.

(2) A number is exactly divisible by 3, if the sum of its digits is exactly divisible by 3.

(3) A number is exactly divisible by 4, if its two right hand figures are zeros or express a number exactly divisible by 4.

(4) A number is exactly divisible by 5, if its right hand figure is 0 or 5.

(5) A number is exactly divisible by 6, if it is even and exactly divisible by 3.

(6) A number is exactly divisible by 8, if its three right hand figures are zeros or express a number exactly divisible by 8.

(7) A number is exactly divisible by 9, if the sum of its digits is exactly divisible by 9.

(8) A number is exactly divisible by 11, if the difference between the sum of the digits in the odd places and the sum of the digits in the even places is either 0 or exactly divisible by 11. Thus, 75691 is exactly divisible by 11, 7+6+1=5+9.

(9) div. by 7. 11. 13 if [illegible] + 1st 4th alike others [illegible]

(b) To find all the divisors of a number.

Ex. Find all the divisors of 360.

(10) div. by 37 if consists [illegible] fig. alike as 333 [illegible] this case the sum of [illegible] is the quotient.

2)360	1, 2, 4, 8
2)180	1, 3, 9
2)90	1, 2, 4, 8, 3, 6, 12, 24, 9, 18, 36, 72
3)45	1, 5
3)15	1, 2, 4, 8, 3, 6, 12, 24, 9, 18, 36, 72,
5	5, 10, 20, 40, 15, 30, 60, 120, 45, 90, 180, 360.

(11) div. by 10.

(12) div. by 25.

(13) div by 125.

Resolve the number into its prime factors.

Form as many series as there are different prime factors, making 1 the first term of each series, the first power of the factor the second term, the second power of that factor the third term, etc. Multiply these series together as shown. The resulting products will be the divisors of the number.

II. CANCELLATION.

Teach the principle on which Cancellation is based, viz. Dividing both divisor and dividend by the same number does not affect the quotient.

Thus, $\frac{72}{12}=\frac{36}{6}=\frac{24}{4}=\frac{18}{3}=\frac{12}{2}=6.$

In cancelling care must be taken to place the uncancelled factor in place of the cancelled number. When this is not done pupils get the idea that 0 replaces the number.

Ex. 1. Divide the continued product of 16, 4 and 5 by the continued product of 8, 2 and 10.

$$\frac{\overset{2}{\cancel{16}}\times\overset{\overset{1}{\cancel{2}}}{\cancel{4}}\times\overset{1}{\cancel{5}}}{\underset{1}{\cancel{8}}\times\underset{1}{\cancel{2}}\times\underset{\underset{1}{\cancel{2}}}{\cancel{10}}}=\frac{2\times1\times1}{1\times1\times1}=2.$$

III. MEASURES.

I. Definitions.

(a) Measure.

Review and teach measure by such problems as the following:—

(1) Find all the units of length which will exactly measure 12 in.; 15 in.; 18 in.; 20 in.; etc.

(2) Find all the units of area which will exactly measure 6 sq. ft.; 16 sq. ft.; 24 sq. ft.; 27 sq. ft.; etc.

(3) Find all the units of value which will exactly measure 25c.; 50c.; 75c.; etc.

(b) Common Measure.

Find all the units of length that will exactly measure 18 ft. and 24 ft.

Blackboard work:

The measures of 18 ft. are 1 ft., 2 ft., 3 ft., 6 ft., 9 ft., 18 ft.

The measures of 24 ft. are 1 ft., 2 ft., 3 ft., 4 ft., 6 ft., 8 ft., 12 ft., 24 ft.

The measures *common* to 18 ft. and 24 ft. are 1 ft., 2 ft., 3 ft., 6 ft.

(c) Greatest Common Measure.

By examples worked as in (*b*) the G.C.M. is seen to be the largest factor common to all the given numbers.

II. To find the G.C.M.

Ex. 1. Find the G.C.M. of 560, 630, and 700.

2)560,	630,	700
$\cancel{2}$)280,	315,	350
$\cancel{2}$)140,	315,	175
$\cancel{2}$)70,	315,	175
5)35,	315,	175
7)7,	63,	35
1	9	5

First find the prime factors of one of the numbers easily factored, as 560.

Then try each factor in turn as a trial factor of the other numbers and strike out such as are not found to be factors of each of the other numbers. The product of all the common divisors will be the G.C.M. required.

NOTE.—The number selected must be resolved into *prime* factors, lest a smaller factor found in one of the other numbers may be overlooked.

In practice the G.C.M. is most readily found by using the following principle:—"The G.C.M. of two numbers must be a factor of their difference or of the difference between a multiple of one of them and the other."

Ex. 2. Find the G.C.M. of 529 and 667.

$667-529=138.$

$138=2\times3\times23.$

By inspection it is seen that neither 2 nor 3 is a common factor. By trial 23 is found to be a factor of 529 and hence must be contained in 667, 6 times more than in 529.

When there are three or more numbers, the G.C.M. is a factor of the *smallest* difference between any two of the numbers.

Ex. 3. Find the G.C.M. of 408, 510, 544, and 595.

The smallest difference is 34.

$34=2\times17.$

2 is seen not to be a common factor.

By trial 17 is found to be a common factor.

With large numbers the work may be set down as follows:

Ex. Find the G.C.M. of 689 and 1573.

689	2	1573
780	4	1378
91	2	195
91	7	182
0		13

The difference between a multiple of 689 and 1573 is 195; the difference between a multiple of 195 and 689 is 91; the difference between a multiple of 91 and 195 is 13; this is contained exactly in 91 and is the G.C.M. of the numbers.

IV. MULTIPLES.

I. Definitions.

(a) Multiple.

Have pupils *multiply* such quantities as 3 in., \$5, 7 successively by 1, 2, 3, 4, 5, etc.

Blackboard work:

Products.	Products.	Products.
1×3 in.= 3 in.	1×\$5=\$5	1×7= 7
2×3 in.= 6 in.	2×\$5=\$10	2×7=14
3×3 in.= 9 in.	3×\$5=\$15	3×7=21
4×3 in.=12 in.	4×\$5=\$20	4×7=28
Multiples.	Multiples.	Multiples.

Have each multiple in the first column measured by 3 in; in the second, by \$5; in the third, by 7.

Thus it is evident that a multiple of a given number may be defined as (1) one or more integral times the given number or (2) as a number which contains the given number exactly.

(b) Common Multiples.

Have pupils write multiples thus:—

The multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, etc.
" " " 3 " 3, 6, 9, 12, 15, 18, etc.

Then have the common multiples selected, viz., 6, 12, 18, etc.

(c) Least Common Multiple.

From among the common multiples of numbers found in (*b*), select the smallest number which is a common multiple of each of the given numbers.

Thus, multiples of 6 are 6, 12, 18, 24, 30, etc.
" " 9 " 9, 18, 27, 36, etc.

The L.C.M. of 6 and 9 is 18.

II. To find the L.C.M.

(*a*) Use the method of resolving the numbers in prime factors.

Ex. 1. Find the L.C.M. of 24, 30 and 35.

24=2×2×2×3.
30=2×3×5.
35=5×7.

To contain 24 the L.C.M. must have the prime factors 2, 2, 2, 3; to contain 30, 2, 3, 5; and to contain 35, 5, 7. Hence, to contain

24, 30, and 35, the L.C.M. must have (2, 2, 2, 3), ($\cancel{2}$, $\cancel{3}$, 5) and ($\cancel{5}$, 7) as factors, or 2, 2, 2, 3, 5, and 7.

L.C.M.$=2\times2\times2\times3\times5\times7=840$.

The pupils should be questioned as to how many times each of the numbers is contained in the L.C.M. and made to see that the quotients are prime to one another.

From a series of examples worked in this way the rule will be derived, viz.: *The L.C.M. of two or more numbers is the product of all the prime factors of the numbers each being taken the greatest number of times it is found as a factor in any of them.*

(*b*) For young pupils the following is a convenient application of the rule:—

Ex. 2. Find the L.C.M. of 15, 18, 24, 40, 50 and 60.

$$\begin{array}{r|rrrrrr}
2 & 15, & 18, & 24, & 40, & 50, & 60 \\ \hline
2 & 15, & 9, & 12, & 20, & 25, & 30 \\ \hline
2 & 15, & 9, & 6, & 10, & 25, & 15 \\ \hline
3 & 15, & 9, & 3, & 5, & 25, & 15 \\ \hline
5 & 5, & 3, & 1, & 5, & 25, & 5 \\ \hline
 & 1, & 3, & 1, & 1, & 5, & 1
\end{array}$$

The numbers are arranged as shown and a prime number that is contained exactly in any two or more is selected and used as a divisor as shown. Then a second prime number is selected and used as a divisor like the first. Thus the work is carried on until there is no prime number greater than one that is contained in any two of the quotients. The product of the divisors and undivided quotients will be the L.C.M. required.

Thus, L.C.M.$=2\times2\times2\times3\times5\times3\times5=1800$.

(*c*) The following method is recommended for entrance pupils.

Ex. 3. Find the L.C.M. of 15, 18, 24, 40, 50, and 60.

$$\begin{array}{r|rrrrrr}
60 & 15, & 18, & 24, & 40, & 50, & 60 \\ \hline
30 & 1, & 3, & 2, & 2, & 5, & 1 \\ \hline
 & 1, & 1, & 1, & 1, & 1, & 1
\end{array}$$

L.C.M.$=60\times30=1800$.

One of the numbers, as 60, having several factors is selected. The G.C.M. of 60 and each of the given numbers is cancelled out of the numbers in succession and all the uncancelled factors and any uncancelled numbers are written in a line. These are then treated in a similar manner unless the L.C.M. can be found by inspection as above. The product of all the numbers selected will be the L.C.M. of the given numbers.

III. Mistakes.

A very common mistake is to speak of one number containing another *evenly* when *exactly* is meant. Thus 35 contains 7 exactly 5 times.

XII. FRACTIONS.

I. DEFINITION, NOTATION AND NUMERATION.

(a) Definition.

There are three steps in teaching the definition:

(1) Division of a unit into equal parts.

(2) Comparison of the parts with one another and with the unit.

(3) A collection of the equal parts.

A fraction is a number and should be treated just like integral numbers. The only difference is that in the integral number, the unit is a whole, as in 7 the unit is 1; in 7-eighths, it is 1-eighth. In a fraction the unit is a *definite* part of the primary unit, as in 3-fourths, the unit is one-fourth of 1; in 5-eighths of a dollar, it is one-eighth of one dollar.

A common mistake is made in speaking of dividing a unit into parts instead of *equal* parts.

Pupils should be required to solve by actual measuring many problems, as find 3-fourths of a foot; 5-sixths of 18 inches; etc.

(b) Notation and Numeration.

As soon as pupils have the notion of fraction, the notation and numeration should be given and explained. The terms, numerator and denominator, should be given and their significance shown. Thus, in $\frac{4}{5}$, the 5 indicates the fractional unit and so gives a name to the number, 4, which is hence called the numberer, or numerator.

The connection between whole and fractional numbers should be impressed by many examples in Addition, Subtraction, Multiplication, and Division of fractions given now. Of course the fractions to be added or subtracted must have a common denominator. Thus,

$\frac{2}{5}+\frac{1}{5}=?$	$\frac{4}{5}-\frac{3}{5}=?$	$2\times\frac{3}{11}=?$	$\frac{6}{7}\div 2=?$
$?+\frac{4}{7}=\frac{6}{7}$	$?-\frac{3}{7}=\frac{2}{7}$	$?\times\frac{4}{9}=\frac{8}{9}$	$\frac{12}{13}\div ?=\frac{6}{13}$
$\frac{2}{7}+?=\frac{6}{7}$	$\frac{5}{7}-?=\frac{4}{7}$	$3\times ?=\frac{12}{13}$	$?\div 2=\frac{4}{9}$

II. REDUCTION.

The terms, proper fraction, mixed number, improper fraction, lowest terms, etc., should be introduced and taught as required.

(a) Reduction of a whole number to a fraction with a given denominator.

Ex. 1. How many ninths are there in 7?

Since 1=9 ninths and

7=7 times 1;

∴ 7=7 times 9 ninths=63 ninths=$\frac{63}{9}$.

(b) Reduction of mixed numbers to improper fractions.

Ex. 2. How many ninths are there in $7\frac{2}{9}$?

The important point here is for the pupil to know that $7\frac{2}{9}$ is $7+\frac{2}{9}$. This should be made clear in teaching the meaning of mixed number by such examples as, Mary bought 7 yards of cloth and then $\frac{2}{9}$ of a yard more. How much did she buy altogether?

Since $7\frac{2}{9}=7+\frac{2}{9}$

and $1=\frac{9}{9}$;

$\therefore\ 7=7\times\frac{9}{9}=\frac{63}{9}$;

$\therefore\ 7\frac{2}{9}=\frac{63}{9}+\frac{2}{9}=\frac{65}{9}$.

(c) Reduction of an improper fraction to a whole or mixed number.

Ex. 3. Express $\frac{35}{4}$ as a mixed number.

4 fourths)35 fourths

8 times and 3 fourths over;

$\therefore\ \frac{35}{4}=8\frac{3}{4}$.

(d) Reduction to lower or higher terms.

Great care should be taken to teach the principle upon which this rests, viz., *Dividing or multiplying both terms of a fraction by the same number does not alter the value of the fraction.*

(1) In teaching this use the knowledge of reduction of compound numbers to show that where the value of the unit is changed, the number of such units is necessarily changed so that the value of the quantity may not be altered; thus, 5s.=60d.; \$3=12 twenty-five-cent pieces; etc.

(2) Use such expedients as:—

(i) Divide a foot into 2 equal parts and represent one-half of it as $\frac{1}{2}$.

(ii) Divide a foot into 4 equal parts and represent one-half of it as $\frac{2}{4}$.

(iii) Divide a foot into 6 equal parts and represent one-half of it as $\frac{3}{6}$.

$\therefore\ \frac{1}{2}$ ft.$=\frac{2}{4}$ ft.$=\frac{3}{6}$ ft.

$\therefore\ \frac{1}{2}=\frac{2}{4}=\frac{3}{6}$.

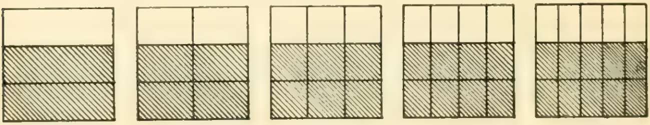
(iv) Divide a foot into 9 equal parts and represent one-third of it as $\frac{3}{9}$.

(v) Divide a foot into 18 equal parts and represent one-third of it as $\frac{6}{18}$.

(vi) Divide a foot into 3 equal parts and represent one-third of it as $\frac{1}{3}$.

$\therefore\ \frac{3}{9}=\frac{6}{18}=\frac{1}{3}$.

(3) Take paper such as is used in Kindergartens for folding and fold it as illustrated below.



$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$

(4) Have pupils rule two diagrams, each 1 in. square and have them show that $\frac{2}{5}=\frac{4}{10}$; $\frac{3}{5}=\frac{6}{10}$; $\frac{1}{4}=\frac{2}{8}$: $\frac{3}{4}=\frac{9}{12}$; etc.

(5) Constantly call for the reasoning used in working Exercises CXIX, CXX, CXXI, CXXII.

(*e*) **Reduction to equivalent fractions with the least common denominator.**

Steps.

(1) Exercises in which one denominator is a multiple of each of the others, as $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{12}$.

(2) Exercises in which the denominators are prime to one another, as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{2}{5}$.

(3) Exercises in which the denominators are not prime to one another, as $\frac{5}{8}$, $\frac{3}{10}$, and $\frac{7}{12}$.

(*f*) **Reduction of Compound to Simple Fractions.**

Steps.

(1) The Definition.

The usual definition, "a fraction of a fraction," is so concise that it is difficult for a beginner to comprehend it; hence, many exercises similar to the following should be worked:—

(i) Find $\frac{2}{3}$ of a foot. Find $\frac{2}{3}$ of half a foot.

(ii) Find $\frac{3}{4}$ of a yard. Find $\frac{3}{4}$ of a third of a yard.

(iii) Find $\frac{3}{4}$ of $\frac{2}{3}$ of a yard; etc.

(iv) Draw an inch square. Mark off $\frac{2}{5}$ of it; then find $\frac{1}{2}$ of the part marked off.

These and similar exercises will make clear that *a compound fraction is a number of the equal parts of a fraction.*

Contrast a compound fraction with a simple one.

(2) Simplifying compound fractions.

The following are suggested as a series of exercises:—

(i) $\frac{1}{2}$ of $\frac{2}{5}$; $\frac{1}{3}$ of $\frac{6}{11}$; $\frac{1}{4}$ of $\frac{12}{13}$.

(ii) $\frac{2}{3}$ of $\frac{6}{11}$; $\frac{3}{4}$ of $\frac{12}{13}$; $\frac{4}{5}$ of $\frac{20}{23}$.

(iii) $\frac{1}{2}$ of $\frac{1}{3}$; $\frac{1}{4}$ of $\frac{1}{5}$: $\frac{1}{3}$ of $\frac{1}{4}$.
(iv) $\frac{3}{5}$ of $\frac{4}{7}$; $\frac{2}{3}$ of $\frac{5}{7}$; $\frac{3}{4}$ of $\frac{7}{11}$.
(v) $\frac{2}{3}$ of $4\frac{1}{2}$; $\frac{2}{5}$ of $3\frac{1}{3}$; $\frac{3}{7}$ of $1\frac{3}{4}$.

In solving (iii) make use of the pupils' knowledge of reduction of denominate numbers to aid them in understanding why the second fraction must be changed.

Thus, to find one-half a dollar, the dollar must be changed into such a quantity that it can be divided into two equal parts, as 100c., or 4 quarters, etc.

Solution of (iii) $\frac{1}{3}=\frac{2\times1}{2\times3}=\frac{2}{6}$;

$$\therefore \tfrac{1}{2} \text{ of } \tfrac{1}{3}=\tfrac{1}{2} \text{ of } \tfrac{2}{6}=\tfrac{1}{6}.$$

Solution of (iv) $\frac{4}{7}=\frac{5\times4}{5\times7}=\frac{20}{35}$;

$$\therefore \tfrac{3}{5} \text{ of } \tfrac{4}{7}=\tfrac{3}{5} \text{ of } \tfrac{20}{35}=\tfrac{12}{35}=\frac{\text{PRODUCT OF NUMERATORS.}}{\text{PRODUCT OF DENOMINATORS.}}$$

NOTE 1.—Draw diagrams and show that

$$\tfrac{3}{5} \text{ of } \tfrac{4}{7}=\tfrac{12}{35}.$$

2.—Show how cancellation shortens the work. Thus,

$$\tfrac{2}{3} \text{ of } 4\tfrac{1}{2}=\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \text{ of } \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{2}}}=\tfrac{3}{1}=3.$$

III. ADDITION OF FRACTIONS.

Steps.

(1) Addition of fractions having a common denominator, as $\frac{2}{7}+\frac{5}{7}+\frac{4}{7}=?$

(2) Addition when one of the denominators is a multiple of each of the others, as $\frac{3}{4}+\frac{2}{5}+\frac{7}{20}=?$

(3) Addition when the denominators are prime to one another, as $\frac{1}{2}+\frac{1}{3}+\frac{3}{5}=?$

(4) Addition when the denominators are not prime to one another, as $\frac{3}{4}+\frac{5}{6}+\frac{3}{8}=?$

(5) Addition of mixed numbers, as $4\frac{2}{5}+2\frac{3}{4}=?$

$$\begin{aligned}4\tfrac{2}{5}+2\tfrac{3}{4}&=4+\tfrac{2}{5}+2+\tfrac{3}{4}\\&=(4+2)+(\tfrac{2}{5}+\tfrac{3}{4})\\&=6+(\tfrac{8}{20}+\tfrac{15}{20})\\&=6+\tfrac{23}{20}\\&=6+1+\tfrac{3}{20}=7\tfrac{3}{20}.\end{aligned}$$

Hence, add the sum of the whole numbers to the sum of the fractions to find the sum of the mixed numbers.

NOTE.—It is easy for the teacher to add two fractions with a common numerator. Thus,

$$\tfrac{2}{7}+\tfrac{2}{9}=\frac{2\times(7+9)}{7\times9}=\tfrac{32}{63}.$$

$$\tfrac{5}{12}+\tfrac{5}{18}=\frac{5\times(12+18)}{12\times18}=\frac{5\times30}{12\times18}=\tfrac{25}{36}.$$

IV. SUBTRACTION OF FRACTIONS.

Steps.

(1) Subtraction of fractions having a common denominator, as $\frac{7}{13}-\frac{3}{13}=?$

(2) Subtraction when one of the denominators is a multiple of the other, as from $\frac{5}{8}$ take $\frac{3}{16}$.

(3) Subtraction when the denominators are prime to each other, as from $\frac{2}{5}$ take $\frac{3}{11}$.

(4) Subtraction when the denominators are not prime to each other, as from $\frac{7}{12}$ take $\frac{3}{16}$.

(5) Subtraction of mixed numbers.

NOTE.—In taking one mixed number from another there are various ways of proceeding when the fraction of the subtrahend exceeds that of the minuend.

Ex. 1. From $7\frac{1}{3}$ take $4\frac{7}{9}$.

$$\begin{array}{ll} (a) & 7\tfrac{1}{3}=7\tfrac{3}{9}=6+1+\tfrac{3}{9}=6\tfrac{12}{9}. \\ & 4\tfrac{7}{9}=4\tfrac{7}{9}= \quad\quad\quad\quad 4\tfrac{7}{9}. \\ & \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \overline{2\tfrac{5}{9}}. \end{array}$$

This is the method of decomposition.

$$\begin{array}{ll} (b) & 7\tfrac{1}{3}=7\tfrac{3}{9};\ 7\tfrac{3}{9}+\tfrac{9}{9}=7\tfrac{12}{9}. \\ & 4\tfrac{7}{9}=4\tfrac{7}{9};\ 4\tfrac{7}{9}+1=5\tfrac{7}{9}. \\ & \quad\quad\quad\quad\quad\quad\quad\quad \overline{2\tfrac{5}{9}}. \end{array}$$

This is the method of equal addition.

$$\begin{array}{ll} (c) & 7\tfrac{1}{3}+\tfrac{2}{9}=7\tfrac{5}{9}. \\ & 4\tfrac{7}{9}+\tfrac{2}{9}=5. \\ & \quad\quad\quad\quad \overline{2\tfrac{5}{9}}. \end{array}$$

This is the method of complementary addition.

In general the last method is the simplest. It consists in adding such a fraction to that in the subtrahend as will make its value 1, and adding the same fraction to the minuend. The subtraction can then be readily performed.

NOTE 2.—Fractions having a common numerator can be readily subtracted by the teacher. Thus,

$$\tfrac{2}{7}-\tfrac{2}{9}=\frac{2\times(9-7)}{7\times9}=\tfrac{4}{63}.$$

$$\tfrac{17}{19}-\tfrac{17}{23}=\frac{17\times(23-19)}{19\times23}=\tfrac{68}{437}.$$

V. MULTIPLICATION AND DIVISION OF FRACTIONS.

Case I. The Multiplication of a fraction by a whole number.

Show by many examples that a fraction is multiplied by a whole number either by multiplying the numerator by the whole number or by dividing the denominator by it. Thus,

$$\tfrac{2}{11}\times5=\tfrac{10}{11} \text{ and } \tfrac{5}{12}\times4=\tfrac{20}{12}=\tfrac{5}{3}=1\tfrac{2}{3}.$$

Case II. The Division of a fraction by a whole number.

By many examples worked as in the reduction of compound fractions, show that a fraction is divided by a whole number either by dividing the numerator or by multiplying the denominator by the whole number.

Case III. The multiplication of a whole number or a fraction by a fraction.

NOTE 1.—Before this step is taken the pupil should be taught that a fraction indicates the division of its numerator by its denominator.

Do this as follows:—

(*a*) Draw two lines, one a foot long and the other two feet long.
Take $\frac{2}{6}$ of the shorter and $\frac{1}{6}$ of the longer.
Compare these parts.
It will be seen that $\frac{2}{6}$ of $1=\frac{1}{6}$ of 2, or $2\div6$.
Similarly show that $\frac{3}{4}$ of $1=\frac{1}{4}$ of 3, or $3\div4$, etc.

(*b*) $1=4$ fourths of a unit;
$3=3$ times 4 fourths of a unit
$=12$ fourths of a unit;
$\therefore \frac{1}{4}$ of $3=\frac{1}{4}$ of 12 fourths of a unit
$=3$ fourths of a unit
$=\frac{3}{4}$ of 1;
$\therefore \frac{1}{4}$ of $3=\frac{3}{4}$ of 1.

Ex. 1. Multiply $\frac{5}{7}$ by $\frac{3}{4}$.

The multiplier is $\frac{1}{4}$ of 3, or $3\div4$.
$\frac{5}{7}\times3=\frac{15}{7}$.

This result is evidently 4 times too great because the multiplier is 4 times too large.

$\frac{15}{7}$ must, hence, be divided by 4.

$$\frac{15}{7}\div 4=\frac{15}{28}$$

Hence, $\frac{5}{7}\times\frac{3}{4}=\frac{15}{28}=\frac{5\times 3}{7\times 4}=\frac{\text{PRODUCT OF NUMERATORS.}}{\text{PRODUCT OF DENOMINATORS.}}$

NOTE 2.—Care must be taken to cancel all the factors common to both numerators and denominators before performing the multiplication.

NOTE 3.—In giving examples for practice in multiplying one mixed number by another, the teacher can make the work easy for himself by having the whole numbers the same and the sum of the fractions 1.

Thus, $7\frac{1}{4}\times 7\frac{3}{4}=8\times 7+\frac{1}{4}\times\frac{3}{4}=56\frac{3}{16}$.

$12\frac{2}{5}\times 12\frac{3}{5}=13\times 12+\frac{2}{5}\times\frac{3}{5}=156\frac{6}{25}$.

$19\frac{1}{2}\times 19\frac{1}{2}=20\times 19+\frac{1}{2}\times\frac{1}{2}=380\frac{1}{4}$.

Case IV. The Division of a whole number or a fraction by a fraction.

In dividing a whole number or a fraction by a fraction there are various ways of proceeding.

Ex. 1. Divide $\frac{3}{5}$ by $\frac{7}{9}$.

(*a*) $\frac{3}{5}=\frac{27}{45}$ and $\frac{7}{9}=\frac{35}{45}$;

$\therefore\ \frac{3}{5}\div\frac{7}{9}=\frac{27}{45}\div\frac{35}{45}=27$ forty-fifths $\div$ 35 forty-fifths

$=\frac{27}{35}=\frac{3\times 9}{5\times 7}=\frac{3}{5}\times\frac{9}{7}=$ *dividend multiplied by the fraction formed by inverting the divisor.*

(*b*) $\frac{7}{9}=7\div 9$.

$\frac{3}{5}\div 7=\frac{3}{35}$.

This is evidently 9 times too small as $\frac{3}{5}$ is not to be divided by 7 but by $\frac{1}{9}$ of 7. Hence, the true quotient will be 9 times $\frac{3}{35}$, or $\frac{27}{35}$.

Thus, $\frac{3}{5}\div\frac{7}{9}=\frac{27}{35}=\frac{3\times 9}{5\times 7}=\frac{3}{5}\times\frac{9}{7}$

$=$ *dividend multiplied by the fraction formed by inverting the divisor.*

(*c*) $\frac{3}{5}\div\frac{7}{9}=(\frac{3}{5}\times\frac{9}{7})\div(\frac{7}{9}\times\frac{9}{7})=\frac{27}{35}\div 1=\frac{27}{35}=\frac{3\times 9}{5\times 7}=\frac{3}{5}\times\frac{9}{7}$

$=$ *dividend multiplied by the fraction formed by inverting the divisor.*

This method depends on the following:—

(i) Any number divided by 1 gives that number for quotient.

(ii) Multiplying both divisor and dividend by the same number does not alter the quotient.

Thus, $3\overline{)15}$ $\quad 6\overline{)30}$ $\quad 15\overline{)75}$
$\quad\quad 5 \quad\quad 5 \quad\quad 5.$

(iii) The divisor when multiplied by the fraction formed by inverting its terms produces 1.

Thus, $\frac{2}{3}\times\frac{3}{2}=1$; $\frac{7}{5}\times\frac{5}{7}=1$, etc.

NOTE.—The inverted divisor is called the reciprocal of the divisor, reciprocal numbers being those which multiplied together will produce unity.

VI. COMPLEX FRACTIONS.

Steps.

(1) *The Definition.*

Use many examples in which one or both terms are fractions as

$$\frac{\frac{1}{2}}{3},\ \frac{3}{1\frac{1}{4}},\ \frac{\frac{3}{5}}{\frac{8}{4}},\ \frac{4\frac{1}{4}}{7},\ \text{etc.}$$

After giving the term *complex fraction* to such forms, the definition will readily follow.

(2) *To simplify a complex fraction.*

(*a*) Use the knowledge that the numerator of a fraction is the dividend and the denominator the divisor.

Thus, $\frac{\frac{1}{2}}{3}=\frac{1}{2}\div3=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$.

$\frac{3}{\frac{5}{4}}=\frac{3}{1}\div\frac{5}{4}=\frac{3}{1}\times\frac{4}{5}=2\frac{2}{5}$.

(*b*) Use the principle that multiplying both terms of a fraction by the same number does not alter its value.

Ex. 1. Simplify $\frac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}}$.

Multiply both terms by 12 and $\frac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}}=\frac{4+3}{6+4}=\frac{7}{10}$.

Ex. 2. Simplify $\frac{2\frac{1}{3}-1\frac{5}{9}}{2\frac{1}{3}+1\frac{5}{9}}$.

Multiply both terms by 18 and $\frac{2\frac{1}{3}-1\frac{5}{9}}{2\frac{1}{3}+1\frac{5}{9}}=\frac{42-28}{42+28}=\frac{14}{70}=\frac{1}{5}$.

In multiplying $2\frac{1}{3}$ by 18, multiply the fractional and integral parts separately and add the results. In multiplying $\frac{5}{9}$ by 18, divide 18 by 9 and multiply the numerator, 5, by the quotient, 2.

VII. THE GREATEST COMMON MEASURE OF FRACTIONS.

Use the pupils' knowledge of the G.C.M. of whole numbers.

Thus, find the G.C.M. of 20s. and 8s.
" " " " £1 and 8s.
" " " " £2 and 15s.
" " " " 12d. and 9d.
" " " " 1s. and 9d.
" " " " 1s. 6d. and 1s.

When the pupils see that the numbers must be reduced to equivalent ones with the same unit, then the application to fractions is easy.

Find the G.C.M. of $\frac{12}{17}$, $\frac{15}{17}$ and $\frac{18}{17}$.

Find the G.C.M. of $\frac{4}{5}$, $\frac{8}{15}$ and $\frac{12}{25}$.

Reduced to a common denominator these become $\frac{60}{75}$, $\frac{40}{75}$ and $\frac{36}{75}$.

The G.C.M. is readily seen to be $\frac{4}{75}$.

On examining the original fractions, 4 is seen to be the G.C.M. of the numerators and 75 is the L.C.M. of the denominators,

or, $\frac{4}{75} = \frac{\text{G.C.M. OF NUMERATORS.}}{\text{L.C.M. OF DENOMINATORS.}}$

Show that each fraction contains the G.C.M. exactly and that the several quotients are prime to one another.

Thus, $\frac{4}{5} \div \frac{4}{75} = 15$; $\frac{8}{15} \div \frac{4}{75} = 10$; $\frac{12}{25} \div \frac{4}{75} = 9$.

15, 10 and 9 are whole numbers and are prime to one another.

VIII. THE LEAST COMMON MULTIPLE OF FRACTIONS.

Use the pupils' knowledge of the L.C.M. of whole numbers.

Thus, find the L.C.M. of 12d. and 8d.
" " " " 1s. and 8d.
" " " " 2s. and 1s. 4d.
" " " " £1 5s. and 15s.

As soon as the pupils see that the numbers must be reduced to equivalent ones with the same unit, then the application to fractions can be made.

Find the L.C.M. of $\frac{5}{12}$ and $\frac{7}{12}$.
" " " " $\frac{8}{13}$ and $\frac{12}{13}$.
" " " " $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{4}{9}$.

Reduced to a common denominator, these become $\frac{12}{18}$, $\frac{15}{18}$ and $\frac{8}{18}$.

The L.C.M. is readily found to be $\frac{120}{18}$, or $\frac{20}{3}$.

Comparing this with the original fractions, the L.C.M. is seen to be $\frac{\text{L.C.M. OF NUMERATORS.}}{\text{G.C.M. OF DENOMINATORS.}}$

Show that this contains each of the given fractions exactly and that the several quotients are prime to one another.

Thus, $\frac{20}{3}\div\frac{2}{3}=10$; $\frac{20}{3}\div\frac{5}{6}=8$; $\frac{20}{3}\div\frac{4}{9}=15$.

10, 8 and 15 are integral numbers and are prime to one another.

IX. THE SIMPLIFYING OF COMPLEX EXPRESSIONS.

In simplifying such expressions as those in Exercise CL, the following is the usual usage regarding the signs "of," $\times$, $-$, $-$, $+$:—

(1) The operations indicated by "of," $\times$ and $\div$ should be performed before adding or subtracting.

Ex. 1. $\frac{2}{3}+\frac{3}{5}$ of $3\frac{1}{3}-\frac{1}{5}\div\frac{1}{4}+\frac{3}{4}\times 2\frac{2}{5}$

$=\frac{2}{3}+(\frac{3}{5}$ of $3\frac{1}{3})-(\frac{1}{5}\div\frac{1}{4})+(\frac{3}{4}\times 2\frac{2}{5})$

$=\frac{2}{3}+2-\frac{4}{5}+\frac{9}{5}$

$=3\frac{2}{3}$.

(2) The operation indicated by "of" should be performed before that indicated by $\div$.

Ex. 2. $1\frac{3}{4}$ of $1\frac{1}{7}\div 1\frac{2}{3}$ of $\frac{6}{11}$

$=(1\frac{3}{4}$ of $1\frac{1}{7})\div(1\frac{2}{3}$ of $\frac{6}{11})$

$= 2 \div \frac{10}{11}$

$=2\frac{1}{5}$.

NOTE.—This is the only case in which usage makes a distinction between "of" and $\times$.

(3) The operations indicated by $\times$ and $\div$ should be performed in the order in which they occur.

Ex. 3. $2\frac{2}{3}\div\frac{1}{2}\times 1\frac{3}{8}$

$=\frac{8}{3}\times\frac{2}{1}\times\frac{11}{8}$

$=7\frac{1}{3}$.

Ex. 4. $2\frac{2}{3}\times\frac{1}{2}\div 1\frac{3}{8}$

$=\frac{8}{3}\times\frac{1}{2}\times\frac{8}{11}$

$=\frac{32}{33}$.

Ex. 5. $\frac{1}{2}\times\frac{1}{3}\div\frac{1}{4}\times\frac{1}{5}$

$=\frac{1}{2}\times\frac{1}{3}\times\frac{4}{1}\times\frac{1}{5}$

$=\frac{2}{15}$.

Ex. 6. $\frac{1}{2}\times\frac{1}{3}\div\frac{1}{4}\div\frac{1}{5}$

$=\frac{1}{2}\times\frac{1}{3}\times\frac{4}{1}\times\frac{5}{1}$

$=3\frac{1}{3}$.

X. DENOMINATE FRACTIONS.

1. The Definition.

Distinguish between the primary and the fractional unit and point out that when the *primary unit is a denominate number* the fraction is said to be a denominate fraction, as $\$\frac{1}{4}$, $\frac{1}{2}$ mi., $\frac{3}{4}$ of a league, etc.

2. To find the value of a fraction of a denominate number.

Ex. 1. Find the value of $\frac{4}{5}$ of a ton.

5)4 t.
 16 cwt.

Since $\frac{4}{5}$ t. $=\frac{1}{5}$ of 4 t. (page 157), 4 t. is divided by 5 as in compound division.

Ex. 2. Find the value of $\frac{3}{4}$ of £1 8s. 4d.

£	S.	D.
1	8	4
		3
4)4	5	0
1	1	3

$\frac{3}{4}$ of £1 8s. 4d. $= \frac{1}{4}$ of 3 times £1 8s. 4d.
$= \frac{1}{4}$ of £4 5s.
$=$ £1 1s. 3d.

3. To express one number as the fraction of another.

Ex. 1. Express \$5 as the fraction of \$8.

\$1 is $\frac{1}{8}$ of \$8;

\$5 is 5 times \$1;

$\therefore$ \$5 is 5 times $\frac{1}{8}$ of \$8, or $\frac{5}{8}$ of \$8.

NOTE.—This example may be worded in any of the following ways:—

(*a*) Reduce \$5 to the fraction of \$8.
(*b*) What fraction of \$8 is \$5?
(*c*) What fraction is \$5 of \$8?
(*d*) What part of \$8 is \$5?
(*e*) If \$8 is the unit, what is the measure of \$5?
(*f*) When pupils understand ratio, it may be expressed as "What ratio is \$5 to \$8?"

Ex. 2. What part of £7 11s. 8d. is £3 8d.?

£3 8d. $=$ 728d.
£7 11s. 8d. $=$ 1820d.
Now, 1d. is $\frac{1}{1820}$ of 1820d.;
But 728d. is 728 times 1d.;
$\therefore$ 728d. is $\frac{728}{1820}$ of 1820d.;
$\frac{728}{1820}=\frac{2}{5}$.
$\therefore$ £3 8d. is $\frac{2}{5}$ of £7 11s. 8d.

Ex. 3. Reduce $\frac{7}{110880}$ mi. to the fraction of a foot.

1 mi. $=$ 5280 ft.;
$\frac{7}{110880}$ mi. $=\frac{7}{110880}$ of 5280 ft.;
Now, $\frac{1}{110880}$ of 5280 ft. $=\frac{1}{21}$ ft.;
and $\frac{7}{110880}$ of 5280 ft. $=\frac{7}{21}$ ft. $=\frac{1}{3}$ ft.

Ex. 4. What fraction of $\frac{3}{7}$ of $4\frac{1}{5}$ is $\frac{2}{3}$ of $\frac{3}{7}$?

$\frac{3}{7}$ of $4\frac{1}{5}=\frac{9}{5}=\frac{63}{35}$;
$\frac{2}{3}$ of $\frac{3}{7}=\frac{2}{7}=\frac{10}{35}$;
Now, $\frac{1}{35}$ is $\frac{1}{63}$ of $\frac{63}{35}$;
$\therefore$ $\frac{10}{35}$ is $\frac{10}{63}$ of $\frac{63}{35}$;
$\therefore$ $\frac{2}{3}$ of $\frac{3}{7}$ is $\frac{10}{63}$ of $\frac{3}{7}$ of $4\frac{1}{5}$.

XIII. DECIMALS.

I. DEFINITION.

For an introduction to teaching the definition of a decimal see the Public School Arithmetic, page 115.

This will make clear why the first place to the right of the point is tenths, the second hundredths, etc.

In introducing the point make use of the notation of dollars and cents. The point fixes the position of units of dollars; now it is used to fix the position of units.

The distinction between common, or vulgar fractions and decimal fractions, or decimals should be made clear. It should be shown that when both numerator and denominator are expressed the fraction is vulgar; whereas when only the numerator is expressed, it is decimal.

Thus, $\frac{3}{4}$, $\frac{7}{10}$, $\frac{19}{100}$ are common fractions,

and .8, .19, .345 are decimals.

II. NOTATION AND NUMERATION OF DECIMALS.

(1) The reading and writing of integral numbers should be thoroughly reviewed.

(2) Pupils should know the following :—

(*a*) There are three places in each period.

(*b*) Reading from right to left, the *first* place in each period is *units;* the *second*, *tens;* the *third*, *hundreds*.

(*c*) The periods from right to left are *Units*, *Thousands*, *Millions*, *Billions*, *Trillions*, etc.

(*d*) The value of any digit in a number depends upon its position in that number. Thus, in 123,456,789, the value of 6 is unit-thousands, or 6 thousands. The value of 2 is 2 ten-millions, or 20 millions.

(*e*) Each digit has thus two values, an absolute value which it has when standing alone, and a relative value depending upon its position in a number. Thus, in the number in (*d*) the absolute value of 5 is five and its relative value is 5 ten-thousand or fifty thousand.

(3) Put a number such as 77777.7777 upon the blackboard.

From the introduction (page 115) the pupil will learn that the names of the places to the right of the point are tenths, hundredths, etc.

(4) Show that the same law of increase from right to left or of decrease from left to right prevails to the right of the point as to the left of it.

Thus, 10 of any place makes one of the next higher place, and 1 of any place makes 10 of the next lower one.

The introduction shows that 10 tenths=1 unit; that 10 hundredths=1 tenth; that 1 hundredth=10 thousandths, etc.

(5) Compare the names of the places immediately left and right of the units' place; the second places to the left and right of units; the third places, etc.

(6) Mark off the number into periods to the left and right from the units. Thus:—

Millions.			Thousands.			Units.				Units.		Thousandths.			Millionths.		
1	2	3,	4	5	6,	7	8	9	.	8	7,	6	5	4,	3	2	1
Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Units.	Tenths.	Hundredths.	Units.	Tenths.	Hundredths.

If instead of writing the point between the 9 and 8, it is placed over the 9, the units' place, to mark this place much of the trouble in connection with reading decimals would disappear.

From this it will be seen that the thousands' and the thousandths' periods occupy corresponding positions to the left and right of the units' period. The same is true of the millions' and the millionths' periods, etc.

(7) Much practice should be given in reading numbers in all possible ways. Thus, 7.256 may be read as 7 and 256 thousandths; as 72 tenths and 56 thousandths; as 725 hundredths and 6 thousandths; as 7256 thousandths.

Teach this as follows:

$$
\begin{aligned}
7.256 &= 7 \text{ units} + 2 \text{ tenths} + 5 \text{ hundredths} + 7 \text{ thousandths} \\
&= 7 + \tfrac{2}{10} + \tfrac{5}{100} + \tfrac{6}{1000} \\
&= 7 + \frac{200+50+6}{1000} \\
&= 7 \text{ and } 256 \text{ thousandths.}
\end{aligned}
$$

Again, 7.256=7 units+2 tenths+5 hundredths+6 thousandths

$$=7+\tfrac{2}{10}+\tfrac{5}{100}+\tfrac{6}{1000}$$

$$=\frac{70+2}{10}+\frac{50+6}{1000}$$

$$=72 \text{ tenths}+56 \text{ thousandths.}$$

Examples similar to the following should be given:—

(1) Read 73.84 as a simple number; as a mixed number; as tenths; as tens; as hundredths.

(2) Read 21.456 as a mixed number and give the value of each digit.

(3) Write 756 as ten-millions.

(4) Write 756 as ten millionths.

(5) How many figures are required to express 2145 hundred thousands?

(6) How many figures are required to express 2145 hundred thousandths?

(7) Read 9 as units; as tenths; as hundredths; etc.

(8) Read 9 as units; as tens; as hundreds; etc.

(9) Show that NAUGHTS affixed to a decimal have no effect on its value.

Ex. 1. $.9=.90=.900$; etc.

$$.9=\tfrac{9}{10}.$$

$$.90=\tfrac{90}{100}=\tfrac{9}{10}.$$

$$.900=\tfrac{900}{1000}=\tfrac{9}{10}.$$

NOTE.—A common and convenient way of reading a decimal such as 7.256 is to say 7 *point* 2, 5, 6, or 7 *decimal* 2, 5, 6.

III. REDUCTION OF DECIMALS TO VULGAR FRACTIONS.

Ex. 1. Reduce .275 to a vulgar fraction in its lowest terms.

$.275=\tfrac{275}{1000}=\tfrac{55}{200}=\tfrac{11}{40}.$

Ex. 2. Reduce .0125 to a vulgar fraction in its lowest terms.

$.0125=\tfrac{125}{10000}=\tfrac{25}{2000}=\tfrac{5}{400}=\tfrac{1}{80}.$

There will be no trouble with this if the notation and numeration of decimals have been properly taught. Appeals to these must be constantly made.

Thus, in example 1:

.275=2 tenths+7 hundredths+5 thousandths.

$$=\tfrac{2}{10}+\tfrac{7}{100}+\tfrac{5}{1000}$$

$$=\frac{200+70+5}{1000}$$

$$=\tfrac{275}{1000}.$$

IV. ADDITION OF DECIMALS.

Introduce addition of Decimals by reviewing briefly:—

(*a*) Decimal notation.

(*b*) The fundamental principle of addition, viz., only numbers of the same unit can be added.

(*c*) How the addends must be written so as to bring units of the same order under one another.

From this the pupils will see why the points are made to range under one another.

The addition follows as in addition of whole numbers.

Ex. 1. Find the sum of 4.8, 47.875, 36, .0897, 3.0069.

```
  4.8
 47.875
 36.
   .0897
  3.0069
 -------
 91.7716
```

Place the addends so that units are under units, tenths under tenths, etc. This is the case when the points are in the same vertical line.

7 ten-thousandths+9 ten-thousandths=16 ten-thousandths=1 thousandth+6 ten-thousandths.

1 thousandth+6 thousandths+9 thousandths+5 thousandths=21 thousandths=2 hundredths+1 thousandth, etc.

V. SUBTRACTION OF DECIMALS.

Whatever method of subtraction has been used in whole numbers should be used in Subtraction of Decimals.

Ex. 1. From 17.3 take 12.876.

```
17.300     or     17.3
12.876            12.876
 4.424             4.424
```

As in subtraction of integers, write units under units, tenths under tenths, hundredths under hundredths, etc. As there are more figures in the subtrahend than in the minuend, annex ciphers so as to make the number of decimal places in each the same. This will not affect the result. (Page 165.)

This expedient for making the subtraction easier for a beginner should be gradually discontinued.

VI. MULTIPLICATION OF DECIMALS.

Make use of multiplication of vulgar fractions to infer the rule for Multiplication of Decimals. Take the following steps:—

(*a*) Multiply a decimal by a whole number.

Ex. 1. Multiply .7 by 9.

$.7=\frac{7}{10}$;

$\therefore\ .7\times9=\frac{7}{10}\times9=\frac{63}{10}=6.3.$

Ex. 2. Multiply .077 by 7.

$.077=\frac{77}{1000}$;

$\therefore\ .077\times 7=\frac{77}{1000}\times 7=\frac{539}{1000}=.539.$

(*b*) Multiply a mixed decimal by a whole number.

Ex. 3. Multiply 17.847 by 3.

$17.847=17\frac{847}{1000}=\frac{17847}{1000}$;

$\therefore\ 17.847\times 3=\frac{17487}{1000}\times 3=\frac{53541}{1000}=53.541.$

(*c*) Multiply a decimal by a decimal.

Ex. 4. Multiply .19 by .7.

$.19=\frac{19}{100}$ and $.7=\frac{7}{10}$;

$\therefore\ .19\times .7=\frac{19}{100}\times\frac{7}{10}=\frac{133}{1000}=.133.$

Ex. 5. Multiply .0017 by .013.

$.0017=\frac{17}{10000}$ and $.013=\frac{13}{1000}$;

$\therefore\ .0017\times .013=\frac{17}{10000}\times\frac{13}{1000}=\frac{221}{10000000}=.0000221.$

(*d*) Multiply a mixed decimal by a decimal.

Ex. 6. Multiply 4.79 by .83.

$4.79=\frac{479}{100}$ and $.83=\frac{83}{100}$;

$\therefore\ 4.79\times .83=\frac{479}{100}\times\frac{83}{100}=\frac{39757}{10000}=3.9757.$

(*e*) Multiply a mixed decimal by a mixed decimal.

Ex. 7. Multiply 7.13 by 5.9.

$7.13=\frac{713}{100}$ and $5.9=\frac{59}{10}$;

$7.13\times 5.9=\frac{713}{100}\times\frac{59}{10}=\frac{42067}{1000}=42.067.$

By examining the products obtained in each case it will be seen that the figures of the product are obtained by regarding both multiplier and multiplicand as whole numbers and multiplying in the usual way.

By comparing the number of decimal places in the product with that in the multiplier and multiplicand together, it will be seen that there are as many decimal places in the product as in the multiplier and multiplicand together.

CAUTION.—Avoid cancelling even if the figures would permit of it.

NOTE.—The pupil should now be given practice in multiplying by 10, 100, 1000, etc.

Ex. Multiply 87.569 by 10, by 1000, by 100000.

$$\begin{array}{r}87.569\\10\\\hline 875.690\end{array}\qquad\begin{array}{r}87.569\\1000\\\hline 87569.000\end{array}\qquad\begin{array}{r}87.569\\100000\\\hline 8756900.000\end{array}$$

Hence, to multiply by a number expressed by 1 followed by naughts, *move the decimal point as many places to the right in the multiplicand as there are naughts in the multiplier.*

This is done to prepare the pupil for making the divisor a whole number in division of decimals.

VII. DIVISION OF DECIMALS.

The difficulty in division of decimals arises from inability to put the decimal point in its proper place in the quotient.

Overcome this as follows:—

(1) Make the divisor a whole number.

(2) Put the point in the quotient as soon as the tenth's place of the dividend is used as a partial dividend.

Illustrate with care the principle upon which (1) is based, *i.e.*, multiplying both divisor and dividend by the same number does not affect the quotient.

Thus, 2)6 8)24 10)30 16)48
 3 3 3 3.

Or, multiplying the numerator and denominator of a fraction by the same number does not alter the value of the fraction (page 153).

$\therefore \frac{6}{2}\times\frac{4}{4}=\frac{24}{8}=3.$

$\frac{6}{2}\times\frac{5}{5}=\frac{30}{10}=3$, etc.

The divisor can always be made a whole number by multiplying it by a number expressed by 1 followed by as many naughts as there are decimal places in the divisor.

Steps in division of decimals :—

(*a*) Division of a mixed decimal by a whole number.

Ex. 1. Divide 62.3 by 5.

5)62.3
 12.46

Divide the whole number by 5. There are 2 over; 2+.3=23 tenths; 23 tenths÷5=4 tenths and 3 tenths over; 3 tenths=30 hundredths; 30 hundredths÷5=6 hundredths.

(*b*) Division of a decimal by a whole number.

Ex. 2. Divide .0236 by 8.

8).0236
 .00295.

(*c*) Division of a decimal by a decimal.

Ex. 3. Divide .00123 by .04.

$$\frac{.00123}{.04}=\frac{.00123\times 100}{.04\times 100}=\frac{.123}{4}=.03075.$$

Ex. 4. Divide .0021 by .000012.

$$\frac{.0021}{.000012}=\frac{002100.}{000012.}=\frac{2100}{12}=175.$$

Ex. 5. Simplify $\frac{.125}{.25}+\frac{1.25}{.025}+\frac{.0125}{.0025}+\frac{12.5}{.00025}$.

$$\frac{.125}{.25}+\frac{1.25}{.025}+\frac{.0125}{.0025}+\frac{12.5}{.00025}$$
$$=\frac{12.5}{25}+\frac{1250}{25}+\frac{125}{25}+\frac{1250000}{25}$$
$$=.5+50+5+50000$$
$$=50055.5.$$

Ex. 6. Divide 1.73546 by 456.7.

$$\begin{array}{r} 4567)17.3546(0.0038 \\ \underline{13701} \\ 36536 \\ \underline{36536} \end{array}$$

4567 is not contained in 17 units. It is not contained in 173 tenths. It is not contained in 1735 hundredths. But 17354 thousandths÷4567=3 thousandths, etc.

NOTE.—Much practice should be given in multiplying and dividing a decimal by 10, 100, 1000, etc., by merely moving the decimal point.

VIII. CONTRACTIONS IN MULTIPLICATION AND DIVISION OF DECIMALS.

When the number of decimal places given is large, and accuracy is not required beyond 4 or 5 places, the labor of multiplication and division may be avoided by a *contraction* of the ordinary process.

Ex. 1. Multiply 33.166245 by 1.4142136 correct to five decimal places.

Ordinary Form.

$$\begin{array}{r|l} 33. & 166245 \\ 1. & 4142136 \\ \hline 1 & 98997470 \\ 9 & 9498735 \\ 33 & 166245 \\ 663 & 32490 \\ 13266 & 4980 \\ 33166 & 245 \\ 1326649 & 80 \\ 3316624 & 5 \\ \hline 46.90415 & 47399320 \end{array}$$

Contracted Form.

$$\begin{array}{rcr} 33.166245 & & \\ 6312414.1 & & \\ \hline 3316625 & = & 3316624\times 1+1 \\ 1326650 & = & 331662\times 4+2 \\ 33166 & = & 33166\times 1 \\ 13266 & = & 3316\times 4+2 \\ 663 & = & 331\times 2+1 \\ 33 & = & 33\times 1 \\ 10 & = & 3\times 3+1 \\ 2 & = & 0\times 6+2 \\ \hline 46.90415 & & \end{array}$$

To explain the contracted method, observe that multiplying any order by units gives that order as product, hence the units' figure of the multiplier is written under the place to be retained. The other figures of the multiplier are written in reverse order.

Now 2, a decimal of the fourth order, multiplied by 4, a decimal of the first order, gives a decimal of the *fifth* order; also, 6, a decimal of the third order, multiplied by 1, a decimal of the second order, gives a decimal of the *fifth* order; etc. The partial products are written with the right-hand figures in the same vertical line. This brings the same orders under one another.

Ex. 2. Divide 6.275349 by 46.28573 correct to five places of decimals.

Common Method.

```
46.28573)6.275349(.13557
         4628573
         ----------
         1646776|0
         1388571|9
         ----------
          258204 10
          231428 65
          ----------
           26775 450
           23142 865
           ----------
            3632 5850
            3240 0011
```

Contracted Method.

```
46.28573)6.275349(.13557
    ....  4628573
          -------
          1646776
          1388572
          -------
           258204
           231429
           ------
            26775
            23143
            -----
             3632
             3240
```

In the contracted method, after the first figure of the quotient has been found in the usual manner, and the first remainder obtained, instead of affixing a 0 or bringing down the next figure, cut off from the divisor the right-hand figure, and divide by the number formed of the remaining figures. At each successive step in the division cut off another figure from the right of the divisor, and continue the division with the numbers formed of the remaining figures. It is, however, necessary to carry from the rejected figures as in multiplication.

IX. THE G.C.M. AND L.C.M. OF DECIMALS.

Ex. 1. Find the G.C.M. and L.C.M. of .25, 1.25 and .0025.

$.25 = \frac{25}{100} = \frac{2500}{10000}$; $1.25 = \frac{125}{100} = \frac{12500}{10000}$; $.0025 = \frac{25}{10000}$.

G.C.M. of $\frac{2500}{10000}$, $\frac{12500}{10000}$ and $\frac{25}{10000}$ is $\frac{25}{10000}$, or

G.C.M. of .2500, 1.2500 and .0025 is .0025.

L.C.M. of $\frac{2500}{10000}$, $\frac{12500}{10000}$ and $\frac{25}{10000}$ is $\frac{12500}{10000}$, or

L.C.M. of .2500, 1.2500 and .0025 is 1.2500, or 1.25.

Hence, to find the G.C.M. or L.C.M. of two or more decimals, reduce the decimals to equivalent ones of the same order and, disregarding the decimal point, find the G.C.M. or L.C.M. of the resulting numbers, and mark off the order by placing the decimal point.

X. REDUCTION OF VULGAR FRACTIONS TO DECIMALS.

It was shown on page 157 that a fraction indicates the division of the numerator by the denominator, so in reducing a vulgar fraction to a decimal merely divide the numerator by the denominator by means of division of decimals.

Ex. 1. Reduce $\frac{7}{20}$ to a decimal.

The pupils should be asked to read $\frac{7}{20}$ as 7 divided by 20.

```
20)7.0(.35
   60
   100
   100
```

$\frac{7}{20}=\frac{1}{20}$ of 7. 7=70 tenths; $\frac{1}{20}$ of 70 tenths is 3 tenths and 10 tenths over.

10 tenths=100 hundredths and $\frac{1}{20}$ of 100 hundredths is 5 hundredths.

Hence, $\frac{7}{20}=.35$.

When pupils can readily reduce vulgar fractions to decimals the following process may be used:—

$$\frac{7}{20}=\frac{7}{2\times2\times5}=\frac{7\times\overset{\overset{1}{\cancel{5}}}{\cancel{10}}\times\overset{5}{\cancel{10}}}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{5}}\times10\times10}=\frac{35}{100}=.35.$$

Ex. 2. Reduce $\frac{7}{125}$ to a decimal.

```
125)7.00(.056
     625
      750
      750
```

$$\text{Or, }\frac{7}{125}=\frac{7}{5\times5\times5}=\frac{7\times\overset{2}{\cancel{10}}\times\overset{2}{\cancel{10}}\times\overset{2}{\cancel{10}}}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{5}}\times\underset{1}{\cancel{5}}\times10\times10\times10}=\frac{56}{1000}=.056.$$

Ex. 3. Reduce $\frac{9}{80}$ to a decimal.

$$\frac{9}{80}=\frac{9}{2\times2\times2\times2\times5}=\frac{9\times\overset{5}{\cancel{10}}\times\overset{5}{\cancel{10}}\times\overset{5}{\cancel{10}}\times\overset{1}{\cancel{10}}}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{5}}\times10\times10\times10\times10}$$

$$=\frac{1125}{10000}=.1125.$$

Ex. 4. Reduce $\frac{2}{3}$ to a decimal.

$\frac{2}{3}=.6666\ldots$ indefinitely.

Ex. 5. Reduce $\frac{7}{9}$ to a decimal.

$\frac{7}{9}=.7777\ldots$ indefinitely.

Ex. 6. Reduce $\frac{4}{33}$ to a decimal.

$\frac{4}{33}=.121212\ldots$ indefinitely.

Ex. 7. Reduce $\frac{5}{6}$ to a decimal.

$$\frac{5}{6}=\frac{5}{2\times3}=\frac{5\times\overset{5}{\cancel{10}}}{\underset{1}{\cancel{2}}\times3\times10}=\frac{25}{30}=.8333\ldots \text{ indefinitely.}$$

Ex. 8. Reduce $\frac{7}{12}$ to a decimal.

$$\frac{7}{12}=\frac{7}{2\times2\times3}=\frac{7\times\overset{5}{\cancel{10}}\times\overset{5}{\cancel{10}}}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times3\times10\times10}=\frac{175}{300}=.58333\ldots \text{ indefinitely.}$$

Ex. 9. Reduce $\frac{7}{88}$ to a decimal.

$$\frac{7}{88}=\frac{7}{2\times2\times2\times11}=\frac{7\times\overset{5}{\cancel{10}}\times\overset{5}{\cancel{10}}\times\overset{5}{\cancel{10}}}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times11\times10\times10\times10}$$

$$=\frac{875}{11000}=.0795454\ldots \text{ indefinitely.}$$

By comparing these nine examples, it will be observed:—

(1) That when the fraction is in its lowest terms and the denominator has no factors other than 2 or 5, the decimal is finite and that there are as many places in the decimal as the greatest number of times 2 or 5 occurs as a factor in the denominator.

(2) That vulgar fractions in their lowest terms whose denominators contain no 2's or 5's as factors produce decimals which never terminate and that the figures repeat from the decimal point.

(3) That vulgar fractions in their lowest terms whose denominators have 2's or 5's and other prime numbers as factors produce decimals of which parts do not repeat and parts repeat and that there are as many figures in the parts which do not repeat as the greatest number of times 2 or 5 occurs as a factor of the denominator.

NOTE.—When a fraction in its lowest terms with a prime number for its denominator is reduced to a recurring decimal, the number of places in the period is either equal to one less than the denominator, or is a factor of the number of units in the denominator less one.

The first of these results arises from the fact that each remainder must be less than the denominator and, hence, the number of remainders cannot be greater than one less than the denominator and whenever a remainder, the same as a previous one, occurs the period will begin to repeat.

$\frac{1}{7}$ will have 6, 3, 2, or 1 places in the period;
$\frac{1}{11}$ " " 10, 5, 2, or 1 " " "
$\frac{1}{13}$ " " 12, 6, 4, 3, 2, or 1 places in the period;
$\frac{1}{17}$ " " 16, 8, 4, 2, or 1 " " "
$\frac{1}{19}$ " " 18, 9, 6, 3, 2, or 1 places in the period, etc.

Hence, decimals fall into two great classes, viz.:—

(*a*) Those which terminate. These are TERMINATING DECIMALS.

(*b*) Those which do not terminate. These are CIRCULATING DECIMALS.

Circulating Decimals fall into two classes, viz.:—

(1) Those which begin to repeat from the decimal point.

These are PURE CIRCULATING DECIMALS.

(2) Those which do not begin to repeat immediately after the point.

These are MIXED CIRCULATING DECIMALS.

Table of Decimals.

$$\text{Decimals.}\begin{cases}(a)\ \text{Terminating.}\\ (b)\ \text{Circulating.}\begin{cases}(1)\ \text{Pure.}\\ (2)\ \text{Mixed.}\end{cases}\end{cases}$$

Exercises similar to the following should be given :—

(1) What kind of decimal will be produced by each of the following:—$\frac{3}{4}$, $\frac{5}{6}$, $\frac{17}{18}$, $\frac{9}{11}$, $\frac{25}{30}$, $\frac{21}{40}$, $\frac{47}{400}$, $\frac{23}{24}$?

(2) How many digits are there in the decimal arising from each of the following:—$\frac{9}{10}$, $\frac{3}{8}$, $\frac{7}{20}$, $\frac{19}{80}$, $\frac{59}{64}$, $\frac{61}{256}$?

(3) Reduce $\frac{9}{20}$, $\frac{53}{70}$, $\frac{11}{15}$, $\frac{17}{18}$ to decimals.

(4) What kind of circulating decimals will the following produce:—$\frac{3}{7}$, $\frac{5}{6}$, $\frac{9}{11}$, $\frac{4}{33}$, $\frac{73}{90}$?

(5) How many figures are there in the finite parts of the decimals arising from $\frac{11}{15}$, $\frac{67}{375}$, $\frac{22}{27}$, $\frac{19}{44}$?

XI. CIRCULATING DECIMALS.

(*a*) The Notation of Circulating Decimals.

Introduction.

Reduce the following to decimals:—

$\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{2}{3}$, $\frac{5}{9}$, $\frac{7}{11}$, $\frac{5}{12}$, $\frac{7}{22}$, $\frac{175}{444}$.

Blackboard shows:—

$\frac{1}{2}=.5$	$\frac{2}{3}=.666\ldots$	$\frac{5}{12}=.41666\ldots$
$\frac{3}{4}=.75$	$\frac{5}{9}=.555\ldots$	$\frac{7}{22}=.3181818\ldots$
$\frac{7}{8}=.875$	$\frac{7}{11}=.6363\ldots$	$\frac{175}{444}=.39414414414\ldots$

The teacher should now explain that the figures which recur are the repetends and show the means adopted to indicate a repetend, viz.: to put a dot over the figure that is repeated and if more than one, to put a dot over the first and last figures which are repeated.

Thus,	$\frac{2}{3}=.\dot{6}$	$\frac{5}{12}=.41\dot{6}$
	$\frac{5}{9}=.\dot{5}$	$\frac{7}{22}=.3\dot{1}\dot{8}$
	$\frac{7}{11}=.\dot{6}\dot{3}$	$\frac{175}{444}=.39\dot{4}1\dot{4}$

Points to be noticed are :—

(1) $.\dot{6}$, $.\dot{5}$, $.\dot{6}\dot{3}$, $.41\dot{6}$, $.3\dot{1}\dot{8}$, and $.39\dot{4}1\dot{4}$ are circulating decimals.

(2) 6, 5, 63, 6, 18, and 414 are the repetends, *i.e.*, the figure or figures that are repeated.

(3) That in some cases the repetend begins at the decimal point, and in others there are one or more figures between the repetend and the point.

(4) That the former are PURE Circulating Decimals and the latter are MIXED Circulating Decimals.

(5) That a Pure Circulating Decimal consists of one part, the infinite part.

(6) That a Mixed Circulating Decimal consists of two parts, a part which is *not* repeated, the finite part, and the part which is repeated, the infinite part.

(*b*) **Reduction of Pure Circulating Decimals to Vulgar Fractions.**

Two modes of procedure are in use :

(1)	$\frac{1}{9}=.\dot{1}$; $\therefore$ $.\dot{1}=\frac{1}{9}$.	$\frac{1}{999}=.\dot{0}0\dot{1}$; $\therefore$ $.\dot{0}0\dot{1}=\frac{1}{999}$.
	$\frac{7}{9}=.\dot{7}$; $\therefore$ $.\dot{7}=\frac{7}{9}$.	$\frac{23}{999}=.\dot{0}2\dot{3}$; $\therefore$ $.\dot{0}2\dot{3}=\frac{23}{999}$.
	$\frac{1}{99}=.\dot{0}\dot{1}$; $\therefore$ $.\dot{0}\dot{1}=\frac{1}{99}$.	$\frac{157}{999}=.\dot{1}5\dot{7}$; $\therefore$ $.\dot{1}5\dot{7}=\frac{157}{999}$.
	$\frac{25}{99}=.\dot{2}\dot{5}$; $\therefore$ $.\dot{2}\dot{5}=\frac{25}{99}$.	$\frac{7142}{9999}=.\dot{7}14\dot{2}$; $\therefore$ $.\dot{7}14\dot{2}=\frac{7142}{9999}$.

From these and other examples it may be inferred that *a pure circulating decimal may be expressed as a vulgar fraction by writing the figures which are repeated for the numerator of the fraction and placing for denominator as many 9's as there are figures in the repetend.*

NOTE.—It is sometimes objected that $.\dot{1}$, $.\dot{2}\dot{6}$, etc., are not decimals as defined, viz., fractions that have 10 or a power of 10 for denominator.

This objection vanishes when it is remembered that when written as $.\dot{1}$, or $.\dot{2}\dot{6}$, the circulating decimal represents A SUM.

$$\text{Thus,}\quad .\dot{1} = .11111\ldots = \tfrac{1}{10}+\tfrac{1}{100}+\tfrac{1}{1000}+\tfrac{1}{10000}+\text{etc.}$$

$$.\dot{2}\dot{6} = .262626\ldots = \tfrac{26}{100}+\tfrac{26}{10000}+\tfrac{26}{1000000}+\text{etc.}$$

(2) Reduce $.\dot{7}85\dot{6}$ to a vulgar fraction.

Let the vulgar fraction $=.\dot{7}85\dot{6}$;

$\therefore$ 10000 times the vulgar fraction $=7856.\dot{7}85\dot{6}$;

and 1 time " " " $=.\dot{7}85\dot{6}$;

$\therefore$ 9999 times " " " $=7856$;

$\therefore$ the vulgar fraction $=\frac{7856}{9999}$.

(c) Reduction of Mixed Circulating Decimals to Vulgar Fractions.

Ex. 1. Reduce $.2\dot{5}$, $.1\dot{2}\dot{5}$, $.12\dot{5}$ and $.01\dot{5}7\dot{9}$ to equivalent vulgar fractions.

$$.2\dot{5}=.2\tfrac{5}{9}=\frac{2\frac{5}{9}}{10}=\tfrac{23}{90}=\frac{25-2}{90}.$$

$$.1\dot{2}\dot{5}=.1\tfrac{25}{99}=\frac{1\frac{25}{99}}{10}=\tfrac{124}{990}=\frac{125-1}{990}.$$

$$.12\dot{5}=.12\tfrac{5}{9}=\frac{12\frac{5}{9}}{100}=\tfrac{113}{900}=\frac{125-12}{900}.$$

$$.015\dot{7}\dot{9}=.015\tfrac{79}{99}=\frac{15\frac{79}{99}}{1000}=\tfrac{1564}{99000}=\frac{1579-15}{99000}.$$

From these examples it will be readily seen that *a Mixed Circulating Decimal is expressed as a vulgar fraction by subtracting the finite part of the decimal from the whole decimal and placing the remainder as numerator, and for the denominator placing as many 9's as there are figures in the repetend, followed by as many ciphers as there are figures in the finite part.*

Ex. 2. Reduce $.34\dot{2}5\dot{7}$ to a vulgar fraction.

Let the vulgar fraction $=.34\dot{2}5\dot{7}$;

100000 times " " " $=34257.\dot{2}5\dot{7}$;

and 100 " " " " $=34.\dot{2}5\dot{7}$;

$\therefore$ 99900 " " " " $=34257-34$;

and " " " $=\dfrac{34257-34}{99900}$

$=\frac{34223}{99900}$.

(d) Addition and Subtraction of Circulating Decimals.

Sufficient accuracy may, in general, be attained by extending the repetend a few places and adding or subtracting as in finite decimals.

When perfect accuracy is required proceed as follows:—

(1) Make the decimals SIMILAR, *i.e.*, make them all begin to repeat at the same place to the right of the point.

This can be done as follows:—

(i) Regard a pure circulating decimal as a mixed one.

Thus, $.\dot{3}=.3\dot{3}=.33\dot{3}$, etc.

(ii) Make a mixed circulating decimal begin to repeat at any place greater than the number in the finite part.

Thus, $.23\dot{4}5\dot{7}=.234\dot{5}7\dot{4}=.2345\dot{7}4\dot{5}$, etc.

Ex. 1. Add $.\dot{3}$, $.3\dot{2}\dot{4}$, $.23\dot{4}\dot{5}$ and $.834\dot{5}6\dot{7}$.

$$\begin{aligned} .\dot{3} &= .333\dot{3} \\ .3\dot{2}\dot{4} &= .324\dot{2}\dot{4} \\ .23\dot{4}\dot{5} &= .234\dot{5}\dot{4} \\ .834\dot{5}6\dot{7} &= .834\dot{5}6\dot{7} \end{aligned}$$

(2) Having made the decimals similar, next proceed to make them COTERMINOUS, *i.e.*, make the repetends end at the same place to the right of the point.

This is done by finding the L.C.M. of the number of places in each repetend and repeating each repetend to that number of places.

Thus, the number of places in the repetends is 1, 2, 2, and 3.

L.C.M. of 1, 2, 2, and 3 is 6.

$$\begin{aligned} \text{Thus,}\quad .\dot{3} &= .333\dot{3}3333\dot{3} \\ .3\dot{2}\dot{4} &= .324\dot{2}4242\dot{4} \\ .23\dot{4}\dot{5} &= .234\dot{5}4545\dot{4} \\ .834\dot{5}6\dot{7} &= \underline{.834\dot{5}6756\dot{7}} \\ & 1.726\dot{6}8877\dot{9} \end{aligned}$$

The number to be carried to the right hand column is found by adding the first column of repeating figures, thus $5+5+2+3=15$; carry 1.

Ex. 2. From $7.9\dot{8}4\dot{5}$ take $5.78\dot{5}638\dot{7}$.

$$\begin{aligned} 7.9\dot{8}4\dot{5} &= 7.984\dot{5}8\dot{4} = 7.984\dot{5}8458458458\dot{4} \\ 5.78\dot{5}638\dot{7} &= 5.78\dot{5}638\dot{7} = \underline{5.785\dot{6}3876387638\dot{7}} \\ & \phantom{= 5.78\dot{5}638\dot{7} =} 2.198\dot{9}4582070819\dot{6} \end{aligned}$$

(e) Multiplication and Division of Circulating Decimals.

Sufficient accuracy can generally be secured by extending the repetends a few places and then multiplying or dividing as in ordinary decimals.

When perfect accuracy is required, the decimals must be reduced to their equivalent vulgar fractions and then these are multiplied or divided and the result is reduced to a circulating decimal.

XII. DECIMALS OF DENOMINATE NUMBERS.

1. To find the value of a decimal of a Denominate Number.

Ex. 1. Find the value of £.875.

$$\begin{array}{r} £.875 \\ \underline{20} \\ 17.500s. \\ \underline{12} \\ 6.000d. \end{array}$$

.875 of £1 = .875 of 20s. = 17.500s.
.5 of 1s. = .5 of 12d. = 6d.
∴ £.875 = 17s. 6d.

Ex. 2. Express 2.375 bu. as a compound number.

$$\begin{array}{r} 2.375 \text{ bu.} \\ \underline{4} \\ 1.500 \text{ pk.} \\ \underline{2} \\ 1.000 \text{ gal.} \end{array}$$

2.375 bu. = 2 bu. + .375 of 1 bu.
.375 of 1 bu. = .375 of 4 pk. = 1.500 pk.
.5 of 1 pk. = .5 of 2 gal. = 1 gal.
∴ 2.375 bu. = 2 bu. 1 pk. 1 gal.

2. To express a compound number as a decimal of a higher denomination.

Ex. 1. Reduce 4 cwt. 3 lb. 8 oz. to the decimal of a ton.

$$\begin{array}{l} 16)\underline{8 \text{ oz.}} \\ 100)\underline{3.5 \text{ lb.}} \\ 20)\underline{4.035 \text{ cwt.}} \\ \quad .20175 \text{ t.} \end{array}$$

8 oz. = $\frac{8}{16}$ lb. = .5 lb.;
∴ 3 lb. 8 oz. = 3.5 lb.

$$3.5 \text{ lb.} = \frac{3.5}{100} \text{ cwt.} = .035 \text{ cwt.};$$

∴ 4 cwt. 3 lb. 8 oz. = 4.035 cwt.;

$$4.035 \text{ cwt.} = \frac{4.035}{20} \text{ t.} = .20175 \text{ t.}$$

Or, 4 cwt. 3 lb. 8 oz. = 6456 oz.;
1 t. = 32000 oz.;
∴ 4 cwt. 3 lb. 8 oz. is $\frac{6456}{32000}$ of 1 t.;
and $\frac{6456}{32000}$ = .20175.

Ex. 2. Express £2 17s. 6d. as the decimal of £5.

£2 17s. 6d. = 690d.;
£5 = 1200d.;
∴ £2 17s. 6d. is $\frac{690}{1200}$ of £5.
Now, $\frac{690}{1200}$ = .575;
∴ £2 17s. 6d. is .575 of £5.

Ex 3. Express $\frac{3}{8}$ of 2 qr. 14 lb. as the decimal of $\frac{2}{25}$ of 1 ton.

$\frac{3}{8}$ of 2 qr. 14 lb. = $\frac{3}{8}$ of 64 lb. = 24 lb.;
$\frac{2}{25}$ of 1 t. = $\frac{2}{25}$ of 2000 lb. = 160 lb.;
∴ $\frac{3}{8}$ of 2 qr. 14 lb. is $\frac{24}{160}$ of $\frac{2}{25}$ of 1 t.
Now, $\frac{24}{160} = \frac{3}{20}$ = .15;
∴ $\frac{3}{8}$ of 2 qr. 14 lb. is .15 of $\frac{2}{25}$ of 1 t.

XIV. PERCENTAGE.

I. THE DEFINITION.

Present many problems as:—

(1) A drover lost 7 out of every 100 sheep he had; how many sheep did he lose out of 900?

(2) A merchant gained \$20 on every \$100 he had in business: how much did he gain on \$1200?

(3) A company charges \$2 for every \$100 of insurance; how much would it charge for \$700?

(4) A boy is absent from school 3 days out of every 100 school days; how many days is he absent in 200 school days?

Tell the class that the usual way of stating these problems is to say 7 per cent. instead of 7 out of every 100;

20 " " " \$20 " " \$100;
2 " " " \$2 " " \$100;
3 " " " 3 da. " 100 da.

Thus it will be seen that per cent. (per centum) means FOR OR BY 100, and being a contraction is to be written with a period.

Explain that business men compute their gains and losses at so much per cent.

Also that the symbol % is used for the words per cent.

Ask many questions as 8 is what per cent. of 100?
8 " " " " " 50?
8 " " " " " 200?
8 " " " " " 25?
8 " " " " " 400?

What is 5% of 100? of 200? of 300? of 400?

II. THE RELATION OF PER CENT. TO DECIMALS.

(*a*) Much practice should be given in reading per cents. as decimals and expressing them as such.

Thus, 5% is read as 5 hundredths and is written .05.
11% " " " 11 " " " " .11.
25% " " " 25 " " " " .25.

EXAMPLES. Write as decimals the following:—

15%, 35%, 40%, 37½%, 5¼%, 8½%, 150%, 112½%, 1¾%.

(*b*) The converse of this should also be given, *i e.*, decimals should be expressed as per cents.

Thus, .25 is read 25 hundredths and is 25%.
.07 " " 7 " " " 7%.
.12½% " 12½ " " " 12½%.

EXAMPLES. Read the following decimals as per cents.:
.15, .20, .75, .16⅔, .05, .06⅔.

III. THE RELATION OF PER CENT. TO VULGAR FRACTIONS.

(*a*) Give practice in expressing per cent., first as decimals and then as common fractions, and finally change the per cents. to common fractions without having them read as decimals.

Thus, 5% is 5 hundredths, or .05, or $\frac{5}{100}$, or $\frac{1}{20}$.
7% " 7 " " .07, or $\frac{7}{100}$.
25% " 25 " " .25, or $\frac{25}{100}$, or $\frac{1}{4}$.

Then $20\% = \frac{20}{100} = \frac{1}{5}$. $50\% = \frac{50}{100} = \frac{1}{2}$.

$12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{1}{8}$. $62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{5}{8}$.

(*b*) Give practice in expressing vulgar fractions in their equivalent per cents.

$\frac{1}{4} = \frac{1}{4}$ of 100 hundredths = 25 hundredths = 25%.
$\frac{3}{8} = \frac{3}{8}$ of 100 " = 37½ " = 37½%.
$\frac{1}{3} = \frac{1}{3}$ of 100 " = 33⅓ " = 33⅓%.

The following shows the principle involved:—

Thus, $\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = 25\%$. $\frac{3}{8} = \frac{3 \times 12\frac{1}{2}}{8 \times 12\frac{1}{2}} = \frac{37\frac{1}{2}}{100} = 37\frac{1}{2}\%$.

$\frac{1}{3} = \frac{1 \times 33\frac{1}{3}}{3 \times 33\frac{1}{3}} = \frac{33\frac{1}{3}}{100} = 33\frac{1}{3}\%$. $\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100} = 40\%$.

NOTE 1.—As the denominator of the fraction must be changed into 100, both terms of the fraction must be multiplied by the quotient arising from dividing 100 by the denominator of the vulgar fraction.

NOTE 2.—It will now be clear that per cent. can be expressed in four ways:—

(1) By using the words per cent.
(2) " " " symbol %.
(3) " " a decimal fraction.
(4) " " a vulgar fraction.

The following should be memorized:—

$12\frac{1}{2}\% = \frac{1}{8}$	$37\frac{1}{2}\% = \frac{3}{8}$	$87\frac{1}{2}\% = \frac{7}{8}$.
$16\frac{2}{3}\% = \frac{1}{6}$	$40\% = \frac{2}{5}$	$6\frac{1}{4}\% = \frac{1}{16}$.
$20\% = \frac{1}{5}$	$50\% = \frac{1}{2}$	$6\frac{2}{3}\% = \frac{1}{15}$.
$25\% = \frac{1}{4}$	$66\frac{2}{3}\% = \frac{2}{3}$	$8\frac{1}{3}\% = \frac{1}{12}$.
$33\frac{1}{3}\% = \frac{1}{3}$	$75\% = \frac{3}{4}$	$10\% = \frac{1}{10}$.

IV. APPLICATION TO TYPE PROBLEMS.

There are three things involved in every example of per cent. Hence, there arise three cases as follows:—

Case I. To find any per cent. of a given quantity.

Case II. To find what per cent. one number is of another.

Case III. To find the quantity when some per cent. of it is given.

Case I. Ex. 1. A drover having 2500 sheep sold 43% of them. How many did he sell?

(*a*) $43\% = \frac{43}{100}$

$\frac{43}{100}$ of 2500 sheep = 1075 sheep.

(*b*) Or, $43\% = .43$;

.43 of 2500 sheep = 2500 sheep × .43 = 1075.00 sheep.

Case II. Ex. 2. A drover having 2500 sheep sold 1075 of them. What per cent. of his sheep did he sell?

(*a*) No. sold from 2500 sheep = 1075;

∴ " " " 100 " $= \frac{1075}{25}$, or 43;

∴ 43% of the sheep were sold.

(*b*) Or, 1075 sheep is $\frac{1075}{2500}$ of 2500 sheep;

and $\frac{1075}{2500} = \frac{1075}{2500}$ of 100 hundredths = 43 hundredths = 43%.

Case III. Ex. 3. A drover sold 1075 sheep; this was 43% of the number he had. How many had he at first?

$43\% = \frac{43}{100}$;

∴ $\frac{43}{100}$ of the number of sheep = 1075;

∴ " " " " $= \frac{100}{43}$ of 1075

$= 100 \times 25$

$= 2500$.

XV. APPLICATIONS OF PERCENTAGE.

NOTE.—The difficulty of applying percentage to commercial transactions lies in the pupil NOT COMPREHENDING THE NATURE OF THE BUSINESS INVOLVED. Hence, before problems are solved, MUCH CARE SHOULD BE GIVEN TO AN EXPLANATION OF THE TRANSACTION. This should be illustrated, as far as possible, by regular business forms, as catalogues, policies of insurance, application forms, interim receipts, etc. The more concrete the illustrations are, the better for the subject. Mere terms should not be taught. The topics involved should be discussed, explained, and illustrated.

These illustrations and explanations will remove the difficulties arising from the language of the problems. The pupils will be readily able to read them, *i.e.*,

picture the transactions and hence will be easily able to make the relations that determine the calculations to be performed.

I. TRADE DISCOUNT.

I. Explanation of Terms.

Manufacturers and wholesale dealers generally catalogue their goods at certain fixed prices. These are usually *the retailers selling price.*

The price is variously called the catalogue, list, gross, or invoice price.

A deduction is made from this price to enable the retailer to make his profit. If the catalogue fell into the hands of the purchaser, he would not think that the retailer was charging him too much for the goods. Whereas, if the net price were printed in the catalogue, the purchaser would naturally feel that he was being overcharged. Such a catalogue, also, enables the manufacturer or wholesale dealer to meet the fluctuations of the market by merely changing the rate of discount and thus the expense of printing a new catalogue is avoided.

Sometimes a second deduction is allowed off what remains after deducting the first one. This is to induce prompt payment by the purchaser.

At times a third deduction is made on certain kinds of goods, but this is a matter of special arrangement between buyer and seller.

These various deductions are known as TRADE OR COMMERCIAL DISCOUNT.

Trade Discount is always reckoned at a certain rate per cent. The first discount is deducted from the list price; the next one from what remains; etc.

The final result is not affected by the order in which the discounts are made. Thus, discounts of 40%, 10%, and 5% are the same as those of 10%, 5%, and 40%; or as those of 5%, 40%, and 10%; etc.

II. Application to Solution of Problems.

Case I. To find the discount having given the amount of the bill and the rates of discount.

EX. 1. Find the discount off a bill of $120, the rates of discount being 25%, 10%, and 5%.

Amount of bill = \$120
First discount at 25% = 30
\$90
Second " " 10% = 9
\$81
Third " " 5% = 4.05
Net price = \$76.95
Total discount = \$(120 − 76.95) = \$43.05.

Case II. To find the bill having given the rates of discount and the net price.

Ex. 2. What was the bill for which \$114 was accepted as payment, the rates of discount being 40% and 5%?

Part of bill remaining after the first discount $= \frac{60}{100}$ of bill.
Second " $= \frac{3}{100}$ "
∴ " " " " " two discounts $= \frac{57}{100}$ "
Hence, $\frac{57}{100}$ of the bill = \$114;
∴ " " $= \frac{100}{57}$ of \$114
= \$200.

Or, part of bill remaining after the first discount $= \frac{60}{100}$ of bill;
" " " " " second " $= \frac{95}{100}$ of $\frac{60}{100}$ of bill;
∴ $\frac{95}{100}$ of $\frac{60}{100}$ of bill = \$114;
∴ the bill $= \frac{100}{95}$ of $\frac{100}{60}$ of \$114
= \$200.

Case III. To find the rate of discount having given the bill and the net price.

Ex. 3. What is the rate of discount when a bill of \$375 is settled by a payment of \$250?

Amount of discount = \$(375 − 250) = \$125.
\$125 is $\frac{125}{375}$ of \$375;
∴ rate of discount $= \frac{125}{375} = \frac{1}{3} = 33\frac{1}{3}\%$.

Case IV. To find a single discount equivalent to two or more rates.

Ex. 4. What single discount is equivalent to discounts of 30% and 10%?

Part of bill remaining after first discount $= \frac{70}{100}$ of bill;
Second discount at 10% $= \frac{7}{100}$ "
∴ part of bill remaining after second discount $= \frac{63}{100}$ "
Hence, single discount $= (\frac{100}{100} - \frac{63}{100})$ of bill $= \frac{37}{100}$ "
= 37%.

II. PROFIT AND LOSS.

In Profit and Loss there are four things to be considered:—

(1) The COST PRICE, or the price at which the goods are bought.

(2) The SELLING PRICE, or the price at which the goods are sold.

(3) The difference between the cost price and the selling price which constitutes THE PROFIT or THE LOSS according as the selling price exceeds or falls short of the cost price.

(4) The GAIN OR LOSS PER CENT.

The gain or loss per cent. is usually expressed as a certain per cent. of the cost price. When this is not the intention, care should be taken to express what is meant.

Before attempting to work problems, the pupil should acquire facility in expressing the selling price, and the gain or the loss, as a certain per cent. of the cost price. Inability to do this readily is a source of trouble in Profit and Loss.

Case I. Given the cost price and the selling price to find the profit or the loss.

Ex. 1. A merchant bought goods for \$257 and sold them for \$310. Find his profit.

Selling price=\$310
Cost " =\$257
Profit= \$53

Case II. Given the cost price and the selling price to find the gain or loss per cent.

Ex. 2. A merchant bought cloth at 5c. per yard and sold it at 8c. per yard. Find his gain per cent.

$$\text{Gain on } 5c. = (8-5)c. = 3c.;$$

$$\therefore \text{ `` `` } 100c. = \frac{100\times 3}{5}c. = 60c.;$$

$$\therefore \text{gain} = 60\%.$$

$$\text{Or, gain} = \tfrac{3}{5} \text{ of cost price};$$

$$\tfrac{3}{5} = \frac{3\times 20}{5\times 20} = \tfrac{60}{100} = 60\%;$$

$$\therefore \text{gain} = 60\%.$$

Case III. Given the cost price and the gain or loss per cent. to find the selling price.

Ex. 3. A merchant bought cloth at 5c. a yard and sold it to gain 60%. Find the selling price.

Selling price $=\frac{160}{100}$ of cost price $=\frac{160}{100}$ of 5c. = 8c.

Case IV. Given the selling price and the gain or loss per cent. to find the cost price.

Ex. 4. A merchant sold cloth at 8c. a yard and thereby gained 60%. Find the cost price.

Selling price $=\frac{160}{100}$ of cost price;

$\therefore$ $\frac{160}{100}$ of cost price = 8c.;

$\therefore$ " " $=\frac{100}{160}$ of 8c. = 5c.

III. COMMISSION.

1. Explanations.

An Agent is a person employed to buy, or sell goods, collect accounts, and transact business for another.

The Principal is the person for whom the business is transacted.

The Agent is variously spoken of as a Commission Merchant, Factor, Collector, Broker and Consignee.

When goods are sent to the agent to be sold, the goods are the Consignment; the person sending them is the Consignor; and the agent is the Consignee.

The Gross Proceeds of a sale or collection is the total amount received by the agent for the goods before deducting his commission and other charges.

The Net Proceeds is what remains after all charges have been deducted.

At times the Agent is called a Broker, and the sum paid to him is known as Brokerage. The term, Commission, is used in some kinds of business, and Brokerage in others. When the Agent is placed in possession of the goods bought or sold, he is usually called a Commission Merchant; but, if the Agent contracts to buy or sell in the name of his Principal and is not put into possession of the goods, he is called a Broker.

The Commission or Brokerage depends upon the amount of the transaction and is stated at so much per cent. of the sum received for sales, expended for purchases, collected from debtors, etc.

Note.—It should be made clear that Commission is to be charged *only on the money actually employed in the business transaction.*

2. Cases of Commission.

There are three things involved in Commission, viz., the SUM involved in the business transaction, the RATE of commission and the COMMISSION.

These give rise to three cases similar to those of Percentage.

Care must, however, be taken with those problems in which the commission is to be deducted from a sum and the remainder invested.

Ex. 1. A sent his agent \$2255 with instructions to deduct his commission at $2\frac{1}{2}\%$ and invest the remainder in sugar. How much was invested in sugar?

Commission on $\$100=\$2\frac{1}{2}$;

$\therefore$ sum invested out of $\$102\frac{1}{2}=\$100$;

$\therefore$ " " " " $\$2255=\$\frac{2255\times100}{102\frac{1}{2}}=\$2200$.

IV. INSURANCE.

1. Explanations.

Companies are organized to insure property of various kinds from loss by fire, water, wind, lightning, or other specified causes.

Insurance Companies are of two main classes, STOCK COMPANIES and MUTUAL COMPANIES.

A STOCK INSURANCE COMPANY is one in which the capital is owned by the members of the company, who are called STOCKHOLDERS. These share the profits and are responsible for the losses, in proportion to the capital which each one owns.

IN A MUTUAL INSURANCE COMPANY the persons whose properties are insured become members of the company and give notes for the premium. These notes are called PREMIUM NOTES and they render the makers liable for their proportionate shares of the salaries of the managers, expenses of the office, and possible losses. After these expenses are met, if there is any surplus it is returned to the members of the company pro rata.

There are various kinds of PROPERTY INSURANCE:—

(1) *Fire Insurance*, or indemnity for loss by fire.

(2) *Marine Insurance*, or indemnity for loss or damage of vessels or their cargoes by the accidents of navigation.

(3) *Insurance of Live Stock*, or indemnity for loss of horses, cattle, etc., by lightning, or other specified causes.

(4) *Transit Insurance*, or indemity for loss or damage to merchandise during transportation from one place to another.

(5) *Accident Insurance*, or indemnity for breakage of plate glass, etc.

Fire Insurance Companies usually insure property for only a part of its value so that the person insured may be interested in its protection.

When a person wishes to effect an insurance, he makes application to a company, usually through an agent who supplies him with a blank form to be filled in and signed.

If the agent approves of the application, he gives the applicant an INTERIM RECEIPT for the premium paid. This binds the company until the policy is issued by the company or the application has been declined by it. In the latter case, the part of the premium proportionate to the unexpired time is returned.

When loss or damage occurs to the property, the insured fills in a CLAIM PAPER stating his loss. This is forwarded to the company. An inspector is sent to the place, who reports to the company the extent of the loss.

If the property is completely destroyed, the sum insured is paid. If the loss is partial, the company agrees to pay the full value of the property destroyed, provided this does not exceed the sum mentioned in the policy.

2. Case of Insurances.

There are three things involved in Insurance, viz., the SUM for which the property is insured, the RATE of insurance, and the PREMIUM.

These give rise to three cases similar to those of Percentage and Commission.

To test the understanding of pupils, there is a fourth kind of problem given at times, viz., **to determine for how much property of a certain value must be insured so that the owner may recover both the value of the property and the premium paid.**

Ex. 1. For what sum must a house worth \$3950 be insured at $1\frac{1}{4}\%$ so that in case it is totally destroyed the owner may sustain no loss?

A policy of \$100 will pay for $\$1\frac{1}{4}$ premium and $\$98\frac{3}{4}$ of property. Hence, sum for which property worth $\$98\frac{3}{4}$ is insured $=\$100$;

$\therefore$ " " " " " \$3950 " " $=\$\left(\frac{3950\times 100}{98\frac{3}{4}}\right)$

$=\$4000.$

V. TAXES.

1. Explanations.

The affairs of Canada are carried on by means of the Dominion, Provincial, and Municipal Governments, or authorities. Money is needed to enable these parties to discharge their duties. The money raised for such purposes is a TAX.

When the tax is paid directly by the person who is to bear the burden, it is a DIRECT TAX and is called a TAX.

When the tax is paid by a person who expects to indemnify himself by charging it to another, the tax is an INDIRECT TAX and is called a DUTY.

Illustrate the use made of the money collected by reference to the cost of making roads, lighting streets, building bridges, erecting and maintaining municipal buildings, paying firemen, policemen, municipal officers as the township clerk, assessor, collector, etc.

For purposes of direct taxation property is regarded as REAL or PERSONAL.

REAL PROPERTY, OR REAL ESTATE, consists of *fixed* property, as lands and houses.

PERSONAL PROPERTY consists of *movable* property, as cash, stocks of banks and companies, merchandise, cattle, income, etc.

Officers called ASSESSORS or ASSESSMENT COMMISSIONERS are appointed by the township, town, and city council to estimate the value of all the taxable property in the municipality.

The report which the Assessor makes to the Council is known as the ASSESSMENT ROLL. On it he has entered the name, address, estimated value of property (real and personal) of each resident of the municipality.

When the municipal authorities know how much taxable property there is in the municipality and also what the expenses for the year will be, the RATE OF TAXATION is found by dividing the sum to be raised by the measure of the value of all the property to be taxed. The quotient is the *tax on the dollar* and is usually expressed in mills, or thousandths of a dollar.

When the tax on the dollar is known, the sum to be paid by each rate-payer is then computed. This sum is collected by a person called the COLLECTOR, appointed by the municipal council.

2. Cases of Direct Taxes.

In direct taxation three things are involved, (1) the ASSESSED VALUE of the property which is taxed, (2) the RATE, and (3) the AMOUNT of the tax.

Case I. Given the assessed value of the property and the rate, to find the tax.

Ex. 1. Find the tax on property assessed for \$5400 at $15\frac{1}{2}$ mills on the dollar.

Tax on \$1 $=\$.0155$;

$\therefore$ " " \$5400 $=\$(5400\times.0155)=\$83.70$.

Case II. Given the assessed value of the property and the amount of the tax, to find the rate.

Ex. 2. What is the rate of taxation when \$56 is the tax upon property assessed for \$3500?

Tax on \$3500 $=\$56$;

$\therefore$ " " \$1 $=\$\frac{56}{3500}=\$.016$.

$\therefore$ the rate is 16 mills on the dollar.

Case III. Given the amount of the tax and the rate, to find the assessed value of the property.

Ex. 3. What is the assessed value of property on which the tax is \$31.25 at $12\frac{1}{2}$ mills on the dollar.

Value of property when \$.0125 is the tax $=\$1$;

$\therefore$ " " " " \$31.25 " " $=\$\frac{31.25\times1}{.0125}=\$2500$.

3. INDIRECT TAXES are of two kinds, CUSTOMS DUTIES and EXCISE DUTIES.

All goods entering Canada from foreign countries are required by law to be landed at certain places called PORTS OF ENTRY.

At each port of entry the Dominion Government has an establishment, called a CUSTOM HOUSE, with one or more officers attached to it, called CUSTOM HOUSE OFFICERS. These officers inspect the goods, examine the invoices, collect the duties, etc.

AN INVOICE is a statement detailing the kind and quality of the articles shipped or sent to a purchaser or agent with their weight or amount, and the cost of each article. Invoices are made out in the currency, and weights and measures of the country from which the importation is made.

Customs Taxes or Duties produce a revenue for the support of the Government and also serve to protect home industries.

Certain articles such as spirituous or malt liquors, cigars, snuff, etc., manufactured in Canada are required by law to pay duty. This is an EXCISE DUTY.

At each place where these are manufactured, there are one or more Government officials, called EXCISE OFFICERS, who check the quantities manufactured, see that none are disposed of without paying duty, etc.

Duties, whether Customs or Excise, may be of two kinds, AD VALOREM and SPECIFIC.

The rate of duty and its kind is stated in the TARIFF which is a schedule of goods and the rate of duty imposed by law upon them.

4. Cases of Indirect Taxes.

In Indirect Taxation three things are involved:—(1) The VALUE of the goods, (2) the RATE of duty, (3) the AMOUNT of the duty.

The three cases arising from finding one of these when the other two are given are similar to those of Percentage.

The following is suggested as headings for teaching this subject:—

TAXATION.

(*a*) Direct.
(*b*) Indirect.

(*a*) Direct Taxes, called Taxes.
(1) By whom levied.
(2) On what levied.
(3) For what purposes levied.
(4) Basis of reckoning—the rate on the dollar and the method of obtaining this rate.
(5) Assessor, Assessment and Collector.
(6) Problems.

(*b*) Indirect Taxes, called Duties.
(1) Customs Duties.
(2) Excise Duties.
(3) By whom levied.
(4) How, when, where collected.
(5) By whom paid.
(6) The Tariff, Ad Valorem, and Specific Duties. Japan has lost
(7) Problems.

VI. INTEREST.

NOTE 1.—Some arithmetics introduce Stocks before Interest, but as transactions in Stocks are difficult for one unacquainted with the subject, it is believed advisable to postpone their consideration until Interest has been studied.

NOTE 2.—Since a new element, time, enters into the calculations in Interest, it is necessary to give more attention to this application of Percentage than to some of the others.

1. Explanations.

Illustrate Interest by calling attention to what is paid when a horse is hired at a livery, when a house or farm is rented, when a man's labor is employed.

Make clear that the money paid is for the USE of the horse, or the house, or the farm, or the labor.

Point out that the rent varies with the *value* of the house and the *time* it is rented.

Apply this to money:—

Instead of getting the use of a house, one who gets the use of money pays for this use in proportion to the *sum* borrowed and the *time* he has its use.

Introduce the terms, Principal and Interest.

Money loaned to another is called PRINCIPAL.

The sum paid for its *use* is the INTEREST.

Money employed by a person in his own business is called CAPITAL.

Interest is computed at a certain RATE PER CENT. per annum.

At this stage give many mental examples in finding the interest.

Introduce the term, Amount, by means of an example, as:— A borrowed \$200 from B for a year at 8% per annum. How much must A pay B at the end of the year?

Make clear how the \$216 is made up, and give the term, AMOUNT.

The AMOUNT consists of the Principal (which is constant) and the Interest (which varies with the Principal, Rate, and Time).

When Interest is paid on the Principal only, it is called SIMPLE INTEREST.

When the Interest remains unpaid after it falls due, it is sometimes added to the Principal and the Interest for the second period is computed on this increased Principal. This is known as COMPOUND INTEREST.

In computing interest for $\frac{1}{2}$ yr., $\frac{1}{3}$ yr., $\frac{1}{4}$ yr., etc., it is usual to reckon a certain rate per cent. per annum, as equivalent to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., this rate for the period specified.

Thus, 8% per annum is regarded as equivalent to 4% each $\frac{1}{2}$ yr., $2\frac{2}{3}$% each $\frac{1}{3}$ yr., or 2% each $\frac{1}{4}$ yr.

This is, however, not strictly accurate.

Thus, the interest on \$100 at 8% for 1 yr. is \$8; at 4% per $\frac{1}{2}$ yr. it is \$8.16.

2. Cases of Interest.

Case I. Given the Principal, the Rate per cent. and the Time, to find the Interest or Amount.

Ex. 1. What is the interest on \$350 for $2\frac{1}{2}$ yr. at 6% per annum?

Interest on \$1 for 1 yr. $=\$\frac{6}{100}$;

$\therefore$ " " \$1 " $2\frac{1}{2}$ yr. $=\$(2\frac{1}{2}\times\frac{6}{100})$;

$\therefore$ " " \$350 " $2\frac{1}{2}$ yr. $=\$(350\times2\frac{1}{2}\times\frac{6}{100})$

$=\$52.50$.

Or, Interest for 1 yr. $=\frac{6}{100}$ of \$350;

$\therefore$ " " $2\frac{1}{2}$ yr. $=2\frac{1}{2}\times\frac{6}{100}$ of \$350

$=\$52.50$.

Case II. Given the Principal, Interest or Amount and Time, to find the Rate per cent.

Ex. 2. At what rate per cent. must \$350 be put out at interest for $2\frac{1}{2}$ yr. to yield \$52.50?

Interest on \$350 for $2\frac{1}{2}$ yr. $=\$52.50$;

$\therefore$ " " \$350 " 1 yr. $=\frac{2}{5}$ of \$52.50;

$\therefore$ " " \$100 " 1 yr. $=\frac{100}{350}$ of $\frac{2}{5}$ of \$52.50;

$=\$6$;

$\therefore$ rate $=6\%$.

Or, Interest for $2\frac{1}{2}$ yr. $=\$52.50$;

$\therefore$ " " 1 yr. $=\frac{2}{5}$ of \$52.50 $=\$21$;

$\therefore$ the interest is $\frac{21}{350}$ of the Principal.

$\frac{21}{350}$ of Principal $=\frac{6}{100}$ of Principal;

$\therefore$ the rate is 6%.

Case III. Given the Principal, Interest or Amount and the Rate per cent., to find the Time.

Ex. 3. In what time will the interest on \$350 at 6% per annum be \$52.50?

The interest on \$350 for 1 yr. at 6% is \$21.

Time for which \$21 is the interest on \$350=1 yr.;

∴ " " " \$52.50 " " " $\$350=\frac{52.50}{21}$ of 1 yr.

$=2\frac{1}{2}$ yr.

Case IV. (*a*) **Given the Interest, Rate per cent. and Time, to find the Principal.**

Ex. 4. What Principal will give \$52.50 interest at 6% per annum in $2\frac{1}{2}$ yr.?

Principal to give \$6 interest in 1 yr. =\$100;

∴ " " " \$6 " " $2\frac{1}{2}$ yr.$=\frac{2}{5}$ of \$100;

∴ " " " \$52.50" " $2\frac{1}{2}$ yr.$=\frac{52.50}{6}\times\frac{2}{5}$ of \$100

=\$350.

(*b*) **Given the Amount, Rate per cent. and Time, to find the Principal.**

Ex. 5. What Principal will amount to \$402.50 at 6 per cent. per annum in $2\frac{1}{2}$ yr.?

The interest on \$100 at 6% for $2\frac{1}{2}$ yr.=\$15;

∴ principal which amounts to \$115 for $2\frac{1}{2}$ yr.=\$100;

∴ " " " " \$402.50 " $2\frac{1}{2}$ yr.$=\$\left(\frac{402.50\times100}{115}\right)$

=\$350.

VII. COMPOUND INTEREST.

In Compound Interest the interest as it falls due at the end of a stated period is added to the principal and this sum becomes the principal for the next period; at the end of this second period the interest on the new principal is added on and the amount forms a new principal for the next period; etc.

When the interest is added at the end of each year, it is said to be compounded annually; when added at the end of each half year, it is compounded half-yearly; when added at the end of each three months, it is compounded quarterly, etc.

Case I. Given the Principal, Rate per cent. and the Time, to find the Compound Interest or Amount.

Ex. 1. A man deposited \$2500 in a savings bank, the interest being compounded half-yearly at 4%. (*a*) How much was at his

credit at the end of 18 months? and (*b*) how much of this is interest?

Original principal=\$2500;
Interest for 1st period=2% of \$2500 = \$50;
Principal " 2nd " =\$2550;
Interest " 2nd " =2% of \$2550 = \$51;
Principal " 3rd " =\$2601;
Interest " 3rd " =2% of \$2601 = \$52.02;
Amount at his credit =\$2653.02;
Interest=\$(2653.02—2500)=\$153.02.

Pupils should see clearly why the interest for the second period is larger than that for the first and by how much; why the interest for the third period is larger than that for the second and by how much.

Examples in compound interest should be regarded as a series of examples in simple interest until the pupil becomes familiar with the subject. Then a shorter method should be introduced.

Thus, at 2% for a period, the amount of any sum at the end of a period is $\frac{102}{100}$ of the sum.

Hence,

Amount of \$1 at end of 1st period $=\frac{102}{100}$ of \$1
=Principal for next period.
" " \$1 " " " 2nd " $=\frac{102}{100}$ of $(\frac{102}{100}$ of \$1$)=(\frac{102}{100})^2$ of \$1
=Principal for next period.
" " \$1 " " " 3rd " $=\frac{102}{100}$ of $[(\frac{102}{100})^2$ of \$1)]
$=(\frac{102}{100})^3$ of \$1;
" " \$2500 " " " 3rd " $=\$[2500\times(\frac{102}{100})^3]$ =\$2653.02;
Compound Interest=\$(2653.02—2500)=\$153.02.

By similar reasoning it may be shown that the amount of \$1 at compound interest for any number of periods is found by finding the amount of \$1 at the given rate per cent. for one period and then raising the measure of this amount to the power indicated by the number of periods. Thus, the amount of \$1 at interest, compounded quarterly at 6% per annum in 15 mos. is $\$(1.015)^5$.

Case II. Given the Amount, Rate per cent. and Time, to find the Principal.

Ex. 2. What sum will amount to \$1378.42 in 1½ yr. at 5%, interest being compounded half-yearly?

Amount of \$1 for 3 periods$=\$(1.025)^3$;

Principal which amounts to $\$(1.025)^3=\$1$:

" " " " \$1378.42 $=\$\frac{1378.42\times 1}{(1.025)^3}=\$1280.$

NOTE.—The other cases of Compound Interest are too difficult for this stage of progress of the pupil.

VIII. TRUE AND BANK DISCOUNTS.

1. Explanations.

If A owes \$212 to B, the money to be paid in 1 yr., it would be fair for A to pay \$200 to B at the beginning of the year, money being worth 6% per annum, for B would have the use of the \$200 for the year. B by accepting the smaller sum would lose nothing.

\$200 is called the PRESENT WORTH of the debt.

\$12 is called the TRUE DISCOUNT; being mathematically exact, it is sometimes called MATHEMATICAL DISCOUNT.

The PRESENT WORTH of a debt at TRUE DISCOUNT is the Principal which will amount to the debt at the time it falls due at the given rate per cent.

TRUE, OR MATHEMATICAL DISCOUNT is the interest on the Present Worth for the given rate and time.

This kind of discount, the finding of which is often treated as a separate rule, is really the case of Interest in which the Amount, Rate per cent. and Time are given, to find the Principal or Interest.

EX. 1. What is the debt on which the Mathematical Discount for 6 mo. at 8% is \$3?

This is the same as finding what principal will give \$3 interest in 6 mo. at 8%. This is readily found to be \$75. Hence the debt is \$78.

EX. 2. If \$25 is the interest on \$325 for a given time and rate, what should be the mathematical discount off \$325 for the same time and rate?

The interest on \$325 for the given time and rate$=\$25$;

∴ " discount off \$350 " " " " " " $=\$25$:

∴ " " " \$325 " " " " " " $=\$\frac{325\times 25}{350}$

$=\$23\frac{3}{14}$.

EX. 3. If \$5 is allowed off a bill of \$105 due 6 mo. hence, what should be the bill from which the same sum is allowed as 12 mo. discount?

$5 is the discount off $105 for 6 mo.;
∴ $5 " " interest on $100 " 6 mo.;
∴ $5 " " " " $50 " 12 mo.;
∴ $5 " " discount off $55 " 12 mo.

If a borrower binds himself by a note or bill to pay $100 in 6 mo. and a banker or other money lender advances money on the security of the note or bill at 8% per annum, he will give to the borrower $96. The difference, $4, is known as Bank Discount.

In making such a transaction clear, the business practice must be specially emphasized. In doing this, the regular forms used in business should be introduced for illustrative purposes. Business transactions should be clearly outlined and carefully carried through.

In addition to what is said under the head of Bank Discount in the Public School Arithmetic, notice the following:—

The value of a note at maturity is the face of the note, *i.e.*, the sum for which it is given, if it does not bear interest; and it is the face plus the accrued interest at maturity, if it draws interest.

COMMERCIAL PAPER embraces *notes*, *checks*, *drafts*, *bills of exchange*, *letters of credit*, etc.

The MAKER of a note is the one primarily liable but each endorser is liable to succeeding holders if the maker fails in payment.

When the PAYEE writes only his name across the back of the note it is a BLANK ENDORSEMENT.

If he writes above his signature, Pay to A. B., or order, it is a FULL ENDORSEMENT, and A. B. would have to endorse it before receiving payment.

If it is made payable to A. B. only, then it is a RESTRICTIVE ENDORSEMENT.

A QUALIFIED ENDORSEMENT is one in which the endorser does not wish to render himself liable for payment and then he writes "Without recourse to me" above his name.

A PROTEST is a declaration made in writing by a Notary Public, giving legal notice to both the maker and the endorser of a note of its non-payment.

A BANK CHEQUE is a written order on a banker or broker to pay money in his keeping belonging to the signer.

A BILL OF EXCHANGE is a written order, addressed to a person in a distant place called the drawee to pay a specified sum of

money to a third person called the payee or to the order of the payee.

Bills of Exchange are of two kinds, viz., (1) Inland Bills of Exchange, or Drafts, and (2) Foreign Bills of Exchange.

An INLAND BILL OF EXCHANGE is one in which the drawer and drawee reside in the same country.

This is usually called a DRAFT.

There are various kinds of Drafts, as *Demand drafts*, payable on presentation; *Sight drafts*, payable after three days of grace; *Time drafts*, payable at a certain time after the date of the draft, or after presentation for acceptance.

After the *Drawee* agrees to pay it and writes "accepted" with his signature and the date, across the face of it, he becomes the *Acceptor*.

A FOREIGN BILL OF EXCHANGE is one in which the Drawer and Drawee reside in different countries.

A LETTER OF CREDIT is a letter or notification issued to persons travelling and addressed by a banker to his agent, informing him that the person named therein is entitled to draw a certain sum of money; when addressed to several agents on whom the money can be drawn in sums to suit the convenience of the traveller, it is called a CIRCULAR LETTER OF CREDIT.

2. Comparison of True and Bank Discounts.

Bank Discount is equal to the interest on the value of the note at maturity.

True Discount is the interest on the sum which will amount to the value of the note at maturity.

Bank Discount is, hence, greater than True Discount by the interest on the True Discount.

3. Cases of Bank Discount.

Case I. Given the note, date of discount and rate of discount, to find the Bank Discount.

$\$644\frac{12}{100}$ TORONTO, April 17, 1901.

Three months after date, I promise to pay to the order of James Jones, six hundred and forty-four $\frac{12}{100}$ dollars with interest at 8% per annum, value received.

JOHN BROWN.

This note was discounted at the Imperial Bank at 7% on May 1st.

Find (1) the bank discount and (2) the net proceeds on May 1st.
On July 20, the day of maturity, the note is worth \$657.
Number of days between May 1st and July 20 is 80.
Bank Discount on \$657 for 80 days at 7% per annum
$=\frac{80}{365}$ of $\frac{7}{100}$ of \$657, or \$10.08.
Net proceeds=\$(657—10.08)=\$646.92.

Case II. Given the value of the note at maturity, the net proceeds of the note or the discount and the terms of discount, to find the rate per cent.

Ex. 2. At maturity, a note is worth \$5621; the discount is \$41.58 and the term of discount 45 da. Find the rate per cent.

Interest on \$5621 for 45 da. =\$41.58;

$$\therefore \quad \text{"} \quad \text{" } \$5621 \text{ " } 365 \text{ da.}=\$\frac{365\times 41.58}{45};$$

$$\therefore \text{ rate}=\frac{100\times 365\times 41.58}{5621\times 45}=6\%.$$

Case III. Given the note, the net proceeds or the Bank Discount and the rate of discount, to find the term of discount.

Ex. 3. At maturity, a note is worth \$116.80. It is discounted at the Imperial Bank at $7\frac{1}{2}\%$ and produces \$115.48. Find the term of discount.

Interest on \$116.80 for term required=\$(116.80—115.48)=\$1.32;
" " \$116.80 " 1 yr. at $7\frac{1}{2}\%$ =\$8.76;
\$8.76 is the interest for 365 da.;

$$\therefore \$1.32 \text{ " " " " } \frac{1.32\times 365}{8.76} \text{ da., or } 55 \text{ da.};$$

∴ term of discount is 55 da.

From this example it is evident that this is merely a modification of Case III of Simple Interest.

Case IV. Given the net proceeds, the rate of discount and the term of discount, to find the value of the note at maturity.

Ex. 4. Find the value of a note at maturity which will realize \$637 when discounted 4 mo. before maturity at 6%.

Interest on maturity-value of note for 4 mo. at $6\%=\frac{2}{100}$ of value;
Net proceeds of note$=\frac{98}{100}$ " "
$\therefore \frac{98}{100}$ of maturity-value=\$637;
$\therefore$ maturity-value$=\frac{100}{98}$ of \$637
=\$650.

IX. STOCKS AND DIVIDENDS.

1. Explanations.

In addition to the explanations given in the Public School Arithmetic on pages 149 and 150 notice the following:—

The chief causes affecting the prices of stocks are:—

(i) The risk of loss owing to the company or government becoming bankrupt.

(ii) The rate of dividend which is paid and the certainty that it will continue.

(iii) The time at which the dividends are due. Prices are higher when the dividends are soon due and usually drop after they have been paid.

(iv) The prospect of a future increase or decrease in the dividends.

(v) The amount of the Rest, or Reserve.

Companies usually reserve a part of their earnings, instead of paying them all in dividends, to enable them to meet unforeseen losses, etc. This is called the REST, OR RESERVE. When the accumulated Rest is large it adds to the value of the Stock.

Bonds bearing a fixed rate of interest, such as Consols, also fluctuate in price but not to the same extent. This fluctuation is due to the variation in the price of money. When money is plentiful and hence cheap, such bonds will increase in price; when money is scarce and hence dear, they will decrease in price.

DEBENTURES are bonds or securities given as acknowledgment of debts, bearing fixed rates of interest, and providing for the repayment of the principal at a specified time.

PREFERENCE, OR PREFERRED STOCK is stock bearing a fixed rate of interest which must be paid before a dividend can be paid on ORDINARY, OR DEFERRED STOCK.

WATERED STOCK is stock which has been issued to stockholders in addition to what they own and for which no payment has been made or is required.

When the Charter of the company forbids the declaring of a dividend exceeding a certain rate per cent. of the par value of the stock or when the company wishes to keep the public in ignorance of its prosperity, its capital stock is increased by issuing additional shares to its present stockholders so that a low rate of dividend on the increased stock will produce as much income as a high rate upon the original capital stock. Stock so increased is said to be watered.

STOCK BROKERS are persons who deal in stocks, debentures, etc. When a stock broker buys or sells stocks for a principal he charges a commission on the *par value of the stock*. This is termed Brokerage. The rate varies according to circumstances.

Problems involving Brokerage often cause difficulty to a beginner because of his inability to know when the brokerage is added to, or subtracted from, the market price. When the nature of the transaction is understood this difficulty disappears. Thus, when stock is purchased for a person, the brokerage must increase its cost to him; on the contrary, when stock is sold for him, the brokerage must diminish what he receives for it.

To buy one share ($100) of Commercial Cable at 172 25, brokerage $\frac{1}{4}$%, the purchaser must pay $172.25 to the person from whom the stock is bought and $.25 to the broker; hence, $100 of C. C. stock will cost him $172.50.

To sell one share of C. C. stock at the price quoted, the broker would receive $172.25 for it. He would keep $.25 for his brokerage and the seller would receive $172 for the $100 C. C. stock.

"Bears" are stock brokers who contract to sell stock at a certain price at a certain time. They therefore operate to depress the market, buy, and then deliver the stock, thus realizing a profit.

"Bulls" are stock brokers who endeavor to raise the price of stock in order to sell to advantage.

To "sell short" is to contract to deliver stock at a fixed price within a fixed time, when the seller has not the stock on hand. Hence, the "Shorts" are the "Bears."

To "buy long" is to contract to purchase stock, deliverable at a stipulated time. The "Longs" are the "Bulls."

2. Cases of Stocks.

The following things are involved in problems in Stocks:—(1) the PRICE of stock, (2) the QUANTITY of stock, (3) the AMOUNT of money invested, (4) the RATE of dividend, (5) the INCOME, (6) RATE PER CENT. of income.

Case I. Given the amount of stock and its price, to find its money value.

Ex. 1. What is $650 stock of the Dominion Bank worth at 245?

Value of $100 stock=$245;

∴ " " $650 " $=\frac{650}{100}$ of $245=$1592.50.

Case II. Given the sum invested in stock and its price, to find the amount of stock.

Ex. 2. How much stock of the Bank of Ottawa at 207 can be bought for $6210?

Stock bought for $207 of money=$100;

∴ " " " $6210 " " $=\frac{6210}{207}$ of $100=$3000.

Case III. Given the quantity of stock and the sum invested, to find the price of the stock.

Ex. 3. How is Bank of Montreal stock quoted when $800 stock costs $2056?

Value of $800 stock=$2056;

∴ " " $100 " $=\$\frac{2056}{8}=\$257$;

∴ B. of M. stock is quoted at 257.

Note.—When a pupil understands these cases, they should be reworked with brokerage included.

Case IV. Given the sum invested and the price of the stock, or the amount of the stock, and the rate of dividend, to find the Income.

Ex. 4. What annual income is derived from investing $2040 in a $7\frac{1}{2}$ per cent. stock at $127\frac{1}{2}$?

Income from $100 stock $=\$7\frac{1}{2}$;

∴ " " $\$127\frac{1}{2}$ money$=\$7\frac{1}{2}$;

∴ " " $2040 " $=\frac{2040}{127\frac{1}{2}}$ of $\$7\frac{1}{2}=\$120$.

Ex. 5. A owns $5200 in a stock which pays a dividend of 5%. What is A's income?

A's income$=\frac{5}{100}$ of $5200=$260.

Case V. Given the amount of stock and the income, to find the rate of dividend paid.

Ex. 6. A who owns $5600 stock receives $238 as the half-yearly dividend. What is the half-yearly rate of dividend?

$5600 stock gives $238 income;

∴ $100 " " $\$\frac{238}{56}$ " , or $\$4\frac{1}{4}$;

∴ the half-yearly rate of dividend is $4\frac{1}{4}\%$.

Case VI. Given the income and the rate of dividend, to find the amount of stock.

Ex. 7. A receives \$14 each quarter from a stock paying $1\frac{3}{4}\%$ quarterly dividends. How much of this stock does he own?

$\frac{7}{400}$ of stock owned = \$14;

$\therefore$ " " $= \frac{400}{7}$ of \$14 = \$800.

Case VII. Given the price of stock and the rate of dividend, to find the rate of interest made.

Ex. 8. A invests in Bank of Commerce stock at 150. This stock pays a yearly dividend of 7%. What rate of interest does he make on his investment?

Income from \$150 = \$7;

$\therefore$ " " \$100 = $\frac{100}{150}$ of \$7 = \$$4\frac{2}{3}$;

$\therefore$ rate of interest = $4\frac{2}{3}\%$.

Case VIII. Given the rate of interest made and the rate of dividend paid, to find the price of the stock.

Ex. 9. By investing in a stock paying 8 per cent., A makes 7 per cent. on his money. What was the price of this stock?

\$7 is the income from investing \$100;

$\therefore$ \$8 " " " " " $\frac{8}{7}$ of \$100, or \$$114\frac{2}{7}$;

$\therefore$ the stock was bought at $114\frac{2}{7}$.

Case IX. Given the price of the stock and the rate of dividend, to find what sum must be invested to produce a fixed income.

Ex. 10. What sum must be invested in the Dominion $3\frac{1}{2}$ per cents. at $97\frac{1}{2}$ so as to produce an income of \$1400?

Since \$3.50 is the income from investing \$$97\frac{1}{2}$;

$\therefore$ \$1400 " " " " " $\frac{1400}{3.50}$ of \$$97\frac{1}{2}$, or \$39000.

X. EQUATION OF PAYMENTS, OR AVERAGE OF ACCOUNTS.

1. Explanations.

Sometimes one person owes another several sums, due at different dates, and he desires to pay all in one sum at such a time that neither person may lose or gain by the transaction.

Thus, suppose A owes B \$200, of which \$100 is due in 6 mo. and \$100 in 12 mo. When might the whole be paid in one sum so that neither A nor B would gain?

Examining this, it becomes evident that if A deferred payment until the last sum is due that he would then owe B \$200 and the interest on \$100 for 6 mo.

Again, if he paid the whole sum when the first payment is due, it is clear that he would lose the interest on $100 for 6 mo.

It thus becomes clear that he must pay the whole at such a time that the interest gained by deferring the first payment may balance the interest lost by pre-paying the second payment. Thus the payment would be made in 9 mo.

Again, suppose on Jan. 1st, 1901, A owes B $1800, of which $300 is payable in 4 mo.; $700 in 6 mo.; and $800 in 18 mo. When might A pay the whole to B without loss or gain to either person?

Examining this as in the last example it will soon become clear that the equitable time of payment is such a time that the interest on the sum whose payment is deferred will be equal to the interest on the sum whose payment is pre-paid.

To enable pupils to find this time, they should be given practice with such mental examples as the following:—

(1) In what time will the use of $1 balance the use of $3 for 1 mo.? of $3 for 2 mo.? of $8 for 4 mo.? etc.

(2) In what time will the use of $1 balance the use of $2 for 3 mo., and of $5 for 4 mo.? etc.

(3) How long should $4 be kept in use to balance the use of $1 for 4 mo.? of $6 for 2 mo.? of $8 for 6 mo.? etc.

(4) How long will it require $8 to balance the use of $5 for 3 mo. and of $3 for 7 mo.? etc.

The example can now be worked as follows:—

The use of $300 for 4 mo.=the use of $1 for 1200 mo.
“ “ “ $700 “ 6 “ = “ “ “ $1 “ 4200 “
“ “ “ $800 “ 18 “ = “ “ “ $1 “ 14400 “
“ “ “ $1800 “ ? “ = “ “ “ $1 “ 19800 “

Number of mo.=$\frac{19800}{1800}$=11.

It should be shown that 11 mo. is correct.

A has the use of $300 for 7 mo. and of $700 for 5 mo., but he loses the use of $800 for 7 mo.

The use of $300 for 7 mo.=the use of $1 for 2100 mo.
“ “ “ $700 “ 5 “ = “ “ “ $1 “ 3500 “
These two deferred payments= “ “ “ $1 “ 5600 “
=A's gain or B's loss.

The use of $800 for 7 mo.=the use of $1 for 5600 mo.
=A's loss or B's gain.

This method of solution is based on the assumption that what is gained by the debtor by deferring certain payments after they are due is lost by him by paying other sums before they become due. This is not strictly correct, for what he gains is the interest on the deferred payments and what he loses is the interest on the present worth of the anticipated payments. The error is, however, so small that it is the method used in ordinary business transactions.

The following is a convenient way of indicating the work:—

$$\begin{array}{l} 300\times 4 = 1200 \\ 700\times 6 = 4200 \\ \underline{800}\times 18 = 14400 \\ 1800 \quad)\underline{19800} \\ \qquad\qquad 11 \end{array}$$

XVI. PARTNERSHIP.

1. Explanations.

When a number of persons unite to carry on some particular branch of business, the connection so formed is called a PARTNERSHIP.

The persons forming a partnership are called unitedly, a COMPANY, FIRM, or HOUSE. Thus there are Railroad Companies, Manufacturing Firms, Wholesale Houses, etc. Usually merchants and manufacturers are known as Firms or Houses.

The persons forming the *Partnership* are called separately, the PARTNERS.

The money or property invested by each is called his STOCK or CAPITAL.

It is usual to make an agreement among the partners to share the gains or losses *in proportion to the capital* contributed by each.

Property of all kinds belonging to the firm and all debts due to it are the RESOURCES, EFFECTS, or ASSETS of the firm.

The debts owed by the firm are the LIABILITIES.

The NET CAPITAL is the excess of resources over liabilities.

The NET INSOLVENCY is the excess of liabilities over resources.

The NET GAIN is the excess of the total gains over the total losses within the period under consideration.

The NET LOSS is the excess of the total losses over the total gains within the period under consideration.

The term, "Limited," placed after the name of a firm means that the responsibility of the partners for the liabilities of the

company does not extend beyond the amount of capital each has in the firm. The law requires the word, "Limited," to be placed after the name of the firm whenever its name is used.

In the case of shareholders in banks, to secure bill holders and depositors, the law makes the liability of stockholders double the amount of the capital held by them. Thus, the owner of five paid-up shares of one hundred dollars each would be liable, in case of the bank's failure, to be called upon to pay $500 as well as lose the money already invested. This is known as *Double Liability*.

When the several shares of capital have been in the business for the *same length of time*, the division of the gains or losses is called *Simple Partnership*, or PARTNERSHIP WITHOUT TIME.

When the several shares of capital have been invested for different lengths of time and the gains or losses are to be divided according to the average investments, *i.e.*, in proportion to the several capitals and the times for which they have been employed, the division of the gains or losses is called COMPOUND PARTNERSHIP OR PARTNERSHIP WITH TIME.

2. Simple Partnership.

Simple Partnership is merely an application of Case II of Sharing. See page 144.

There are three things to be considered, (1) the CAPITAL of the firm, (2) the CAPITAL of each partner, and (3) the GAIN OR LOSS to be shared.

Case I. Given the capital of each partner and the gain or loss, to find each partner's share of the gain or loss.

Ex. 1. A and B formed a partnership. A invested $9000; B, $7000. They gained $4200. What share of the gain should each get?

$$\begin{array}{r} \$9000 \\ 7000 \\ \hline \$16000 \end{array}$$, the whole capital.

Sum gained by $16000=$4200;

∴ " " " $\$9000=\frac{9000}{16000}$ of $4200=$2362.50=A's share.

∴ " " " $\$7000=\frac{7000}{16000}$ of $4200=$1837.50=B's "

Case II. Given the total capital and the share of each partner of the gain or loss, to find the capital each invested.

Ex. 2. A, B, and C formed a partnership. Their total capital was $29000. A received $246 of the gain; B, $314; and C, $440. Find the capital of each.

$\$(246+314+440)=\$1000;$

A's capital $=\frac{246}{1000}$ of $\$29000=\$\ 7134.$

B's " $=\frac{314}{1000}$ of $\$29000=\$\ 9106.$

C's " $=\frac{440}{1000}$ of $\$29000=\$12760.$

3. Compound Partnership or Partnership with Time.

Three things are to be considered, (1) the CAPITAL of each partner, (2) the TIME the capital of each partner is employed in the business, and (3) the GAIN OR LOSS to be shared.

It is plain that each partner's share will depend upon two things:—

First, the amount of his capital.

Second, the length of time it is continued in the business.

It is evident that when the times are equal, the gain or loss must be shared in proportion to their capitals; when the capitals are equal the gain or loss must be shared in proportion to the times; and when neither the times nor the capitals are equal it must be shared as their product.

Ex. 3. A commenced business with a capital of \$8000; 3 mo. after he took in B with a capital of \$4000; and 4 mo. after this he took in C with a capital of \$1200. At the end of the year the firm had gained \$5175. Find the share of each partner.

Gain on \$8000 for 12 mo. = gain on \$96000 for 1 mo.
" " \$4000 " 9 " = " " \$36000 " 1 "
" " \$1200 " 5 " = " " \$6000 " 1 "
∴ " " Capital = " " \$138000 " 1 "

Gain on \$138000 for 1 mo. = \$5175;

∴ " " \$96000 " 1 " $=\frac{96}{138}$ of \$5175 = \$3600 = A's share.
and " " \$36000 " 1 " $=\frac{36}{138}$ of \$5175 = \$1350 = B's "
and " " \$6000 " 1 " $=\frac{6}{138}$ of \$5175 = \$225 = C's "

The following is a convenient way of indicating the work:—

$$\begin{aligned}8000\times 12 &= 96000\\ 4000\times 9 &= 36000\\ 1200\times 5 &= \underline{\ \ 6000}\\ &\ 138000\end{aligned}$$

A's share $=\frac{96}{138}$ of $\$5175=\$3600.$

B's " $=\frac{36}{138}$ of $\$5175=\$1350.$

C's " $=\frac{6}{138}$ of $\$5175=\$225.$

XVII. INVOLUTION AND EVOLUTION.

I. INVOLUTION.

Steps.

1. Teach Power.

In doing this emphasize *the number of factors in the power.*

The definition of power is often inaccurately expressed as the number of times the number is multiplied by itself. The third power is not obtained by multiplying a number by itself twice but by multiplying the square of the number by the number; the fourth power is not obtained by multiplying a number by itself three times but by multiplying the square by the number to obtain its cube and this cube by the number to obtain its fourth power.

2. Teach Exponent or Index.

Drill Exercises.

(*a*) Have a number of factors indicated as powers.

(*b*) Have powers expanded into factors.

3. Raise numbers to the powers indicated by the exponent.

(*a*) Begin by squaring (1) whole numbers, (2) decimals, (3) vulgar fractions.

NOTE 1.—Call attention to a convenient way of squaring a mixed number with $\frac{1}{2}$ as the fractional part. Thus,

$$(5\tfrac{1}{2})^2=5\times6+\tfrac{1}{4}=30\tfrac{1}{4};$$
$$(7\tfrac{1}{2})^2=7\times8+\tfrac{1}{4}=56\tfrac{1}{4};$$
$$(9\tfrac{1}{2})^2=9\times10+\tfrac{1}{4}=90\tfrac{1}{4}.$$

2.—Show how to square a number of two figures. Thus,

$$35^2=30\times40+5^2=1225;$$
$$57^2=60\times54+3^2=3249;$$
$$82^2=80\times84+2^2=6724.$$

(*b*) Proceed to higher powers with (1) whole numbers, (2) decimals, (3) vulgar fractions.

4. Have pupils learn the following:—

(*a*) The squares of all the numbers from 1 to 12 inclusive.

(*b*) The squares of any number of tens up to 10 tens. Thus, 20^2, 30^2, 40^2, 50^2, etc.

(*c*) Squares of frequent occurrence as 15^2, 16^2, 25^2, etc.

NOTE.—The second power of a number is called its *square*, because the product of the number of units in the length of a square by itself is equal to the number of units of area of the square.

2.—The third power of a number is called its *cube*, because the continued product of the number of units in the three equal sides of a cube gives the number of units in its cubic content.

3.—The square of a whole number is called *a perfect square.*

4.—The cube of a whole number is called *a perfect cube.*

II. SQUARE ROOT.

Steps.

1. Introduction.

Review Involution in so far as it applies to the *squares* of whole numbers.

2. Teach the meaning of Square Root and how it is indicated.

Take the squares up to 81 and resolve each into *two* equal factors, as,

$$\because\ 2^2=2\times2=4;\quad \therefore\ 4=2\times2=2^2.$$
$$\because\ 3^2=3\times3=9;\quad \therefore\ 9=3\times3=3^2.$$
$$\because\ 4^2=4\times4=16;\ \therefore\ 16=4\times4=4^2.$$

Explain that 2 is the square root of 4; 3 is the square root of 9; etc.

Explain also that there are two ways of indicating the square root of a number, (1) by using the radical sign $\sqrt{\ }$, and (2) by using $\frac{1}{2}$ as an index.

Thus, $\sqrt{4}=2$; $\sqrt{9}=3$; $\sqrt{16}=4$;

Or, $4^{\frac{1}{2}}=2$; $9^{\frac{1}{2}}=3$; $16^{\frac{1}{2}}=4$.

Fix the meaning by drilling on such mental examples as, find the square root of 36, 64, 81, $\frac{9}{16}$, $\frac{9}{64}$, $\frac{25}{49}$, $\frac{9}{100}$, etc.

3. To ascertain how many figures there will be in the square root of a perfect square.

Examine the following:—

Since, $1^2=1$ and $9^2=81$;

$\therefore$ the square of a number of 1 digit is a number of 1 or 2 digits.

Conversely, the square root of a number of 1 or 2 figures is a number of 1 digit.

Again, since $10^2=100$ and $99^2=9801$;

$\therefore$ the square of a number of 2 figures is a number of 3 or 4 figures.

Conversely, the square root of a number of 3 or 4 figures is a number of 2 figures.

Again, since $100^2=10000$ and $999^2=998001$;

$\therefore$ the square of a number of 3 figures is a number of 5 or 6 figures.

Conversely, the square root of a number of 5 or 6 figures is a number of 3 figures; etc.

Hence, to ascertain how many figures there will be in the square root of a given number, divide the number into periods of

two figures each and there will be as many figures in the square root as there are periods or part of a period.

4. To analyze Perfect Squares.

$$\begin{array}{rcl} 15 &=& 10+5 \\ \underline{15} &=& \underline{10+5} \\ 75 &=& 10\times5+5^2 \\ \underline{150} &=& \underline{10^2+\quad 10\times5} \\ 225 &=& 10^2+2\times10\times5+5^2 \\ &=& 10^2+(2\times10+5)5 \end{array} \qquad \begin{array}{rcl} 43 &=& 40+3 \\ \underline{43} &=& \underline{40+3} \\ 129 &=& 40\times3+3^2 \\ \underline{1720} &=& \underline{40^2+\quad 40\times3} \\ 1849 &=& 40^2+2\times40\times3+3^2 \\ &=& 40^2+(2\times40\quad+3)3 \end{array}$$

From these examples, it is evident that *the square of a number is equal to the square of the tens+twice the product of the tens by the units+the square of the units, or is equal to the square of the tens+the product of twice the tens plus the units by the units.*

5. To find a number from its square.

Ex. 1. Find the square root of 1849.

Formal Method.

$$\begin{array}{rl} & 1849(40+3 \\ 40^2= & \underline{1600} \\ & 249 \\ (2\times40+3)3= & \underline{249} \\ \end{array} \qquad \begin{array}{rl} & 1849(43 \\ & \underline{16} \\ 83 & 249 \\ & \underline{249} \\ \end{array}$$

Square root=43. Square root=43.

Since the square of tens is hundreds, the part of 1849 expressed by 49 contains no part of the square of tens. These figures are, therefore, disregarded for the present. The greatest square in 18 hundred is 16 hundred, the square root of which is 4 tens.

The remainder 249 equals twice the tens plus the units, multiplied by the units. Twice the tens is 8 tens; 8 tens is contained in 24 tens, 3 times.

Twice the tens plus the units is 83, and 83 multiplied by 3 is 249.

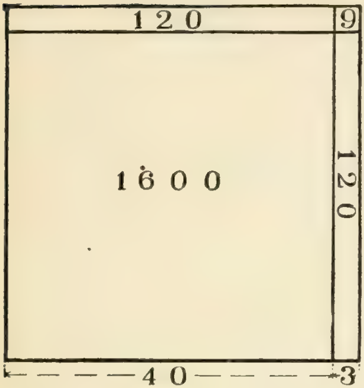
Another method of explaining square root is often given:—

Ex. 2. A has 1849 square disks and wishes to arrange them into a large square. How many disks will there be in each side of the square?

First, he ascertains that there will be between 40 and 50 disks in the side of the square.

Next he arranges a square with 40 disks in the side and places 1600 of them in position. He has still 249 to arrange.

To preserve the square already formed he must add disks equally to *two* sides as shown in the diagram.



One row along the two sides would take twice forty (2×40) disks.

To ascertain how many rows he can place he divides 80 into 249 and finds he can place 3 rows along each of the two sides. He still has 9 disks which are needed to fill in the small square.

The formal solution is the same as that given above under Example 1.

Ex. 3. Extract the square root of 481636.

$$\begin{array}{rl}
 & 481636\,(600+90+4 \\
600^2 = & \underline{360000} \\
 & 121636 \\
(2\times600+90)90 = & \underline{116100} \\
 & 5536 \\
(2\times690+4)4 = & \underline{5536}
\end{array}$$

The square root $=694$.

$$\begin{array}{rl}
 & 481636\,(694 \\
 & \underline{36} \\
129 & 1216 \\
 & \underline{1161} \\
1384 & 5536 \\
 & \underline{5536}
\end{array}$$

The square root $=694$.

Since the square of hundreds is ten-thousands, the part of 481636 expressed by 1636 contains no part of the square of the hundreds. These figures are, therefore, disregarded for the present.

For the reason given under Ex. 1, the figures 36 are disregarded in finding the tens' figure.

Hence, when the number of figures in the square root of a number is known, it is at once evident what part of it must be considered in finding the left-hand digit of the root; what part must be considered in finding the next figure; etc.

Hence, the steps in the extraction of Square Root are:—

(1) Divide the number into periods of two figures each beginning at the decimal point.

(2) Find the largest square contained in the left-hand period. Place the square root of this number as the left-hand figure of the root. Subtract its square from the left-hand period and annex the next period to the remainder.

(3) Double the part of the root found, place it to the left of the remainder, and using it as a trial divisor divide the remainder, omitting the last figure, by it. The quotient is the next figure of the root and must be annexed to the trial divisor as well as to the root. Multiply the divisor, completed as it now stands, by the quotient. Place the product under the remainder and subtract. To the new remainder annex the next period.

(4) Proceed as in (3) to find the next figure of the root.

6. To extract the square root of a decimal.

Since the square of a number of tenths is a number of hundredths; and the square of a number of hundredths is a number of ten-thousandths; and the square of a number of thousandths is a number of millionths; etc., *i.e.*, are decimals of 2nd, 4th, 6th, etc., orders, respectively, hence to find the square root of a decimal it must be of the 2nd, 4th, 6th . . . order, *i.e.*, there must be 2 figures in each period counting from the point.

Ex. 4. Extract the square root of .4225.

```
     .4225(.65
     .36
 125  625
      625
```

When the number of decimal places is even, point off the decimal into periods of two figures each, counting from the point and extract the square root as in whole numbers, placing the point in the root as soon as the left-hand period is dealt with.

Ex. 4. Extract the square root of .4.

```
     .400000(.632 . . .
      36
123   400
      369
1262  3100
      2524
       576
```

When the number of decimal places is not even, make them even by affixing a cipher, and extract the square root as in whole numbers.

7. To extract the square root of a vulgar fraction.

(*a*) If both terms are exact squares, extract the square root of each. The result will be the terms of the root. Thus, $\sqrt{\frac{9}{25}}=\frac{3}{5}$; $\sqrt{\frac{49}{100}}=\frac{7}{10}$.

(*b*) If both terms are not exact squares, reduce the fraction to a decimal and extract the square root of the decimal. Thus, $\sqrt{\frac{3}{4}}=\sqrt{.75}=.866\ldots$; $\sqrt{\frac{9}{10}}=\sqrt{.9}=.9486\ldots$

8. Applications of Theory.

(1) To find the square root of a number by resolving it into prime factors.

(2) To find by what factor a number must be multiplied to make it a perfect square.

(3) To find what number must be subtracted from a number which is not a perfect square, so that the remainder may be a perfect square.

(4) To find what number must be added to a number which is not a perfect square, so that the sum may be a perfect square.

(5) To find the fourth root of a number.

9. Practical Applications.

(1) To find the side of a square, when the area is given.

(2) To find the sides of a rectangle, when the relation between the sides, and the area are given.

(3) To find the radius of a circle, when the area is given.

(4) To find one side of a right-angled triangle when the other two are given.

(5) To find the corresponding dimensions of similar surfaces, when their areas and the dimensions of one are given.

III. CUBE ROOT.

Steps.

1. Introduction.

Review Involution in so far as it applies to the *cubes* of whole numbers.

2. Teach the meaning of Cube Root and how it is indicated.

Take the perfect cubes up to 729 and resolve each into *three* equal factors, as,

$$\because 2^3=2\times2\times2=8;\quad \therefore 8=2\times2\times2=2^3;$$
$$\because 3^3=3\times3\times3=27;\quad \therefore 27=3\times3\times3=3^3;$$
$$\because 4^3=4\times4\times4=64;\quad \therefore 64=4\times4\times4=4^3;\text{ etc.}$$

Explain that 2 is the cube root of 8; 3 is the cube root of 27; etc.

Explain also that there are two ways of indicating the cube root of a number, (1) by placing $\sqrt[3]{\ }$ before the number, and (2) by using $\frac{1}{3}$ as an index.

Thus, $\sqrt[3]{8}=2$; $\sqrt[3]{27}=3$; $\sqrt[3]{64}=4$;

or, $8^{\frac{1}{3}}=2$; $27^{\frac{1}{3}}=3$; $64^{\frac{1}{3}}=4$.

3. To ascertain how many figures there will be in the cube root of a perfect cube.

Since, $1^3=1$; $2^3=8$; $3^3=27$; $4^3=64$; $5^3=125$; $6^3=216$; $7^3=343$, $8^3=512$; $9^3=729$;

∴ the cube of a number of 1 digit is a number of 1, 2, or 3 digits.

Conversely, the cube root of a number of 1, 2, or 3 digits is a number of 1 digit.

Again, since $10^3=1000$ and $99^3=970299$;

∴ the cube of a number of 2 figures is one of 4, 5, or 6 figures.

Conversely, the cube root of a number of 4, 5 or 6 figures is a number of 2 figures.

Again, since $100^3=1000000$ and $999^3=997002999$, therefore, the cube of a number of 3 figures is a number of 7, 8 or 9 figures; conversely, the cube root of a number of 7, 8 or 9 figures is a number of 3 figures, etc.

Hence, to ascertain how many places there will be in the cube root of a given number, divide the number into periods of *three* figures each and there will be as many places in the cube root as there are periods and part of a period.

4. To analyze perfect cubes.

$85^3=614125$;

$\therefore\ 614125=85^3=(80+5)^3.$

$$
\begin{array}{rcr}
85 & = & 80+5 \\
\underline{85} & = & \underline{80+5} \\
425 & = & 80\times5\ +5^2 \\
\underline{6800} & = & \underline{80^2+80\times5\qquad} \\
7225 & = & 80^2+2\times80\times5\ +5^2 \\
\underline{85} & = & \underline{80\ +5} \\
36125 & & 80^2\times5+2\times80\times5^2+5^3 \\
\underline{578000} & = & \underline{80^3+\ 2\times80^2\times5+\quad 80\times5^2\qquad} \\
614125 & = & 80^3+\ 3\times80^2\times5+3\times80\times5^2+5^3 \\
 & = & 80^3+(3\times80^2\quad +3\times80\times5\ +5^2)5
\end{array}
$$

By treating other numbers in a similar way it will be evident that a perfect cube having 4, 5, or 6 places consists of

$$tens^3+3\times tens^2\times units+3\times tens\times units^2+units^3.$$

5. To find a number from its cube.

Ex. 1. Find the cube root of 614125.

$$
\begin{array}{llr}
 & & 614125(80+5 \\
 & 80^3 \qquad\qquad\qquad = & \underline{512000} \\
\text{Trial divisor} & 3\times80^2=\ 19200 & 102125 \\
 & 3\times80\times5\ =\ \ 1200 & \\
 & 5^2=\ \underline{\qquad 25} & \\
 & \quad 20425\times5= & \underline{102125}
\end{array}
$$

The greatest cube in 614 thousand is 512 thousand, the cube root of which is 8 tens.

The remainder 102125 is the product of two factors. One of these is largely made up of the term 3×80^2. The other factor is found by using 3×80^2 as a *trial divisor.*

Many devices of arrangement have been adopted in indicating the operation of obtaining the cube root of a number of more than one period; but all involve the principle as developed above.

The formal method.

$$\begin{array}{rrr|r}
8 & & & 614125 \\
 & & & 512 \\
\hline
24 & 5 & 19200 & 102125 \\
 & & 1225 & \\
\cline{3-3}
 & & 20425 & 102125 \\
\cline{4-4}
\end{array}$$

Ex. 2. Find the cube root of 14706125.

$$\begin{array}{rrr|r}
2 & & & 14|706|125 \\
 & & & 8 \\
\cline{4-4}
6 & 4 & 1200 & 6706 \\
 & & 256 & \\
\cline{3-3}
 & & 1456 & 5824 \\
\cline{4-4}
 & & 16 & 882125 \\
\cline{3-3}
72 & 5 & 172800 & \\
 & & 3625 & \\
\cline{3-3}
 & & 176425 & 882125 \\
\cline{4-4}
\end{array}$$

The number is separated into 3 periods.

The greatest cube in the left hand period is 8. Set down 2 its cube root and subtract 8 from 14 and to the remainder 6 annex 706, the next period.

Then set down three times 2, which is 6, and three times the square of 2, which is 12 and annex two cyphers to it.

Then 6706 is divided by 1200. On trial 5 is found to be too large so 4 is set down under 2 and midway between 6 and 1200.

Then 6 and 4 are regarded as 64 and multiplied by 4. The product, 256, is added to 1200; the result, 1456, is multiplied by 4 and the product subtracted from 6706; and to the remainder the last period, 125, is annexed.

Next set down three times 24 and three times the square of 24 which is 1728.

NOTE.—This last result can be obtained by placing 16, the square of 4, the second figure of the root under the divisor and adding together the three numbers coupled by the brace.

Two cyphers are then annexed to 1728 and the last figure of the root 5 is found by dividing 882125 by 172800. Then the divisor is completed by adding 5 times 725 to 172800.

6. To extract the cube root of a Decimal.

Since the cube of a number of tenths is a number of thousandths; and the cube of a number of hundredths is a number of millionths; and the cube of a number of thousandths is a number of billionths; etc., *i.e.*, are decimals of the 3rd, 6th, 9th, etc., orders, respectively, hence to find the cube root of a decimal it must be of the 3rd, 6th, 9th . . . order, *i.e.*, there must be three figures in each period counting from the point.

Ex. 3. Find the cube root of .008.

$$\sqrt[3]{.008}=\sqrt[3]{\tfrac{8}{1000}}=\tfrac{2}{10}=.2.$$

Ex. 4. Find the cube root of .079507.

$$\sqrt[3]{.079507}=\sqrt[3]{\tfrac{79507}{1000000}}=\tfrac{43}{100}=.43.$$

Ex. 5. Find the cube root of .8.

```
              9                    .800|000|000|000
                                   729
       27     2     24300          71000
                      544 )
                    24844 }         49688
                        4 )        21312000
      276     8    2539200
                     22144 )
                   2561344 }        20490752
                        64 )          82124800 0
     2784     3    258355200
                       83529
                   258438729        775316187
                                     45931813
```

Hence $\sqrt[3]{.8}=.9283\ldots$

7. To extract the cube root of a Vulgar Fraction.

(*a*) If both terms are exact cubes, extract the cube root of each. The results will be the corresponding terms of the cube root of the fraction.

(*b*) If both terms are not exact cubes, reduce the fraction to a decimal and extract the cube root of the resulting decimal.

8. To find the cube root of a perfect cube consisting of 4, 5, or 6 figures by inspection.

The tens' digit is readily found.

The units' digit is found from the following:—

If a No. ending in 1 is cubed, the cube ends in 1;
" " " " 2 " " " " " " 8;
" " " " 3 " " " " " " 7;
" " " " 4 " " " " " " 4;
" " " " 5 " " " " " " 5;
" " " " 6 " " " " " " 6;
" " " " 7 " " " " " " 3;
" " " " 8 " " " " " " 2;
" " " " 9 " " " " " " 9;

Hence, when the last digit of the cube is 1 the units figure is 1;
" " " " " " 8 " " " 2;
" " " " " " 7 " " " 3; etc.

Thus, $\sqrt[3]{13824}=24$; $\sqrt[3]{681472}=88$; $\sqrt[3]{103823}=47$.

9. Applications of Theory.

(1) To find the cube root of a number by resolving it into prime factors.

(2) To find what number must be subtracted from a number which is not a perfect cube so that the remainder may be a perfect cube.

(3) To find what number must be added to a number which is not a perfect cube, so that the sum may be a perfect cube.

(4) To find the sixth root of a number.

10. Practical Applications.

(1) To find the side of a cube, when its cubic content is given.

(2) To find the radius of a sphere, when the cubic content is given.

(3) To find the dimensions of a body, when the cubic content and dimensions of a *similar* one are given.

XVIII. MENSURATION.

I. THE RECTANGLE AND SQUARE.

See pages 135 and 141.

II. THE PARALLELOGRAM.

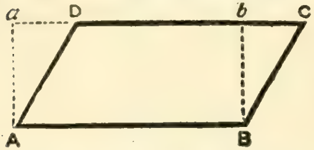
Steps.

1. The Definition.

Draw parallelograms. Have pupils draw others of various sizes and shapes.

Show that the area depends upon (1) the length or width, and (2) the perpendicular distance between the parallel sides.

Case I. Given the length of a side and the perpendicular distance between this side and the opposite one, to find the area.



Show by figures similar to A B C D that every parallelogram may be changed into a rectangle whose length is that of the parallelogram and whose breadth is the perpendicular width, or altitude of the parallelogram. These two measurements are the dimensions of the parallelogram.

A good test of the pupil's comprehension, is for the teacher to draw a parallelogram and give the length of a certain side, and then to require the pupil to point out the other dimension which must be given to enable the area to be found.

Case II. Given the area and one dimension, to find the other dimension of the parallelogram.

This is treated under the Rectangle, page 141.

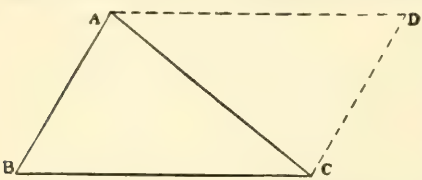
III. THE TRIANGLE.

Steps.

1. The Definition.

(*a*) Draw and have drawn various kinds of triangles.

(*b*) Show that the area of a triangle is always half the area of a parallelogram of the same base and altitude.



(*c*) Draw triangles and give the length of one of the sides, have pupils draw the figure showing the other dimension to enable the area to be determined.

Case I. Given the length of a side and the perpendicular from the opposite angle on that side or that side produced, to find the area of the triangle.

In teaching this Case, the following order is suggested:—

(*a*) Right angled triangles.

(*b*) Acute " "

(*c*) Obtuse " "

If it is made clear that the triangle is half the area of a parallelogram of the same base and altitude, there will be no difficulty with this Case.

Case II. Given the area and one dimension of the triangle to find the other.

All that is necessary is to convert the triangle into a parallelogram and this becomes Case II of the parallelogram.

IV. TRAPEZOID.

Steps.

1. Teach the meaning by drawing trapezoids and comparing and contrasting them with parallelograms, rectangles and squares.

Have pupils take rulers and draw trapezoids of certain dimensions, as draw a trapezoid the parallel sides being 2 in. and 3 in. long respectively, and the distance between them, $2\frac{1}{2}$ in.

Case I. Given the length of the two parallel sides of a trapezoid, and the distance between them, to find the area.

The simplest way to teach this is to draw a diagonal dividing the trapezoid into two triangles of which the length of the bases and the perpendicular heights are given.

The area can be readily found.

Another way is to take half the sum of the areas of two rectangles one being on the longer side and the other on the shorter side of the trapezoid; each being of the same altitude as the trapezoid.

Ex. 1. Find the area of a trapezoid the parallel sides of which are 25 in. and 15 in. in length and the perpendicular distance between them 1 ft.

Area of one triangle $=\frac{25\times12}{2}$ sq. in. $=150$ sq. in.;

Area of the other triangle $=\frac{15\times12}{2}$ sq. in. $=90$ sq. in.;

$\therefore$ area of trapezoid $=(150+90)$ sq. in. $=240$ sq. in.

Or area of both triangles $=\frac{40\times12}{2}$ sq. in. $=240$ sq. in.

Case II. Given the area and the sum of the two parallel sides, to find the perpendicular distance between them.

Ex. 2. The area of a garden with two parallel sides which measure 25 ft. and 17 ft., respectively, is 567 sq. ft. What is the perpendicular distance between the parallel sides?

Since $\frac{25\times p}{2}+\frac{17\times p}{2}=567$;

$$\therefore p=\frac{2\times567}{42}=27;$$

$\therefore$ perpendicular height $=27$ ft.

V. THE TRAPEZIUM.

Steps.

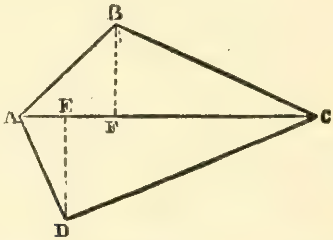
1. Teach the meaning by drawing trapeziums and compare and contrast them with trapezoids, parallelograms, rectangles, and squares. Notice that the trapezium is the simplest four-sided figure to define.

Case I. Given the length of a diagonal and the length of the perpendiculars let fall upon this diagonal from the opposite angles, to find the area ot the trapezium.

Show that the diagonal divides the trapezium into two triangles.

Have pupils point out the measurements necessary to enable their areas to be found.

Ex. 1. A B C D is a trapezium. A C is 48 yd. long; B F and D E are respectively 17 yd. and 19 yd. long. Find the area of the trapezium.



$$\text{Area of triangle A B C}=\frac{48\times 17}{2}\text{ sq. yd.}=408\text{ sq. yd.}$$

$$\text{" " " A D C}=\frac{48\times 19}{2}\text{ sq. yd.}=456\text{ sq. yd.}$$

$$\therefore\text{ area of A B C D}=(408+456)\text{ sq. yd.}=864\text{ sq. yd.}$$

$$\text{Or, area of A B C D}=\left(\frac{48\times 17}{2}+\frac{48\times 19}{2}\right)\text{ sq. yd.}$$

$$=\frac{48\times 36}{2}\text{ sq. yd.}=864\text{ sq. yd.}$$

Case II. Given the area and either the length of the diagonal, to find the sum of the lengths of the perpendiculars, or the sum of the lengths of the perpendiculars, to find the length of the diagonal.

Ex. 2. The diagonal of a trapezium measuring 1 acre is 88 yd. long. Find the sum of the lengths of the perpendiculars upon this diagonal from the opposite angles.

$$\text{Area of trapezium} = \frac{88 \times \text{sum of p's}}{2} = 4840 \text{ sq. yd.};$$

$$\therefore \text{sum of p's} = \frac{2 \times 4840}{88} \text{ yd.} = 110 \text{ yd.}$$

VI. THE CIRCLE.

Steps.

1. Teach Circle, Centre, Circumference, Diameter, Radius, and Semicircle.

In teaching circle make clear that it is a surface, not a ring, nor a ball.

Have pupils describe circles, using a string for radius rather than a pair of compasses.

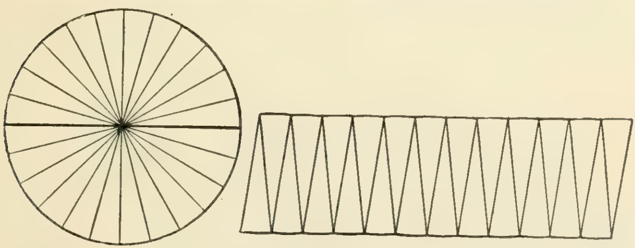
2. Teach the relation between the diameter of a circle and its circumference.

This can be done approximately by comparing the lengths of the diameter and circumference of two or three cylinders.

3. Fix this relation by having many applications of it as follows:—

(*a*) Given the diameter of the circle, to find its circumference.
(*b*) Given the circumference of a circle, to find its diameter.
(*c*) Given the radius of a circle, to find its circumference.
(*d*) Given the circumference of a circle, to find the radius.

Case I. Given the radius of a circle, to find its area.



To teach this, cut a circle about 10 in. in diameter out of a piece of leather and mark it into an even number of triangles—24 is a convenient number—as shown in the figure. Then with a sharp knife cut the triangles almost to the circumference leaving a slight margin to connect the triangles. Now separate the circle into semi-circles and fit the triangles together as shown in the

diagram. The more numerous the triangles are, the more nearly the figure will approximate a rectangle.

The length of the rectangle is evidently half the circumference of the circle and its breadth is half the diameter, or radius.

Hence, the number of units of area is found by multiplying the number of corresponding units in half the circumference by the number in the radius.

Hence, area$=\frac{1}{2}\ c\times r=\frac{2\ \pi\ r}{2}\times r=\pi\ r^2$.

NOTE.—When other dimensions are given, as the diameter, or the circumference, it is better to express these in terms of the radius, rather than to teach a new way of finding the area of the circle.

Case II. Given the area, to find the radius.

Ex. 1. The area of a circle is 3850 sq. in.; find its radius.

Since, $\frac{22}{7}\times r^2=3850$;

$\therefore\ r^2=\frac{7}{22}$ of $3850=1225$;

$\therefore\ r=\sqrt{1225}=35$.

Hence, radius=35 in.

Case III. To find the Area of an Annulus.

Ex. 2. A road runs round a circular shrubbery; the diameter of the shrubbery is 478 ft. and the width of the road is 19 ft.; find the area of the road.

Radius of shrubbery$=\frac{478}{2}$ ft. $=239$ ft.;

Radius of road and shrubbery$=(239+19)$ ft.$=258$ ft.;

Area of large circle$=(\frac{22}{7}\times 258^2)$ sq. ft.;

" " small " $=(\frac{22}{7}\times 239^2)$ sq. ft.;

$\therefore$ area of road$=[\frac{22}{7}\times(258^2-239^2)]$ sq. ft.

$=[\frac{22}{7}\times(258+239)\ (258-239)]$ sq. ft.

$=(\frac{22}{7}\times 497\times 19)$ sq. ft.$=29678$ sq. ft.

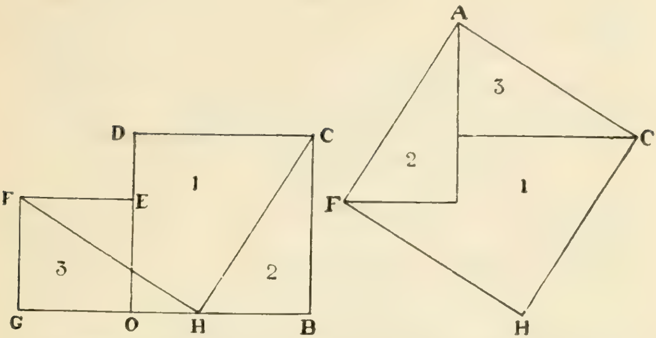
VII. THE RIGHT-ANGLED TRIANGLE.

Steps.

1. Teach the definition and the names of the sides.

Do this by drawing right-angled triangles of various sizes and shapes and give names to the sides. Then have pupils draw others and name the sides.

2. Teach that the square on the hypothenuse is equal to the sum of the squares on the base and perpendicular of a right-angled triangle.



To teach this, let B C D E F G be a figure composed of two squares placed side by side. Take B H equal to G O and join F H and C H. Cut the whole figure out of cardboard and then divide it into three pieces. Fit the pieces together as in the figure A F H C. A single square is thus obtained equal to the sum of the two squares. B C H is a right-angled triangle. B C D O is the square on the side B C; G O E F is equal to the square described on B H; and H C A F is the square on the hypothenuse. Hence, the truth of the proposition becomes evident.

Pupils should be asked to cut similar figures for themselves.

The following is suggested, have pupils draw right-angled triangles of the following dimensions:—

Sides containing the right-angle 3 in. and 4 in., or 5 in. and 12 in., or 7 in. and 24 in., or 8 in. and 15 in.

Then, by actually measuring, they should discover that the hypothenuse is 5 in., 13 in., 25 in., and 17 in., respectively.

They should then compare the sum of the squares on the two shorter sides with that on the hypothenuse.

NOTE.—It is desirable that pupils should not be confronted with mechanical difficulties of calculation at this stage. The following formulæ will enable the teacher to use such examples as will give perfect squares.

(1) $(a^2+b^2)^2=(a^2-b^2)^2+(2ab)^2$.

(2) $(2n^2+2n+1)^2=(2n^2+2n)^2+(2n+1)^2$.

In the first let $a=7$ and $b=5$, then

$(7^2+5^2)^2=(7^2-5^2)^2+(2\times7\times5)^2$, or

74, 24, and 70 are the sides of the right-angled triangle.

Case I. Given the base and perpendicular, to find the hypothenuse.

Ex. 1. Find the hypothenuse of a right-angled triangle whose base and perpendicular are 40 in. and 42 in. respectively.

(The measure of the hypothenuse)$^2 = 40^2 + 42^2$;

$\therefore$ the measure of the hypothenuse $= \sqrt{(40^2+42^2)} = 58$;

$\therefore$ length of hypothenuse $= 58$ in.

Case II. Given the hypothenuse and one side, to find the other side.

Ex. 2. The hypothenuse of a right-angled triangle is 73 in. long and the base is 55 in. long; find the length of the perpendicular.

(The measure of the perpendicular)$^2 = 73^2 - 55^2$;

$\therefore$ the measure of the perpendicular $= \sqrt{(73^2-55^2)} = 48$;

$\therefore$ the length of the perpendicular $= 48$ in.

Case III. Given any two sides of a right-angled triangle to find the length of the perpendicular from the right angle upon the hypothenuse.

Ex. 3. The base and perpendicular of a right-angled triangle are respectively, 55 in. and 48 in.; find the length of the perpendicular from the right-angle upon the hypothenuse.

Length of hypothenuse $= \sqrt{(55^2+48^2)}$ in. $= 73$ in.

Twice the area of the triangle $= (55\times 48)$ sq. in.;

Also, " " " " " " $= (p\times 73)$ sq. in.;

$\therefore p\times 73 = 55\times 48$;

$$\therefore p = \frac{55\times 48}{73} = 36\tfrac{12}{73};$$

Length of perpendicular $= 36\frac{12}{73}$ in.

VIII. SIMILAR SURFACES.

Steps.

1. Teach the Definition.

By drawings show that similar surfaces must be of the same shape. Thus, circles, squares, and equilateral triangles are similar. Show that when they are rectilineal they are (1) equiangular and (2) have the sides about the equal angles proportional.

Illustrate this by drawing many parallelograms and rectangles of the same shape but of different sizes.

2. Show that the areas of similar surfaces are to one another as the squares of the measures of corresponding lines of the surface.

Do this by drawing similar figures and comparing their areas.

Thus, compare the areas of two circles 1 in. and 2 in. in radius respectively.

The first=$3\frac{1}{7}$ sq. in.; the second=$12\frac{4}{7}$ sq. in.

The areas are as 1 to 4, or as 1^2 to 2^2.

Compare the areas of two circles 3 in. and 4 in. in radius.

Area of first=$(9\times3\frac{1}{7})$ sq. in.; area of second=$(16\times3\frac{1}{7})$ sq. in.

Areas are as 9 to 16, or as 3^2 to 4^2.

Compare the area of a rectangle 6 in. by 5 in. with another twice as long and wide.

Show by a diagram that the latter can be divided into 4 rectangles, each of which is equal to the former.

Hence, the areas are as 1 to 4 or as 1^2 to 2^2.

Ex. 1. A map is constructed on the scale of $\frac{1}{10}$ in. to a mile. What area on this map will represent a lake 4000 acres in area?

1 mile is represented by $\frac{1}{10}$ inches;

$\therefore$ 1 sq. mi. is represented by $\frac{1}{100}$ sq. in.;

$\therefore$ $\frac{4000}{640}$ sq. mi. is represented by $(\frac{4000}{640}\times\frac{1}{100})$ sq. in., or by $\frac{1}{16}$ sq. in.

Ex. 2. A water pipe $\frac{1}{2}$ in. in diameter will fill a cistern in 20 min. How long a time will be required to fill it when there is a discharge pipe $\frac{1}{3}$ in. in diameter opened at the same time.

The diameters are as $\frac{1}{2}$ to $\frac{1}{3}$, or as 3 to 2;

$\therefore$ the areas of the openings in the pipes are as 9 to 4.

The area of a pipe that would fill the cistern as fast as the two would be represented by 9—4, or 5.

A pipe whose area is represented by 9 fills cistern in 20 min.;

$\therefore$ " " " " " " 1 " " " 9×20 min.;

$\therefore$ " " " " " " 5 " " " $\frac{9\times20}{5}$ min., or 36 min.

IX. THE CUBE AND RECTANGULAR SOLID.

Pages 136 and 142.

X. THE CYLINDER.

Steps.

1. Teach the Definition.

Do this by showing cylinders of various dimensions.

Have the three bounding surfaces described. Two are circles of the same area. The third is a curved surface every part of which is equally distant from a straight line joining the centres of the circular faces.

2. Given the dimensions, to find the surface area.

First show how to find the area of the curved surface.

This is most readily done by cutting a sheet of paper to fit the curved surface. On removal the paper is seen to be a rectangle, one dimension of which is equal to the circumference of the cylinder and the other is equal to its height.

Hence, the area of the curved surface is found by multiplying the number of units in the circumference by the number of like units in the length to find the number of corresponding units of area.

The total surface can now be easily found by the pupil who can find the area of the bounding circles.

3. Given the dimensions, to find the Cubic Content.

Ex. 1. Find the cubic content of a cylinder $3\frac{1}{2}$ in. in radius and 1 ft. in length.

Area of one end $=[\frac{22}{7}\times(3\frac{1}{2})^2]$ sq. in. $=38\frac{1}{2}$ sq. in.

It is evident that a block from this cylinder 1 inch deep will contain $38\frac{1}{2}$ c. in. The cylinder is 12 in. deep, hence, the cubic content is $(12\times 38\frac{1}{2})$ c. in., or 462 c. in.

Ex. 2. Find the number of cubic feet of iron in a water pipe 3 ft. in diameter, 12 ft. long, the iron being 1 in. thick.

This is to be found by computing the cubic content of *two* cylinders, one the whole cylinder as though it were solid, and the other the cylindrical opening, and then taking one from the other.

Thus, cubic content of a solid cylinder $=[12\times(\frac{18}{12})^2\,\pi\,]$ c. ft.

" " " the opening $=[12\times(\frac{17}{12})^2\,\pi\,]$ c. ft.

Quantity of iron in cylinder $=(12\times\frac{35}{12}\times\frac{1}{12}\times\pi\,)$ c. ft.

$=9\frac{1}{6}$ c. ft.

The popular way of doing this example is to conceive the pipe to be cut open and flattened out. This is inaccurate for the resulting shape is not a *rectangular* solid, the upper and lower surfaces not being of the same area.

4. Given the cubic content and one dimension, to find the other.

Ex. 3. Find the radius of a cylinder 28 in. long which contains 1782 c. in.

Cubic content $=(28\times r^2\times\pi\,)$ c. in. $=1782$ c. in.;

$$\therefore\ r^2=\frac{1782}{28\times\pi}=\frac{81}{4};$$

$$\therefore\ r=\frac{9}{2}=4\frac{1}{2}.$$

Hence, radius $=4\frac{1}{2}$ in.

XI. THE PYRAMID.

Steps.

1. The Definition.

Teach the definition by showing pyramids of various kinds as those having bases which are triangular, quadrilateral, pentagonal, etc.

Call attention to the base—a plane surface bounded by straight lines.

Then examine the other faces. There are as many triangular faces as the base has sides and all meet in a point.

Hence, a pyramid is a solid bounded by a plane face enclosed by three or more straight lines, which is called the base, and as many triangular faces as the base has sides.

2. Given the slant height and the length of each side of the base, to find the surface of the pyramid.

Ex. 1. Find the total surface of a square-based pyramid whose edge measures 2 ft. 8 in. and slant height is 3 ft. 6 in.

$$\text{Area of base} = (\tfrac{8}{3}\times\tfrac{8}{3}) \text{ sq. ft.} \qquad = 7\tfrac{1}{9} \text{ sq. ft.};$$

$$\text{Area of 4 faces} = \left(4\times\frac{2\frac{2}{3}\times3\frac{1}{2}}{2}\right) \text{ sq. ft.} = 18\tfrac{2}{3} \text{ sq. ft.};$$

$$\text{Total surface} = 25\tfrac{7}{9} \text{ sq. ft.}$$

3. Given the base and altitude, to find the cubic content of a pyramid.

To illustrate this procure a hollow vessel with a rectilinear base and sides at right angles to it and also a hollow pyramid with a base equal to the base of the other vessel and of the same altitude. Fill the pyramid with dry sand and empty it when exactly full into the vessel. Continue to do this until the vessel is full. It will be found that the vessel holds just THREE times as much as the pyramid.

Hence, to find the cubic content of a pyramid multiply the number of units of area in the base by the number of corresponding units of altitude and one-third of the product will be the number of corresponding units of cubic content.

Ex. 2. Find the cubic content of a pyramidal tent which covers a rectangular piece of ground 15 ft. by 18 ft. and is 23 ft. high.

$$\text{Area of base} = (15\times18) \text{ sq. ft.}:$$

$$\text{Cubic content} = (\tfrac{1}{3}\times15\times18\times23) \text{ c. ft.} = 2070 \text{ c. ft.}$$

XII. THE CONE.

Steps.

1. The Definition.

Teach this by showing cones of various sizes.

Call attention to the base, a circle, and to the curved surface tapering to a point.

Hence, a cone is a solid bounded by a circular plane face, the base, and a curved face tapering from the circumference of the base to a point.

Show that a cone may be projected by revolving a right-angled triangle about one of the sides enclosing the right-angle.

2. Given the dimensions of the base and the slant height to find the area of the surface.

If a piece of paper be cut to fit the curved surface and then spread out, it will form a sector of a circle.

Ex. 1. Find the total surface of a cone 7 in. in radius and 18 in. in slant height.

The curved surface, the sector of a circle, may be supposed to be made up of triangles the sum of the bases of which is the circumference of the base of the cone and the altitude of which is the slant height.

$$\text{Circumference of base} = (\tfrac{22}{7} \times 14) \text{ in.} = 44 \text{ in.};$$

$$\text{Area of curved surface} = \frac{44 \times 18}{2} \text{ sq. in.} = 396 \text{ sq. in.}$$

$$\text{Area of base} = (\tfrac{22}{7} \times 7^2) \text{ sq. in.} = 154 \text{ sq. in.}$$

$$\text{Total area} = 550 \text{ sq. in.}$$

3. Given the base and altitude, to find the cubic content of a cone.

To illustrate this procure a hollow cylinder and a hollow cone of the same area of base as the end of the cylinder, and of equal altitude. Then proceed as in the case of the pyramid. It will be found that the cubic content of the cylinder is just THREE times that of the cone.

Ex. 1. What is the volume of the largest possible cone turned from a cubic block of wood whose edge is $10\frac{1}{2}$ in.?

$$\text{Diameter of cone} = 10\tfrac{1}{2} \text{ in.}:$$

$$\text{Area of base of cone} = [\tfrac{22}{7} \times (5\tfrac{1}{4})^2] \text{ sq. in.} = 86\tfrac{5}{8} \text{ sq. in.}:$$

$$\text{Altitude of cone} = 10\tfrac{1}{2} \text{ in.};$$

$$\text{Volume of cone} = (\tfrac{1}{3} \times 10\tfrac{1}{2} \times 86\tfrac{5}{8}) \text{ c. in.} = 303\tfrac{3}{16} \text{ cu. in.}$$

XIII. THE SPHERE.

Steps.

1. The Definition.

Present spheres of various sizes. Call attention to the position of the bounding surface. It is equally distant from a point within called the centre.

Hence, a sphere is a solid, bounded by a curved surface every point of which is equally distant from a point within, called the centre.

2. Given the radius, to find the area of the sphere.

How to find the area of a sphere cannot be shown to a class of beginners. The rule should be told. However, it may be approximately determined as follows:—

Bore a hole through the centre of a circular piece of board. Tie over one of the circular faces a rubber membrane and mark on the rubber any small area. Insert a cork and tube into the hole and introduce water until the rubber is bulged into the form of a hemisphere. Measure the area which was marked on the rubber. It is now just twice as large as before the water was introduced. Thus the curved surface of a hemisphere is just twice the area of the plane face.

Or, take a croquet ball and with a saw divide it into hemispheres. Place one with the flat part down and drive a tack into the centre of the curved part. Attach a ball of twine to the tack and wind it round the surface of the hemisphere as a cord is wound round a spinning top so that it will exactly cover the surface. Then drive a tack into the centre of the flat part of the other hemisphere and attach the twine to the tack and wind it around pressing it down on the flat surface until it is entirely covered. It will be found that the string covering the flat surface is just half as long as that covering the round surface.

The number of units of area in plane face $= \pi r^2$;
∴ " " " " " " " curved " $= 2\pi r^2$;
∴ " " " " " " " sphere " $= 4\pi r^2$;
$= \pi d^2$.

3. Given the radius, to find the cubic content of a sphere.

A sphere may be regarded as composed of a large number of pyramids, whose apexes unite at the centre of the sphere and whose bases form the curved surface.

Hence, number of units of area of base $=4\pi r^2$;

" " " " height $=r$;

$\therefore$ " " " " cubic content $=\frac{1}{3}\times r\times 4\pi r^2$

$=\frac{4}{3}\pi r^3$.

Ex. 1. How much iron is there in a hollow sphere 12 in. in diameter, the iron being 1 in. thick?

Cubic content of sphere $=(\frac{4}{3}\times\frac{22}{7}\times 6^3)$ c. in.;

" " " cavity $=(\frac{4}{3}\times\frac{22}{7}\times 5^3)$ c. in.;

$\therefore$ number of c. in. of iron $=\frac{4}{3}\times\frac{22}{7}(6^3-5^3)=381\frac{1}{3}$.

XIV. SIMILAR SOLIDS.

Steps.

1. The Definition.

Present similar solids as spheres, cubes, rectangular solids, etc.

Show that when similar they are of the same shape and have the same number of *similar surfaces* but they are not necessarily of the same size.

2. The volumes of similar solids are proportional to the cubes of the measures of the corresponding lines of measurement.

Illustrate this by comparing the volumes of a number of similar solids.

Thus, compare the volumes of two cubes with edges 1 in. and 2 in. long, respectively; of two spheres with radii 3 in. and 4 in., respectively; etc.

Compare the volume of a rectangular block 2 in. by 3 in. by 4 in. with another three times as long, three times as wide and three times as deep.

Show by drawing that the latter can be divided into 27 blocks each equal to former.

Ex. 1. A sphere 2 in. in diameter weighs 16 oz.; find the weight of a sphere made of the same material $2\frac{1}{2}$ in. in diameter.

Weight of 2nd sphere $=\left(\frac{2\frac{1}{2}}{2}\right)^3\times 16$ oz. $=31\frac{1}{4}$ oz.

Ex. 2. The breadth of a rectangular solid is 12 ft.; what must be the breadth of a similar solid whose volume is three times as great?

Here, $\left(\frac{\text{measure of breadth}}{12}\right)^3=3$;

$\therefore$ (measure of breadth)$^3=12^3\times 3$;

$\therefore$ measure of breadth $=12\times 1.44225=17.307$;

$\therefore$ breadth $=17.307$ ft.

XIX. THE METRIC SYSTEM.

1. Explanations.

In 1795 France adopted the Metric System of Weights and Measures. It is based on the decimal system of notation, all the units used being multiples or submultiples by 10 or by powers of 10, of the primary unit.

The standard unit is the metre which is legally defined to be the length of a platinum rod at the temperature of melting ice, and which was supposed to represent one ten-millionth part of the distance from the equator to the pole of the earth at the level of the sea. It has since been ascertained that this is not quite correct. Hence, the metric standard unit of length is Borda's platinum rod and not an exact fraction of a meridian of the earth. As will be seen from the diagram in the Public School Arithmetic the decimetre or tenth part of a metre is nearly 4 inches long. The metre itself is 39.37 inches long, nearly $1\frac{1}{10}$ yards.

2. Primary Units.

(1) The unit of length is the METRE.

(2) The unit of volume is the LITRE. This is the capacity of a cubical box 1 decimetre long, wide, and deep. It is equal to 1.761 Imperial Pints.

(3) The unit of weight is the GRAM. This is the weight of as much distilled water as at the temperature of 4°C. (its greatest density) will fill a cubical measure 1 centimetre long, wide, and deep. It equals 15.4323 grains.

(4) The unit of surface is the ARE. This is equal to a square surface 10 metres long, and wide and contains 119.601 sq. yards.

3. Apparatus needed.

Certain apparatus is necessary to teach the tables properly:—

(*a*) FOR LINEAR MEASURE :—

A divided metre stick, a metrical tape line, a metrical chain.

(*b*) FOR SQUARE MEASURE:—

A square decimetre sub-divided into square centimetres.

(*c*) FOR MEASURE OF CAPACITY:—

A litre; a cubical box 1 decimetre long, wide and deep; a centilitre.

(*d*) FOR MEASURE OF WEIGHT:—

A pair of scales and the following weights:—A gram wt., 2 two gram wts., a five gram wt., 2 ten gram wts., a twenty gram wt., a fifty gram wt., a hectogram, two hectogram wts., a half kilogram, and a kilogram.

4. Steps.

(*a*) Teach Linear Measure first.

(*b*) Begin with the metre. Show that 10 decimetres make 1 metre, and that 10 centimetres make 1 decimetre, and infer that 100 centimetres make a metre.

(*c*) Have pupils measure 2, 3, 4, 5, etc., decimetres and express each of these in centimetres. If they understand decimals have these expressed also as metres.

(*d*) Show by measuring that 10 millimetres make a centimetre and once more use reduction to fix the relative lengths of the measures already taught.

(*e*) From the metre proceed by measuring to the dekametre; then to the hectometre and kilometre and use in each case many mental problems in reduction to fix the relative lengths of these measures.

A kilometre should be measured and marked off for future reference. This can be done after school with the class assisting. Select a straight road or street for this purpose.

(*f*) At this time make no attempt to infer the meanings of the prefixes, milli, centi, deci, deka, hecto, and kilo. This may be done after Long Measure is well understood.

XX. MISCELLANEOUS EXERCISES.

I. CIRCULATING DECIMALS.

See Page 173.

II. PROBLEMS ON WORK.

(1) Show the relation between the time of doing a work and the part done in one unit of time.

Thus, a work can be done in 2 da., hence $\frac{1}{2}$ of it is done in 1 da.; etc.

(2) Show the relation between the part done in one unit of time and the time necessary to complete the work.

Thus, if $\frac{2}{3}$ of a work is done in 1 da., in what time can the whole work be done?

Time to do $\frac{2}{3}$ of work $= 1$ da.;
$\therefore$ " " " $\frac{1}{3}$ " " $= \frac{1}{2}$ da.;
$\therefore$ " " " $\frac{3}{3}$ " " $= \frac{3}{2}$ da. $= 1\frac{1}{2}$ da.

(3) Show how to find the amount of work done by a number of men in a unit of time.

(4) Apply these principles to solving problems. See page 74.

III. CLOCK PROBLEMS.

(1) By means of a clock or watch show that the minute-hand moves 12 times as fast as the hour-hand. It moves over 60

minute-spaces while the hour-hand moves over 5 minute-spaces and thus 55 minute-spaces are gained by it in 60 minutes of time, hence 1 minute-space is gained by the minute-hand in $1\frac{1}{11}$ minutes of time.

(2) Next train the pupils to find the relative position of the hands at the time given.

(3) Then ascertain the number of minute-spaces to be gained by the minute-hand to bring them into the position required.

(4) Now calculate the time required.

Ex. 1. See page 78.

Ex. 2. What is the second time after four that the hands of a watch are 2 minute-spaces apart?

(1) At four they are 20 min.-spaces apart.

(2) These 20 min.-spaces must be gained and 2 more; hence, 22 min.-spaces must be gained.

(3) 22 min.-spaces are gained in 22 times $1\frac{1}{11}$ min. of time, or in 24 min.

Hence, the time is 24 min. past 4.

Ex. 3. What is the first time after 7 that the hands of a watch are at right angles?

(1) At 7 the hands are 35 min.-spaces apart.

(2) When at right angles they are 15 min.-spaces apart; hence, 20 min.-spaces must be gained.

(3) 20 min.-spaces are gained in 20 times $1\frac{1}{11}$ min. of time, or $21\frac{9}{11}$ min.

Hence, the time is $21\frac{9}{11}$ min. past 7.

Ex. 4. What is the first time after 5 that the hands of a watch form an angle of 72°?

$72° = \frac{1}{5}$ of the circumference;

$\therefore 72° = \frac{1}{5}$ of 60 min.-spaces $=$ 12 min.-spaces;

(1) At 5 the hands are 25 min.-spaces apart.

(2) 13 min.-spaces must be gained.

(3) 13 min.-spaces are gained in 13 times $1\frac{1}{11}$ of time, or $14\frac{2}{11}$ min.

Hence, the time is $14\frac{2}{11}$ min. past 5.

Ex. 5. What is the first time after 5 that the hands of a watch are equally distant from the figure 5?

(1) The hands are 25 min.-spaces apart.

(2) The hour-hand is a certain distance from 5.

(3) The min.-hand is the same distance from 5 and 12 times that distance from 12.

(4) Hence, 13 times that distance $=$ 25 min.-spaces;

$\therefore$ once that distance $= 1\frac{12}{13}$ min.-spaces:

$\therefore$ the min.-hand is $(25-1\frac{12}{13})$ min.-spaces past 12, or $23\frac{1}{13}$ min.-spaces past 12.

Time is $23\frac{1}{13}$ min. past. 5.

IV. PROBLEMS INVOLVING VELOCITY.

These are quite simple. The main point is to state clearly what is given. There are just two cases as follows:—

Case I. Given the distance a body moves in a given time, to find the distance covered by it in another given time.

Ex. 1. How many feet per second is equal to a rate of 45 miles per hour?

In (60×60) sec. the distance gone$=(45\times5280)$ ft.;

$\therefore$ " 1 " " " " $=\dfrac{45\times5280}{60\times60}$ ft.$=66$ ft.

Case II. Given the distance a body moves in a given time, to find how long it will require to move a certain distance.

Ex. 2. How long will a train 80 yards in length, moving 20 miles per hour, take to cross a bridge 184 yards long?

Time to move (20×1760) yd.$=(60\times60)$ sec.;

$\therefore$ " " " $(80+184)$ yd. $=\dfrac{264\times60\times60}{20\times1760}$ sec.$=27$ sec.

V. PROBLEMS INVOLVING THE SUM AND DIFFERENCE OF TWO QUANTITIES.

1. Given the sum of two numbers and the difference of the same two numbers, to find them.

By means of many examples show

(1) that the sum of two numbers added to their difference gives a sum which is equal to *twice the greater* number, and

(2) that the difference of two numbers taken from their sum gives a remainder equal to *twice the smaller* number.

E.g.

$$\begin{array}{rcl} 9+5 & = & 14 \\ 9-5 & = & 4 \\ \hline 2\times9 & = & 14+4 = 18 \\ 2\times5 & = & 14-4 = 10 \end{array}$$

2. Stream Problems.

Problems should not be given until the forces affecting a boat on a stream are understood. Show the following by such questions as, "What forces are causing a boat to move *down* a stream?" "How do these forces affect the boat in going up the stream?" etc.:

(1) The rate down the stream is the *sum* of two rates, viz., the rower's rate in still water and the rate of the stream.

(2) The rate up stream is the difference of the same two rates.

(3) The sum of the rower's rate down and up stream is equal to twice his rate of rowing in still water.

(4) The difference between the rates down and up stream is equal to twice the rate of the stream.

Ex. 1. A can row 12 miles down stream in 3 hours and the same distance back in 4 hours. Find the rate of the stream in miles per hour.

Rate down per hour=4 ml.
" up " " =3 ml.
Hence, rate in still water+rate of str.=4 ml.
" " " " − " " " =3 ml.;
∴ 2×rate of str.=1 ml.;
∴ rate of str. $=\frac{1}{2}$ ml.

Ex. 2. A can row $4\frac{1}{2}$ ml. down stream in an hour. Without the aid of the stream, it would have taken him $1\frac{1}{2}$ hr. How long will it take him to return?

Rate in still water per hour$=\dfrac{4\frac{1}{2}}{1\frac{1}{2}}$ ml.=3 ml.;
∴ " of stream " " $=(4\frac{1}{2}-3)$ ml.$=1\frac{1}{2}$ ml.;
∴ rate of boat up stream$=(3-1\frac{1}{2})$ ml.$=1\frac{1}{2}$ ml.;
Time to row $4\frac{1}{2}$ ml. up stream$=\dfrac{4\frac{1}{2}}{1\frac{1}{2}}=3$ hr.

Ex. 3. A can row 6 ml. down stream and back again in 2 hr. 40 min., and his rate of rowing in still water is twice the rate of the stream. Find his rate of rowing in still water.

Rate *down* stream=3 times rate of str.
" *up* " =1 " " " "

Hence, in going the round trip he spends $\frac{1}{4}$ of the time in going down and $\frac{3}{4}$ in rowing back.

$\frac{1}{4}$ of 160 min.=40 min.
Rate per hour down stream$=\frac{60}{40}$ of 6 ml.=9 ml.
" " " up " $=\frac{6}{2}$ ml. =3 ml.
Hence, rate in still water$=\dfrac{9+3}{2}$ml. =6 ml.

Ex. 4. A boat's crew can row over a course of $1\frac{1}{4}$ ml. against a stream which flows at the rate of 2 ml. per hr. in 10 min. The usual rate of the stream is $\frac{1}{2}$ ml. per hr. How long would the crew take in the usual state of the river?

Rate up stream per hr.$=(6\times1\frac{1}{4})$ ml.$=7\frac{1}{2}$ ml.
Rate of stream=2 " ;
∴ rate in still water$=9\frac{1}{2}$ " ;
∴ rate in usual state of river=9 "
Hence, time to row 9 ml.=60 min.;
∴ " " " $1\frac{1}{4}$ " $=\dfrac{1\frac{1}{4}\times60}{9}$ min.$=8\frac{1}{3}$ min.

PART IV.

Review Problems.

SIMPLE RULES.

1. A boy rides round a field 80 rods long and 48 rods wide. How many rods will he travel in going round it three times? Ans. 768 rods.

2. A room is 64 ft. long and 27 ft. wide. How long will a string be that will go around it twice? Ans. 364 ft.

3. Find the sum of all the numbers ending in 73 between 9272 and 10087. Ans. 87057.

4. A has $3120 which is $934 more than B has. If C has $1306 less than A and B together, how much have all? Ans. $9306.

5. A man bought a house and lot for $4500; he expended $1290 on it. The house was burned; he got $3780 insurance, and sold the lot for $1960. Find his loss. Ans. $50.

6. Find a number that exceeds 14967 as much as 47058 exceeds 31407. Ans. 30618.

7. In a school there are 318 scholars: 177 are boys. The girls occupy two rooms, in one there are 65. How many are there in the other? Ans. 76 girls.

8. A boy caught 178 fish and another caught 262. How many must the first get from the second that both may have an equal number? Ans. 42 fish.

9. A drover bought 10431 sheep; he sold 5736 and then bought 763 less than he had left. How many had he then? Ans. 8627 sheep.

10. A is worth $15687; B $573 less than A; C $947 more than B, and D $1568 more than C. How much have they all? Ans. $64491.

11. John has $1650 and James has $764 less than John. If Chas. has $2347 more than John and James together, how much have they all? Ans. $7419.

12. A man lost $2750 on the race track; his father gave him $3470 and he borrowed $1550, he then spent $3895 and had $12000 left. What had he at first? ANS. $13625.

13. Three brothers A, B and C had given to them the following amounts: A, $12500; B, $18000; C, $11500. B gave C $5375, who gave $2875 to A, who gave $1375 to B. How much had each then? ANS. $14000 each.

14. A sold a horse to B and gained $118. B sold it to C for $550 and gained $72. What did the horse cost A? ANS. $360.

15. A wholesale merchant's sales for the first six months were as follows: Jan., $21972; Feb., $36489; March, $17437; April, $21008; May, $14567; June, $21497. Out of which his gain was $35217. Find the cost of his goods. ANS. $97753.

16. A man gave away $37.90, and then borrowed $26.75, spent $75.50 and had $13.35 left. What had he at first? ANS. $100.

17. What number increased by the difference between 1875 and 2368 will make the sum of 416, 2487 and 2097? ANS. 4507.

18. A lady paid $26 for a dress, $3.50 for a parasol, $4.50 for boots, and $7 for a hat. How much had she left of $50? ANS. $9.

19. Find the difference between the sum of the six numbers of 3 figures each that can be expressed by the figures 5, 8, 7; and the sum of six numbers of two figures each that can be expressed by the same figures. ANS. 4000.

20. A man has $372.50 in the bank, he takes out $26.95, he then puts in $109.41, and five days after he draws out $57.64. How much has he still in the bank? ANS. $397.32.

21. A, B, C, D bought a farm for $14000; A paid $3750, B $525 less than A, C $800 more than B. How much less than A did D pay? ANS. $550.

22. A man spends on Monday $1.80, on Tuesday $1.15, Wednesday $1.05, and as much more for the rest of the week. He earned $21.80. How much did he save? ANS. $13.80.

23. Find the product of all the numbers ending in 4 between 21 and 61. ANS. 1938816.

24. Two trains start from Hamilton at 5 p.m. and travel in the same direction, one at 27 miles and the other at 36 miles per hour. How far apart will they be at 5 a.m. next day? ANS. 108 miles.

25. A farmer bought a horse for $175, which he traded for some cattle and $21 in cash. Some of the cattle died and he sold the rest for $137. How much did he lose altogether? ANS. $17.

26. A man earns \$2.40 a day. What will he earn in the month of July if the month begins on Tuesday? Ans. \$64.80.

27. A man earning \$2.25 per day of 9 hrs., lost 17 hours in a week. What did his earnings amount to for that week of 6 days? Ans. \$9.25.

28. A man bought 375 acres of land at \$18 per acre and sold it for \$7000. What did he gain? Ans. \$250.

29. A man earns \$18 a week and pays \$27 per month for board. How much can he save in a year? 1 yr.=12 months=52 weeks. Ans. \$612.

30. What number is that to which if 17 be added the result will be 15 times 128? Ans. 1903.

31. Find the difference in the cost of 275 horses at \$93.20 each and 483 cattle at \$56.10 each. Ans. \$1466.30.

32. A farmer exchanged 1840 bu. of wheat at \$1.10 per bu. for 164 bbl. flour at \$9.20 per bbl. and the balance in cash. How much cash should he receive? Ans. \$515.20.

33. Find the difference in the value of two farms one of 250 acres at \$74 an acre and the other 188 acres at \$125 per acre. Ans. \$5000.

34. What is the difference between the values of 84 horses at \$129 each and 168 cattle at \$64 each? Ans. \$84.

35. Find a number such that when the sum of the odd numbers between 68 and 78 is taken from it the remainder will be 25 times 296. Ans. 7765.

36. What number is that from which if 47 times 19 be taken, the remainder will be 35 times 137? Ans. 5688.

37. If I buy coffee for 65c., tea for 93c. and butter for 127c., and give a \$5 bill in payment. What change should I receive? Ans. \$2.15.

38. 398 acres of wheat in Manitoba yielded 52 bu. per acre, which sold at 40c. a bu. What was the value of the crop? Ans. \$8278.40.

39. A farmer sold 234 cattle at \$64 each and then bought 960 acres of land at \$25 an acre, borrowing the balance to pay for it. How much did he borrow? Ans. \$9024.

40. A southern trader sold 673 bales of cotton of 417 lb. each at 23c. a lb. What did he get for it? Ans. \$64547.43.

41. A farmer has 168 sheep in one field, 3 times as many less 247 in another, and in a third 149 more than in both of the others. How many has he altogether? ANS. 999 sheep.

42. A, B, C and D bought property for $100000. A's share was $25625; B's $7250 more than A, and C $5210 less than B. How much had D? ANS. $13835.

43. A farmer bought 48 cows for $1056; he fed them at an expense of $2.45 each, and sold them all for $1496. Find his total gain. ANS. $322.40.

44. A boy lives 96 rods from school, he goes regularly for 37 weeks, 5 days each week, and goes home to dinner every day. How many rods will be travel altogether? ANS. 71040 rods.

45. Four men bought mining property for $25200. A put in $6525; B $845 more than A, and C $160 less than B. What did D give? ANS. $4095.

46. A drover bought 468 sheep at $4 each; he paid freight $117. Fifteen sheep died and he sold the remainder at $6.19 each. Find his net gain. ANS. $815.07.

47. In a factory there are 25 men at $2.40 per day and 38 women at $1.60 per day. How much more will be paid out to the women than to the men in 18 days? ANS. $14.40.

48. A man earns $62.75 a month and spends $7.25 a week, how much will he save in 5 years. 1 year=12 months=52 weeks. ANS. $1880.

49. Divide the product of 784 and 348 by the difference between 784 and 436. ANS. 784.

50. 231 men earn $1732.50 in a week. What does each earn in a day? ANS. $1.25.

51. Divide the product of 22, 23, 24, 25 by the product of their right hand digits. ANS. 2530.

52. Find the remainder when the product of the first 10 numbers is divided by the difference between the product of the even numbers and the product of the odd numbers. ANS. 1365 remainder.

53. The exports of cotton from the U. S. during 1885 was 1889514368 lbs. How many bales of 476 lb. each were exported? ANS. 3969568 bales.

54. In a paper mill 475 sq. ft. are manufactured per minute. How many sq. ft. will be made in 8 weeks, working 5 days each week and 600 minutes each day? ANS. 11400000 sq. ft.

55. I sold 56 horses for \$8680, gaining \$25 on each. Find the cost of each. ANS. \$130.

56. A Manitoba farmer bought 640 acres of land for \$14000, he sold part of it for \$9600 at \$32 per acre and the rest at \$65 an acre. What did he gain? ANS. \$17700.

57. A merchant bought flour for \$1061.50 at \$5.50 per bbl. and sold it to gain 40c. on each bbl., find the number of bbl. and the total gain. ANS. 193 bbl.; \$77.20.

58. If 36 horses cost \$4950, at what rate must each be sold to gain \$12.50 per head? ANS. \$150.

59. Multiply 41768 by 630 and divide the product by $2 \times 5 \times 7 \times 9$. ANS. 41768.

60. A fruit dealer bought 6 bbl. of apples at \$2.25 per bbl. Each bbl. contained 560 apples, which he sold at 3 for 5c. How much did he gain? ANS. \$42.50.

61. A man bought 30 oxen at \$45 each; 4 died and he sold the rest so as to gain \$80 on the transaction. Find the selling price of each. ANS. \$55.

62. A man sold 213 bbl. of flour for \$5112 and gained \$1278. Find the cost per bbl. ANS. \$18.

63. What is the smallest number divisible by 9 which added to 4392 makes the sum divisible by 12? ANS. 36.

64. A certain number increased by 4376 and the sum multiplied by 427 gives 3846416 for product. What is the number? ANS. 4632.

65. What is the least number which taken from 292463 leaves a remainder of which 84 is an exact divisor? ANS. 59.

66. The dividend is $24 \times 9 \times 15 \times 84$ and the divisor $5 \times 7 \times 36 \times 108$. Find the quotient. ANS. 2.

67. I sell my home for \$4500; village lots for \$500; Loan Co. stock for \$1150; furniture for \$1050; and receive in payment \$6000 in cash and the balance in N. W. lands at \$2.50 per acre. How many acres do I get? ANS. 480 acres.

68. Find the amount of the following bill:—

147 cords hard wood @ \$5.75 per cord;
206 " soft " @ \$4.25 " "
4 car loads of slabs 16 cords each @ \$2.75 per cord;
816 tons hard coal @ \$5.15 per ton;
536 " soft " @ \$3.85 " " ANS. \$8162.75.

69. A merchant bought 63 bbl. of apples and 47 bbl. of flour for $722. He paid $10 a bbl. for the flour; what did he pay per bbl. for the apples? Ans. $4.

70. A bought 100 acres of land for $3750; he allowed 15 acres for streets, and divided the rest into village lots and sold them for $17920 at $320 each. How many lots did he sell? Ans. 56 lots.

71. In a certain city there are 5500 children in the P. S. and the cost of maintaining them is $39050. What is the cost for each child? Ans. $7.10.

72. In a block of stone there are 454464 c. in. How many c. ft. are there if 1 c. ft.=1728 c. in.? Ans. 263 c. ft.

73. The steel rails on a railway weigh 384 lb. each. How many will there be in 2211840 lb. Ans. 5760.

74. A history has 496 pages in a volume and 84 lines on a page. If the total number of lines is 249984, how many volumes are there? Ans. 6 volumes.

75. A coal dealer sold 552 tons coal for $2456.40 and lost $110.40. At what rate per ton should he have sold to gain $138.00. Ans. $4.90.

76. In South Africa a soldier's rations were 36 oz. per day. How many lb. of 16 oz. each would it take to supply Lord Robert's army for 64 days if he had 156000 men? Ans. 22464000 lb.

77. A fruit grower in California packed 340000 lemons in boxes and sold them for $6528 at $4.80 a box. How many lemons are there in a box? Ans. 250 lemons.

78. A drover exchanged 245 horses at $75 each for 468 cattle and $2463 cash. What was the price of the cattle per head? Ans. $34.

79. How many doz. eggs at 14c. a doz. must be given for 15 yd. of cloth at $14 for 5 yd.? Ans. 300 doz.

80. I sold 2048 yd. of cloth for $1249.28 and by so doing lost 4c. a yd. At what price per yd. must I have sold it to gain $102.40? Ans. 70c.

81. If I gain $1428 by selling 119 acres for $9163, what would 221 acres cost me to buy at the same rate? Ans. $14365.

82. A man sold his farm in village lots for $750 each and bought 300 horses at $125 each with the money. How many lots did he sell? Ans. 50 lots.

83. From the sum of $49086 \times 7 \times 9$ and $50914 \times 7 \times 9$ take the difference between $76936842 - 9$ and $22739766 \div 7$. ANS. 1000000.

84. How many bushels of wheat at 72c. a bu. should be given for 900 bu. of oats at 40c. a bu.? ANS. 500 bu.

85. A train goes from Sarnia to London, 62 miles, in 2 hours. At the same rate in how many hours after leaving London will it reach Montreal, a distance of 434 miles? ANS. 14 hr.

86. If 45 lb. of tea cost $28.80, what amount of butter at 16c. per lb. will pay for 8 lb. of tea? ANS. 32 lb.

87. Find the value of 6360 lb. of wheat at 71c. a bu. ANS. $75.26.

88. How many boxes of tea of 48 lb. each at 60c. a lb. must be given for 16 tubs of butter of 50 lb. each at 18c. a lb.? ANS. 5 boxes.

89. A man bought 60 acres at $56 per acre; he paid out for repairs $754 and sold it for $5374. How much did he gain per acre? ANS. $21.

90. A boy buys 240 papers a week at 2c. each and sells them at 4 for 18c. How much did he gain in 7 weeks? ANS. $42.

91. Find the amount of the following bill:—

216 bu. potatoes @ 45c. a bu.;
150 bu. turnips @ 25c. a bu.;
170 bu. peas @ 80c. a bu. ANS. $270.70.

92. A man sold 1476 bbl. of flour for $11583. 288 bbl. were sold @ $7 per bbl. and 450 bbl. at $6.50 per bbl. At what rate per bbl. did he sell the remainder? ANS. $9.

93. A sold to B 350 acres at $120 per acre; and gained $5775. B sold it at a loss of $47.25 per acre. How much less per acre did B receive than A paid? ANS. $30.75.

94. A store keeper sold hats at $3 each and an equal number at $2 each. How many did he sell altogether for $9835? ANS. 3934 hats.

95. What number is that to which if 247 and 4 times 247 be added, and 127 and 4 times 127 be taken from the result, the remainder will be 9164? ANS. 8564.

96. A farmer raised 512 bu. wheat in 1898 and twice as much in 1899, and in 1900 as much as in the other two years. What was the value of all at $1.25 per bu.? ANS. $3840.

97. If I take 23778 from the sum of 8861 and 24867, divide the remainder by 25 and multiply the quotient by 9, what will the product be? Ans. 3582.

98. If 17 gal. of wine at $3.75 per gal. are mixed with 51 gal. at $2.88 per gal. at what price per gal. must the mixture be sold to gain $34.17? Ans. $3.60.

99. A reservoir holds 396000 gal. How many hours will it take to fill it if 1890 gal. are pumped in every minute and 1340 taken out in the same time. Ans. 12 hr.

100. A certain number is multiplied by 65 and 748 subtracted from the product, and when the remainder is divided by 264 the quotient is 32 and the remainder 34. Find the number. Ans. 142.

101. A dealer sold pork for $32832 and gained $5184 by selling it at $19 a bbl. What was the cost per bbl. Ans. $16.

102. I paid $30432 for horses at $96 each; 17 died and the rest were sold for $31500. What did I gain on each one sold? Ans. $9.

103. A farmer planted 864 rows of corn, with 297 hills in each row; had he put 243 hills in each row how many more rows would be necessary to have the same number of hills? Ans. 192 rows.

104. If 27 men can plant 11664 hills of corn in 13 hours, how many hills can 63 men plant in the same time? Ans. 27216 hills.

105. A man sells 250 bags of flour at 7c. a lb. and receives in payment 34 chests of tea of 75 lb. each at 70c. a lb. Find the weight of one bag of flour. Ans. 102 lb.

106. A cattle drover paid $22464 for cattle at $26 each; 25 head were stolen and 56 died; he sold the rest at a gain of $5724 above the cost of all. Find the selling price per head. Ans. $36.

107. 5700 persons each paying 2c. toll and 728 cabs each paying 6c. toll pass over a bridge in 12 hours. Find the average amount of money paid per hour in toll. Ans. $13.14.

108. A certain number increased by 12968 and the sum divided by 247 will give a quotient of 138 and a remainder; if the remainder is multiplied by 225 the product will be 55350. What is the original number? Ans. 21364.

109. If flour is $4.75 a bbl. and apples $2.40 a bbl., how many bbl. can be bought for $343.20 to have the same number of each? Ans. 48 bbl. of each.

110. A farmer spent $7.25 for groceries; $13.50 for a stove; $3.75 for a pair of boots; $17.75 for clothes, and $6.25 for

harness. How many bu. of wheat at 97c. will pay the bill. ANS. 50 bu.

111. A farmer sold an equal number of bu. of wheat, oats and barley for $270; the wheat sold at 90c., the oats at 32c. and the barley at 58c. How many bu. of each were sold? ANS. 150 bu.

112. A grocer bought 48 bbl. of oil and sold it at a gain of 6c. a gal. and thus gained $144. How many gal. were there in a bbl.? ANS. 50 gal.

113. A drover with $6600 bought 34 horses at $70 each; 55 cows at $26 each; 192 pigs at $5.25 each, and sheep at $6 each. How many sheep did he buy? ANS. 297 sheep.

114. 36 bbl. of vinegar cost $594. If it was sold at 40c. a gal. and there were 54 gal. in a bbl., how much was gained? ANS. $183.60.

115. A speculator bought town lots for $7200, and sold them to gain $810 at $90 each. How many did he sell? ANS. 89 lots.

116. If 40 sq. rods of land will produce 15 bu. of potatoes, what will be the value of the crop grown on 17 acres at 50c. a bu.? ANS. $510.

117. I sold 1600 bu. of wheat for $2048 and thus lost $80. If I had sold it to gain $96, what should I get per bu.? ANS. $1.39.

118. How many bbl. of apples containing 3 bu. each at 40c. a bu. will be equal in value to 16 boxes of oranges each containing 288 oranges at 12 for 15c.? ANS. 48 bbl.

119. How many bbl. of sugar of 300 lb. each worth 4c. a lb. must be given for 29 bbl. of flour at $4.40 per bbl. and $52.40 cash? ANS. 15 bbl.

120. A farmer sold 1440 bu. of oats at 28c. a bu. and received in payment 8640 ft. of lumber at 3c. a foot and pigs at $8.00 each. How many pigs did he get? ANS. 18 pigs.

121. A field containing 48 acres yielded 26 bu. per acre. The grain was sold at 65c. a bu. and the money invested in sheep at $5.20 each. How many sheep were bought? ANS. 156 sheep.

122. A dealer bought 840 bu. of wheat at 18 bu. for $22.50 and sold it at 11 bu. for $16.50. How much did he gain? ANS. $210.

123. The smaller of two numbers is contained 48 times in 864 and the larger number is 23 times the smaller. Find the product of the two numbers. ANS. 7452.

124. A man buys 364 lb. of sugar for $14.56. He uses 48 lb. At what price must he sell the remainder to gain $4.40? Ans. 6c. per lb.

125. A and B have equal shares in a load of potatoes of 86 bu. A takes 25 bu. and B the rest, giving A $10.80. What are potatoes worth per bu.? Ans. 60c.

126. What is the cost of 296 head of cattle if by selling 247 head for $19219 the gain is $1435? Ans. $21312.

127. Extend the following and find the sum of the results:—

(a)		(b)	
$1\times9+2=$		$1\times8+1=$	
$12\times9+3=$		$12\times8+2=$	
$123\times9+4=$		$123\times8+3=$	
$1234\times9+5=$		$1234\times8+4=$	
$12345\times9+6=$		$12345\times8+5=$	
$123456\times9+7=$		$123456\times8+6=$	
$1234567\times9+8=$		$1234567\times8+7=$	
$12345678\times9+9=$		$12345678\times8+8=$	
	123456788	$123456789\times8+9=$	
			1097393685

II. COMPOUND NUMBERS.

1. A farmer had 38 cwt. 52 lb. of pork; he kept 4 cwt. and 86 lb. for his own use, and sold the rest in barrels of 198 lb. How many bbl. did he sell? Ans. 17 bbl.

2. In walking 1 mi. 212 rd. 4 yd. 2 feet James took 3768 steps, and Tom 5913 steps in 3 mi. 85 rd. 1 yd. 0 ft. 9 in. Find the difference in the length of their steps. Ans. 7 inches.

3. If 72 cords of wood be taken from a pile 320 ft. long 12 ft. wide and 8 ft. high. How many cords will be left? Ans. 168 cords.

4. A farmer sold 3776 boxes of fruit averaging 1 qt. each. How many bushels did he sell? Ans. 118 bu.

5. At the rate of 10 rods in a minute, how many miles and rods can a boy walk in an hour? Ans. 1 mi. 280 rods.

6. A field contains 348480 sq. ft. What is it worth at $500 an acre? Ans. $4000.

7. If 32 sq. rd. of land cost $96, what will be the value of 7 ac. 136 sq. rd.? Ans. $3768.

8. If a man earns £4 11s. 6d. in 6 days, what amount will he earn in 273 days. Ans. £208 3s. 3d.

9. If a cup weighing 11 oz. 15 dwt. cost £3 8s. 6½d., what must be paid for one weighing 17 oz. 10 dwt. 12 gr.? Ans. £5 2s. 2⅘d.

10. The latitude of Brantford is 42° 21′ 22″, and of Quebec 46° 50′ 10″. How many degrees is Quebec north of Brantford? Ans. 4° 28′ 48″.

11. If 122 lb. 3 oz. 15 dwt. 16 gr. of gold be made into 14 ornaments of equal weight. How much will one ornament weigh? Ans. 8 lb. 8 oz. 16 dwt. 20 gr.

12. I sold 124 loads of wood, each containing 96 cu. ft. for $418.50. What did I get per cord? Ans. $4.50.

13. A farmer has 111 bu. 2 pk. 4 qt. of wheat which he wishes to put in bags which hold 2 bu. 1 pk. 4 qt. each. How many bags will be required? Ans. 47 bags.

14. How long will it take a man to walk 147840 yd. at 3 miles an hour? Ans. 28 hours.

15. If a boy goes to bed at half-past 8 p.m. and gets up at 7 a.m. How long is he in bed? Ans. 10 hr. 30 min.

16. From a bbl. of vinegar containing 31 gal. 2 qt., 5 gal. 2 qt. were taken out, and 30 qt. were wasted. How much still remained in the barrel? Ans. 18 gal. 2 qt.

17. How many lb. oz. dr. Avoir. wt. are there in 4 lb. 4 oz. 10 dwt. Troy? 16 drams=1 oz. Ans. 3 lb. 9 oz. 9⅗ dr.

18. Multiply the difference between 17 miles and 15 mi. 319 rd. 5 yd. 1 ft. 6 in, by 27. Ans. 27 miles.

19. There are 650 posts (including those at the end of the road) on a railway 18 mi. 445 yd. 1 ft. 6 in. long. How many *rods* apart are the posts? Ans. 9 rods.

20. A butcher bought 2 t. 8 cwt. of beef at $190 a ton and sold it at 2 lb. for 25c. How much did he gain? Ans. $144.

21. How many steps will a man take in travelling 20 mi. and 450 yd., allowing 2 ft. 6 in. to each step? Ans. 42780 steps.

22. Find the cost of 25 lb. 14 oz. of quinine when $192.50 is paid for 5 lb. 8 oz. Ans. $905.62½.

23. A cellar 42 ft. long and 24 ft. wide is flooded to the depth of 8 inches. How many barrels of water are there in it? 1 bbl.= 31½ gallons. Ans. 133⅓ bbl.

24. I have a lot 90 ft. long and 75 ft. wide, which is enclosed by a tight board fence 6 ft. 3 in. high. Find the cost of painting both sides at 15c. per sq. yd. Ans. $68.75.

25. A shed 32 ft. long, 24 ft. wide and 14 ft. high is filled with tan bark. What is the value of the bark at $3.25 per cord? ANS. $273.

26. A farm containing 123 ac. 80 sq. rd. has 80 ac. 100 sq. rd. cleared. What is the uncleared part worth at $80 per acre? ANS. $3430.

27. If water weighs 770 times as much as air, and 1 c. ft. of water weighs 1000 oz., how many c. ft. of air weigh 100 lb.? ANS. 1232 c. ft.

28. How many bu. of wheat will weigh as much as 180 bu. of Indian corn? ANS. 168 bu.

29. From a pile of wood 28 ft. long, 4 ft. wide, 4 ft. high 1 cd. 64 c. ft. is sold at one time and 96 c. ft. at another. What is the remainder worth at $6.40 a cord? ANS. $8.00.

30. Mary's birthday is on Aug. 14th. When is Jane's, which is 137 days later? ANS. Dec. 29th.

31. How many times will a wheel 5 ft. 6 in. in circumference turn in going 4 miles? ANS. 3840 times.

32. 7 cwt. 29 lb. of sugar is taken in exchange for 1 cwt. 62 lb. of coffee at 3s. 9d. a lb. Find the price of the sugar per lb. ANS. 10d.

33. Find the cost of sodding a lawn 40 yd. long and 10 ft. wide with sods each containing 3 sq. ft.; the sods when laid costing 60c. per hundred. ANS. $2.40.

34. A barrel of flour weighs 196 lb. If 2 bu. and 2 pk. of wheat make 100 lb. of flour, how many bbl. can be made from 490 bu. of wheat? ANS. 100 bbl.

35. What is the cost of 25 joists each 6 in. by 4 in. and 15 ft. long at $24 per M. board measure? ANS. $18.00.

36. How many tons of wire will it require to fence a rectangular farm 180 rods long and 120 rods wide, the fence being 6 wires high and 3 ft. of wire weighing 1 lb.? ANS. 9 tons 1800 lb.

III. AVERAGES AND SHARING.

1. The scores made by a side at cricket were 20, 5, 14, 0, 45, 16, 22, 0, 0, 10, 0; find the average score of each batter. ANS. 12.

2. A butcher sold 12 lb. beef at 11c. a lb. and 7 lb. at 8c. a lb. and 11 lb. at 12c. What was the average price per lb.? ANS. $10\frac{2}{3}$c.

3. A traveller visiting England finds his average expenses for 56 days to be 10s. 6d. a day; for the first 42 days they averaged 10s. a day. What was the average for the last 14 days?

Total expense for 56 days is 588s. and for 42 days it is 420s. Expense for 14 days is (588−420)s., or 168s. ANS. 12s.

4. The average number of runs made by 11 men at cricket was 12. The 10 of these who got out made respectively 22, 3, 7, 4, 18, 0, 21, 0, 0, 20. Find the score of the 11th man. ANS. 37 runs.

5. A farmer pays a rent of \$5 an acre for 150 acres, \$6.50 per acre for 120 acres, and \$8.30 per acre for 90 acres. What is the average rent per acre? ANS. \$6.32½.

6. Three schools educate 75, 33 and 31 children at an average cost annually of £2 10s., £3 10s. and £3 15s., respectively; they are united and an annual saving is made of £141 5s. Find the average cost after union. ANS. £2.

7. Divide \$3608 among three boys in the proportion of 1, 4, 6. ANS. \$328, \$1312, \$1968.

8. Divide an estate of 3374 acres among A, B and C: giving B 4 times A's share and C 9 times A's. ANS. A, 241 ac; B, 964 ac; C, 2169 ac.

9. An examiner wishes to mark 3 questions in the proportion of 5, 8 and 12. The sum of the marks is 150. How must he distribute them? ANS. 30, 48 and 72.

10. A bag contains a certain number of sovereigns, three times as many shillings and four times as many pence. The whole sum amounts to £560. How many coins are there of each kind? ANS. 480 sov., 1440s. and 1920d.

11. A, B and C have \$200, B has twice as much as A, and the difference between the shares of A and C is \$3.92, C's being the greater. Find the share of each. ANS. A, \$49.02; B, \$98.04; C, \$52.94.

12. A sum of money is divided among A, B and C. C gets twice as much as A; A and B get \$200; B and C get \$240. How much does each get? ANS. A, \$40; B, \$160; C, \$80.

13. Divide \$81 among 5 persons giving two of them each \$2.50 less than each of the others.

If the two got the same as the others the sum distributed would be (2×\$2.50)+\$81=\$86.

\$86÷5=\$17.20 for each of three;
\$(17.20−2.50)=\$14.70 " " " two.

14. Divide 2583 ac. among A, B and C, giving B 228 ac. more than A and C 438 ac. more than B. ANS. A, 563 ac.; B, 791 ac.; C, 1229 ac.

15. A father divides his property, which amounts to $56000, among his 4 boys in the proportion of the numbers 1, 2, 3, 4. Find the share of each. ANS. $5600; $11200; $16800; $22400.

16. Three brothers divide an estate of $16000 in proportion to 3, 4, 5. The expenses of settlement are $120, which are paid first from the estate. Find the amount received by the first. ANS. $3970.

17. $26000 is divided among 3 brothers. The eldest got half as much again as the second and twice as much as the youngest. Find the share of each.

The youngest receives $3, the second $4, and the eldest $6 out of every $13. ANS. $12000, $8000, $6000.

IV. APPLICATIONS OF SIMPLE AND COMPOUND NUMBERS.

1. Make out a bill for 5 days' work at 60c. a day, 4 days' work at $1.25 a day, and 4 bu. potatoes at 40c. a bu., crediting the person against whom you make the bill with 2 days' work of 8 hr. each at 25c. an hr. ANS. Balance $5.60.

2. Make out a bill for the following and receipt it:—27 yd. of linen at 1s. 6d. a yd.; 35 yd. of silk at 2s. 11d. a yd.; 30 pairs of gloves at 2s. 6d. a pair; 48 yd. of velvet at 4s. 11d. a yd.; 4 doz. collars at 6s. 3d. a dozen, and 2 doz. neckties at 12s. 6d. a dozen. ANS. £25 3s. 7d.

3. Find the amount of the following bill:—13 yd. silk at $1.85 a yd.; 27 yd. flannel at 75c. a yd.; 45 yd. muslin at 12c. a yd., and 65 yd. of calico at 7c. a yd. ANS. $54.25.

4. Make out the following bill:—15 quires of paper at $1.40 a ream; 25 quires at $4.20 a ream; 2500 envelopes at 70c. per M; 750 envelopes at 18c. per C, and 120 Christmas cards at 15c. a dozen. ANS. $10.90.

5. What will it cost to carpet a room 48 ft. long and 33 ft. 9 in. wide with carpet 27 inches wide at 4s. 6d. a yd.? ANS. £54.

6. The carpeting of an assembly room cost $420 at $1.75 per sq. yd. If the room was 36 ft. wide, find its length.

No. sq. yd. $=\$420 \div \$1.75 = 240$;

$\therefore$ length of room $= (2160 \div 36)$ ft. $= 60$ ft.

7. A parlor is 27 ft. long 18 ft. wide; what will it cost for rugs 4 ft. 6 in. long and 2 ft. 3 in. wide to cover it, if each costs $7.50?

No. of rugs is 48; cost, $360.

8. Find the number of sq. ft. of plastering in a room 12 ft. 6 in. long 10 ft. 6 in. wide and 10 ft. high. ANS. 460 sq. ft.

9. What will it cost to plaster the walls of a room 40 ft. long 22 ft. wide and 10 ft. high at $4.25 per 100 sq. ft. if an allowance of 140 sq. ft. is made for openings?

Perimeter, 124 ft.; height, 10 ft.; area of walls, 1240 sq. ft.; surface to be plastered, 1100 sq. ft.; cost, $46.75.

10. A room 11 ft. high takes 143 yd. of paper 2 ft. wide for the four walls; how many feet of moulding will be required to go around the room? ANS. 78 ft.

11. A room is 12 ft. 6 in. long 10 ft. 6 in. wide and 10 ft. high. It has one door 7 ft. by 4 ft., one window 6 ft. by $3\frac{1}{2}$ ft. How much paper 2 ft. 6 in. wide would be required to paper the walls?

Area of walls is 460 sq. ft.; area of openings 49 sq. ft.; number of sq. ft. of paper is 411 sq. ft.; length of paper $164\frac{2}{5}$ ft.

12. A board fence 9 ft. high is built around a racing park 84 rd. long and 76 rd. wide at $1.50 per 120 sq. ft. Find the cost.

Perimeter is 320 rd., or 5280 ft.; surface is (5280×9) sq. ft., or 47520 sq. ft.; cost of 47520 ft. at $1\frac{1}{4}$c. per foot., is $594.

13. A pasture is 48 rd. wide and contains 15 ac. What will it cost to fence it at 68c. per rd.? ANS. $133.28.

14. What will it cost for lumber to fence a lot 125 rd. long and 100 rd. wide, 5 boards high, the boards being 6 inches wide, at $9.50 per M? ANS. $$176.34\frac{3}{8}$.

15. How many scantling each 16 ft. long, 4 inches by 5 inches, will be required to make 40 c. yd.?

No. of scantling $=1080\div2\frac{2}{9}=486$.

16. 27 c. ft. 1296 c. in. of lumber are required to floor a room 13 ft. 6 in. by 12 ft. 4 in. What is the thickness of the lumber?

Cubical content of floor $=47952$ c. in.;

Area of " $=(162\times148)$ sq. in;

$\therefore$ thickness $=[47952\div(162\times148)]$ in. $=2$ in.

17. 110 c. yd. of gravel are put on a road 40 rd. long 9 ft. wide and of uniform depth. How deep is it? ANS. 6 inches.

18. A rectangular shaft of marble contains 18 c. yd. 4 c. ft. and it is 3 ft. 6 in. square; find its height.

Cubical content of shaft, 490 c. ft.; area of end, 12 sq. ft. 432 sq. in.; length of shaft, 40 ft.

19. The ceiling of a room 27 ft. long 24 ft. wide and 12 ft. high is to be raised to increase the space 48 c. yd. How many feet must it be raised?

Cubical content of room=7776 c. ft,; 48 c. yd.=1296 c. ft., or $\frac{1}{6}$ of 7776. ∴ height must be increased $\frac{1}{6}$ of 12 ft., or 2 ft.

20. 116 c. yd. are taken from a cellar in 108 loads; how many c. ft. are there in each load? ANS. 29 c. ft.

21. Two school rooms have equal cubical content. One has a floor area of 360 sq. ft. and is 12 ft. high. The other is 15 ft. high; find its floor area.

Cubical content is 4320 c. ft.; area of floor is (4320÷15) sq. ft. ANS. 288 sq. ft.

22. A log 24 ft. long and 2 ft. square is cut into scantlings 4 inches square and 8 ft. long. What will they be worth at 22c. each. No allowance for waste.

Cubical content of log is 96 c. ft.; cubical content of scantling is 1536 c. in.; number of scantling is 108; cost of 108 scantling a 22c. each is $23.76.

23. A farmer's granary 13 ft. long 12 ft. wide is filled with wheat to the depth of 6 ft. Find its value at 96c. a bu.

25 qt.=1 c. ft.; 1 qt. is worth 3c.; 1 c. ft. is worth 75c. ANS. $702.

24. What length must be cut off a beam 9 in. broad and 15 in. deep to contain 2 c. ft. 864 c. in.? ANS. 2 ft. 8 in.

25. A stick of timber 27 ft. long and 12 in. square weighs 14 cwt. 112 lb. What will be the length of another stick which weighs 1 t. 520 lb. and which is 9 in. wide and 8 in. thick?

1 c. ft. weighs 56 lb.; the second stick weighs 2520 lb. It contains 45 c. ft. and its length is 90 ft.

26. A cubic foot of water weighs 1000 oz.; find the weight of rain-fall on one acre 1 in. deep. ANS. 113 t. 875 lb.

27. A street 1000 ft. long and 40 ft. wide costs $1728 for paving. What will it cost to pave one 1320 ft. long and 66 ft. wide?

Cost of 40000 ft.=$1728;

∴ " " 87120 ft.$=\$\left(\frac{87120\times 1728}{40000}\right)=\$3763.58\frac{2}{5}$.

28. A reservoir is 49 ft. 4 in. long 25 ft. 6 in. wide. How many gallons of water will be required to fill it to the depth of 18 inches? Ans. $11793\frac{3}{4}$ gal.

29. A reservoir is 280 ft. long 192 ft. wide and 48 ft. deep. How long will it take to fill it at 288 gal. per minute? Ans. 38 da. 21 hr. 20 min.

30. A courtyard 46 yd. sq. is to have two pavements of brick across it through the centre, one at right angles to the other. How many bricks 8 in. long $4\frac{1}{2}$ in. wide will be required if the pavement is 3 ft. wide?

Pavement contains 91 sq. yd., or $(91\times9\times144)$ sq. in.; 1 brick $=36$ sq. in. of surface; $\therefore$ number required is 3276.

31. How long will it take to ride around a field 233 yd. wide and containing 44 ac. 4196 sq. yd. at 5 miles 520 yd. an hour?

The area is 217156 sq. yd.: width is 233 yd.; $\therefore$ length is 932 yd.; perimeter is 2330 yd.; one rides 9320 yd. in 60 min.; hence, he rides 2330 yd. in 15 min.

32. Two rectangular plots of ground are offered for sale; one 80 yd. long and 56 yd. wide for \$1120, and the other 96 yd. long and 60 yd. wide for \$1800. Which is the better purchase and by how much per sq. yd.? Ans. The latter; $6\frac{1}{4}$c.

33. A man has a rectangular block of wood 20 ft. long 6 ft. wide and 3 ft. thick; how many blocks 8 in. long 4 in wide and 2 in. thick can be cut out of it? No allowance for waste.

Cubical content of the block $=(240\times72\times36)$ c. in.
" " " one small block$=(8\times4\times2)$ c. in.;
$\therefore$ number of blocks$=9720$ Ans.

34. A pond 121 ft. by 180 ft. is covered with ice 2 in. thick. Find the weight in tons of the ice if 1 c. ft. weighs 56 lb. Ans. 101 t. 1280 lb.

35. A rectangular cistern is 12 ft. long and 8 ft. wide inside measurements: and the water is 7 ft. deep. How many tons of water are there in the cistern. Ans. 21 tons.

36. A street 20 rods long and 60 ft. wide is to be paved with brick laid flat-wise, each 8 in. by $4\frac{1}{2}$ in. Find the cost of the brick at \$8.50 per M.

Surface to be covered$=(20\times16\frac{1}{2}\times12\times60\times12)$ sq. in.;
Surface of 1 brick$=(8\times4\frac{1}{2})$ sq. in.;
Number of bricks$=79200$;
Cost$=\$673.20$.

37. A man has a field 112 rd. long, and 63 rd. wide. How many more rods of fencing will be required to enclose it than a field of the same area and 84 rd. wide?

$$\text{Area of field} = (112 \times 63) \text{ sq. rd.};$$
$$\therefore \text{ length in rods} = [(112 \times 63) \div 84] \text{ rd.} = 84 \text{ rd.};$$
$$\text{Perimeter of first} = 2 \times (112 + 63) \text{ rd.} \quad = 350 \text{ rd.};$$
$$\text{" second} = 2 \times (84 + 84) \text{ rd.} \quad = 336 \text{ rd.}$$
$$\text{Difference} = 14 \text{ rd.}$$

38. A room is 69 ft. 4 in. long and if it were 15 ft. wider the area would be 3484 sq. ft.; find its width.

$$\text{Width of supposed room} = \frac{3484 \times 144}{832} \text{ in.} = 603 \text{ in.}$$
$$\text{Width of room} = (603 - 180) \text{ in.} = 35 \text{ ft. } 3 \text{ in.} \quad \text{Ans.}$$

39. How many c. ft. of lead are there in a leaden cistern 15 ft. long 10 ft. wide and 8 ft. 6 in. deep, the lead being 3 in. thick?

Cubical content with outside measurement = 1275 c. ft.;
" " " inside " = 1136 c. ft. 756 c. in.
∴ number of c. ft. of lead = 138 c. ft. 972 c. in.

40. If a c. ft. of marble weighs 2.716 times as much as an equal volume of water, find the weight of a block of marble 9 ft. long 3 ft. wide and 4 ft. thick. Ans. 9 t. 333 lb.

41. If 6 c. in. of iron weigh as much as 45 c. in. of water, find the weight of 26 c. ft. of iron. Ans. 6 t. 187½ lb.

42. How many lb. Troy are there in 8640 lb. Avoirdupois? Ans. 10500 lb. Troy?

43. A man mixes oats and peas in the proportion of 11 bu. of oats to 4 bu. of peas and the mixture is worth 55c. per bu. If the oats are worth 35c. per bu., what are the peas worth per bu.? Ans. \$1.10 per bu.

44. The floor of a room which is 15 ft. high is a square and costs \$54 for carpeting it with carpet 2 ft. wide and worth \$1.00 a yd. What will it cost to paint the walls at 30c. a sq. yd.?

Area of floor is 324 sq. ft.; length, 18 ft.; area of walls, $[2 \times (18+18) \times 15]$ sq. ft., or 120 sq. yd. Ans. \$36.

45. How many miles are there in 2640 knots? Ans. 3043 mi.

46. What is the value of a block of marble 5 ft. 3 in. long 2 ft. 4 in. wide and 1 ft. 2 in. thick @ \$10.56 per c. ft.? Ans. \$150.92.

47. A carload of wheat weighing 14 t. 6 cwt. was sold for \$429. How much is that per bu.? Ans. 90c.

48. How many days are there from June 1st to October 6th inclusive? ANS. 128 days.

49. If there are 2 reams in a bundle and 5 bundles in a bale and 73 reams are sold out of twenty bales, what is the remainder worth at 10c. a quire? ANS. $254.00.

50. A tub of butter weighed 40 lb. 12 oz.; the tub alone weighed 4 lb. 3 oz. What is the butter worth at 32c. a lb.? ANS. $11.70.

51. I buy pass books at $5.04 a gross and sell them at 60c. a dozen. What is my gain on 120 books? ANS. $1.80.

52. A hound running 64 rd. a min. is after a fox running 56 rd. a min. The fox has a start of 240 rd. In what time will it be caught? ANS. 30 min.

53. If steel rails weigh 25 lb. to the foot in length, what weight in tons will be required for 1 mi. of railroad? ANS. 66 t.

54. A company melted 588 lb. 6 oz. 10 dwt. of silver into 125 bars of equal weight. Find the weight of each bar. ANS. 4 lb. 8 oz. 10 dwt.

55. A barrel of beef weighs 200 lb. Find the value of the beef required for an army of 2000 men for 15 da. at $18 per bbl., if each man receives 1 lb. 4 oz. daily. ANS. $3375.

56. If 15 flags of the same size and quality cost $324 at 15c. a sq. ft., find the length of a flag if the width is 8 ft.

1 flag costs $21.60; number of sq. ft. in 1 flag 144; length of flag $=\frac{144}{8}$ ft. $=18$ ft.

57. A farmer sold 58 bu. barley and 87 bu. oats for $84.68 receiving 26c. per bu. more for barley than for oats. Find the price of each per bu.

If all were barley the cost $=\$84.68+(87\times26)$c. $=\$107.30$;
∴ price of 1 bu. barley $=\$107.30\div(58+87)=74$c.
" " 1 bu. oats $=(74-26)$c. $=48$c.

58. Multiply 18 mi. 120 rd. 4 yd. 2 ft. 7 in. by 59. ANS. 1084 mi. 92 rd. 2 ft. 5 in.

59. The fore wheel of a carriage is 8 ft. in circumference and makes 810 revolutions more than the hind one in going 4 mi. 160 rd. Find the circumference of the hind wheel.

4 mi. 160 rd. $=23760$ ft.;
Number of revolutions by fore wheel $=\frac{23760}{8}=2970$;
" " " " hind " $=(2970-810)=2160$;
Circumference of " " $=23760$ ft. $\div2160=11$ ft.

60. If a number divided by 247 leaves a remainder of 73, what will be the remainder when the same number is divided by 13?

$$\text{Number}=247\times\text{Quotient}+73;$$
$$\therefore\ \frac{\text{Number}}{13}=\tfrac{247}{13}\times\text{Quotient}+\tfrac{73}{13}$$
$$=19\times\text{Quotient}+5\tfrac{8}{13}.$$ Remainder is 8.

61. From a cask containing 252 gal., 36 gal. were taken out at one time, 93 gal. at another and 17 qt. at another. How many gal. and qt. are left? ANS. 118 gal. 3 qt.

62. A milk man had 12 cans of milk, 4 of 8 gal. each; 5 of 3 gal. 2 qt. each and the rest 6 gal. 1 qt. each. What is the milk worth at 5c. a qt.? ANS. $13.65.

63. A and B have equal shares in a field of wheat which yields 284 bu. A takes 97 bu. and B the rest, paying A $48.15. What is the value of the field of wheat? ANS. $151.94.

64. By what number must 66 mi. 272 rd. be divided to give for quotient 2 mi. 123 rd. 5 yd. 1 ft. 6 in.? ANS. 28.

65. A party of 25 persons dined together; each man paid 50c. and each woman 43c. The bill was $11.87. How many women were in the party? ANS. 9 women.

66. A man mixes 56 gal. of spirits at $5 a gal. with 17 gal. at $1.60 per qt., and then adds 8 gal. of water. He wishes to gain $81 on the transaction; at what rate per gallon must he sell it? ANS. $5.80.

67. If 3 turkeys are worth 4 geese, and 6 geese are worth 20 chickens, how many chickens can be had for 9 turkeys? ANS. 40 chickens.

68. A dealer sold 26 M shingles at $3 per M; 75 doors at $2 each; 20 windows at $1.50 each and received in payment 35 tons of coal at $5.25 per ton and 8 cords of wood at $4.50 per cord. How much cash must he receive to balance the account. ANS. $38.25.

69. A man buys 30 lb. coffee at 24c. a lb. and 50 lb. at 32c. a lb., and after mixing them sells 40 lb. at 28c. a lb. At what price must he sell the remainder to save him from loss? ANS. 30c.

70. A man drew 215 loads of gravel at 75c. a load. He had a boy helping him to whom he paid 37c. a load for 114 loads. If it took him 63 days altogether, find his average daily wages. ANS. $1.89.

71. A man sold 100 animals, turkeys and geese, for \$104, the turkeys at \$1.25 and geese at 75c. each. How many geese did he sell. See question 57, page 252. ANS. 42 geese.

72. Silver is $10\frac{1}{2}$ times as heavy as water, find the weight in tons of 28 c. ft. of silver. ANS. 9 tons 375 lb.

73. How many gallons of water will a cistern hold whose outside measurements are 8 ft. 4 in. long 4 ft. 7 in. wide and 2 ft. 8 in. deep (no lid). The plank of which the cistern is made is 2 in. thick.

No. gallons $=(8\times 4\frac{1}{4}\times 2\frac{1}{2}\times 6\frac{1}{4})$ gal. $=531\frac{1}{4}$ gal.

74. Twenty-seven men earned a certain sum in 13 days. Had they received \$38.61 more than they did, each would have earned \$1.31 per day. Find the daily wages of each.

Hint \$38.61 for all = 11c. for each per day ;

$\therefore$ daily wages $=\$(1.31-.11)=\$1.20$.

75. A man bought 120 bbl. of apples for \$393. Some at \$3 and some at \$3.50 per bbl. How many at each price did he buy. ANS. 54 at \$3; 66 at \$3.50.

76. A man buys 5000 bu. of wheat at 60c. per bu. He sells half of it at 68c. per bu. At what rate per bu. must he sell the remainder to gain 12c. a bu. on the whole quantity?

His total gain $=5000\times 12$c. $=\$600$;
His gain on half $=2500\times 8$c. $=\$200$;
Gain on 2nd " = \$400;
Gain on 2nd half per bu. $=\$400\div 2500=16$c.;
Selling price $=76$c.

77. A stick of timber is 24 ft. long 14 inches wide and 16 in. thick. If a cubic yard is cut off the end, find the length of the remainder.

Area of end $=\frac{14}{9}$ sq. ft.;
$\therefore$ length to contain 1 c. yd. $=(27\div\frac{14}{9})$ ft. $=17\frac{5}{14}$ ft.;
Hence, $(24-17\frac{5}{14})$ ft. $=6\frac{9}{14}$ ft. = remainder.

78. I bought apples at 5 for 7c. and sold them at 7 for 10c. and gained \$3.25. How many did I sell?

49c. is paid for 35 apples; 50c. is received for 35 apples; 1c. is gained on 35 apples; 325c. is gained on 11375 apples.

79. A man is to receive \$425 and a watch for 12 months' work. At the same rate he receives \$204 and 2 watches for 7 months' work. Find the value of the watch.

In 24 months he earns \$850 and 2 watches; in 7 months he earns \$204 and 2 watches; in 17 months he earns \$646 and in 12 months he earns \$456; value of watch is $\$(456-425)=\$31$.

80. How many palings each 3 ft. 6 in. long and 2 in. wide can be made from 20 boards, each 14 ft. long and 1 ft. wide, no deduction being made for waste in cutting? Ans. 480.

81. A man bought 24 horses for a certain sum, had he got 11 more at the same rate he would have paid out \$5145 altogether. What did he pay for 24 horses?

Cost of 35 horses=\$5145;
" " 1 " =\$147;
" " 24 " =24×\$147=\$3528.

82. A man has a field 96 rd. long and 48 rd. wide, how many rods of fencing will be required for a square field of half the area?

A field half the length and same width would give half the area; ∴ the side of square field is 48 rd. long and the perimeter is 192 rd.

83. What is the difference between the smallest number of 4 figures exactly divisible by 34, and the largest number of 4 figures exactly divisible by 43?

Smallest No. is 1020; the largest No. is 9976. Ans. 8956.

84. An equal number of men and boys earned \$532.50 in 6 weeks. Each man earned \$7.50 in 2 weeks. Each boy earned \$6.50 in 3 weeks. How many boys were employed?

1 man earns \$22.50 in 6 weeks;
1 boy earns \$13.00 in 6 weeks;
1 man and 1 boy earn \$35.50 in 6 weeks;
No. of each employed$=\frac{532.50}{35.50}=15$. Ans. 15 boys.

85. A pile of wood was 96 ft. long 8 ft. high 4 ft. wide and 5 cords 80 c. ft. have been sold. What is the length of the remainder of the pile?

A pile 4 ft. long contains a cord; ∴ length to contain 5 cords 80 c. ft. is $22\frac{1}{2}$ ft.; 96 ft.$-22\frac{1}{2}$ ft.$=73\frac{1}{2}$ ft.

86. A carpenter earns \$2.20 a day. His expenses are \$7.20 a week. How long will it take to save \$312?

In a week he saves \$(13.20−7.20). Ans. 52 weeks.

87. How many feet board measure are there in 20 scantling 24 ft. long 5 in. wide and 7 in. thick. Ans. 1400 ft.

88. A farmer sold 54 bu. of wheat and 28 bu. of oats for \$54.40. The oats were worth 40c. a bu. less than the wheat; find the value of 2 bu. of wheat and 3 bu. of oats.

See example 57, page 252. Ans. \$2.80.

89. A man bought 248 head of cattle. He sold half of them for \$4960 and gained \$5 each. Find the whole gain if he sells the remainder at \$12 each above cost.

Gain is $\$[124\times(12+5)]$, or \$2108.

90. Two men divided equal sums of money. One gave his to 19 men and the other to a number of women. Each man received \$4.75 and each woman \$1.14 less. How many women were there? ANS. 25 women.

91. A number is divided by the factors 3, 11, 5; the first remainder is 2, the second is 7 and the third is 4; if the quotient is 285 find the number. ANS. 47180.

92. A man sells 60 chickens at a profit of 30c. each; what additional number must he sell at a profit of 40c. each to realize an average profit of 37c. on each?

The additional profit on 60 at 7c. each is \$4.20; hence, number required is 420c.÷3c., or 140.

93. A certain sum of money was divided among A, B and C. A and B got \$37.80, A and C \$35.35, B and C \$31.25. What did A get less than B and C together? ANS. \$10.30.

V. FACTORS, MEASURES AND MULTIPLES.

1. Find the prime factors of 889056 and 213962 and multiply the common factors in each together.

Prime factors of 1st$=2^5\times3^4\times7^3$;
" " " 2nd=2, 7, 17, 29, 31; 14.

2. Simplify $\frac{385\times468\times95\times87\times171}{55\times78\times19\times57\times29}$. ANS. 1890.

3. What number multiplied by 144×37 will give the product of $864\times96\times74$? ANS. 1152.

4. Affix a figure to the right of each of the following so that the resulting number may be exactly divisible by 9:—473; 295; 66719; 8712. ANS. 4734; 2952; 667197; 87120 or 87129.

5. Affix a figure to the right of each of the following so as to make the resulting number exactly divisible by 11:—7428; 1739; 7535; 2473. ANS. 74283, 17391, 75350, 24739.

6. Determine which of the following numbers are prime:—103, 2197, 619, 4393, 7111, 997, 499, 1793 and 581. ANS. 103, 619, 997, 499.

7. What is the largest prime factor common to 1561 and 1477. ANS. 7.

8. From a heap of shot weighing 54345 grains, a heap weighing 12321 grains is taken, show that a single shot cannot weigh more than 3 grains.

$12321=3\times3\times37\times37$;

$54345=3\times5\times3623$. Hence, the heaviest weight is 3 gr.

9. A coal dealer has 5 bins holding 102 bu., 114 bu., 255 bu., 285 bu. and 323 bu. respectively. What is the smallest number of bushels that can be exactly measured in each?

L.C.M. of the numbers=9690 bu.

10. A railway Co. built three switches 6699 ft., 8671 ft. and 9367 ft. long. Find the length of the longest rails that could be used without cutting them. ANS. 29 ft.

11. What is the least sum of money with which I can buy horses at \$75; mules at \$50; cows at \$30, or sheep at \$12, and spend all my money? ANS. \$300.

12. A man has three farms of 56 ac., 72 ac. and 88 ac., and wishes to divide them into fields of the largest possible equal size. How many fields will he have?

Size of field, 8 ac. ANS. 27 fields.

13. Find the largest number that will exactly divide 11496, 20597 and 24429 after 81 is added to it.

H.C.F. of the Nos.=479; $\therefore$ No.$=(479-81)=398$.

14. Find the largest number from which if 97 be taken the remainder will exactly divide 3696, 5808 and 8448.

H.C.F.=528; $\therefore$ No.$=528+97=625$.

15. Find the largest number that will divide 2235, 3231 and 4615, leaving as remainders 68, 79 and 84 respectively. ANS. 197.

16. The L.C.M. of 12, 15, 18, 24 and another number prime to them is 16920. Find the number.

L.C.M. of 12, 15, 18, 24=360; $16920\div360=47$.

17. Find the L.C.M. of all the numbers that exactly contain 16 between 48 and 128 inclusive. ANS. 13440.

18. Find all the integral divisors of 168. ANS. 1, 2, 3, 4, 6, 7, 8, 12, 14, 21, 24, 28, 42, 56, 84, 168. See page 147.

19. Find the G.C.M. of 192 per. and 223 per. 1 yd. 2 ft. 6 in. ANS. 11 ft.

20. Find the smallest sum of money that can be divided into sums of 54c. 63c. or 99c. ANS. $41.58.

21. Find the two lowest numbers that can be divided by either 102 or 136 without a remainder. ANS. 408 and 816.

22. Find the greatest length of which 48 per. 1 ft., and 55 per. 5 yd. 6 in. are multiples. ANS. 13 ft.

23. A company bought 3 blocks of land containing 1029 ac.; 1176 ac.; 1372 ac. respectively. They divided them into farms of the largest possible equal size. What did they get for all at $1200 for each farm? ANS. $87600.

24. Find the least number which, divided by any integral number between 6 and 13, will leave a remainder of 2. ANS. 27722.

25. Three bells strike at intervals of 6, 7 and 8 min. They strike together at 8 p.m. At what time will they strike together again. ANS. 48 min. past 10 p.m.

26. Find all the numbers that are exactly contained in 2280 and 2808.

H.C.F.=24: the measures of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

27. A street 10 miles long has telegraph poles 150 ft. and telephone poles every 120 ft. How many telegraph and telephone poles will come together?

Total distance 52800 ft.; L.C.M. of 150 ft. and 120 ft.=600 ft.; $\therefore \frac{52800}{600}$ ft. =88; hence, 88+1 (starting point)=89 poles.

28. Find the L.C.M. of all the composite numbers from 40 to 50 inclusive. ANS. 44629200.

29. The H.C.F. of two numbers is 395; L.C.M. is 20145; one number is 6715, find the other.

The factors of the second number not found in the first must be 20145÷6715, or 3. $\therefore$ 395×3=1185=the other number.

30. Find the two smallest numbers whose H.C.F. is 79 and whose difference is 237.

Since 79 is H.C.F. and 237÷79=3; hence, one number is 4 times 79. ANS. 79 and 316.

31. The L.C.M. of two numbers is 28035; the only factor of the first not in the second is 7, and the only factor in the second not in the first is 5. Find the H.C.F.

The H.C.F. is the product of all the common prime factors; $\therefore$ 28035÷(5×7)=801=H.C.F.

32. The H.C.F. of two numbers is 316 and the L.C.M. 3476. Find the numbers.

$3476 \div 316 = 11$ = product of the factors not common to the numbers; numbers are 1×316 and 11×316, or 316 and 3476.

33. The product of two numbers is 2304 and the L.C.M. is 144. Find the H.C.F.

The product of the two numbers is the product of the L.C.M. and the H.C.F.; $2304 \div 144 = 16$. ANS. 16 and 144.

34. Find the three lowest common multiples of 24 and 36.

Their L.C.M. = 72; ∴ the three lowest common multiples are 72×1, or 72; 72×2, or 144; 72×3, or 216.

35. The factors 7, 3, 5 are common to two numbers, the other factors in one are 3, 2, 2, and in the other 5, 7, 11. Find the H.C.F. and L.C.M. of the numbers.

H.C.F. $= 7 \times 3 \times 5 = 105$; L.C.M. = H.C.F. $\times 3 \times 2 \times 2 \times 5 \times 7 \times 11 = 105 \times 12 \times 385 = 485100$.

36. The H.C.F. of two numbers is 30; the factors in the first not in the second are 7 and 11, and those in the second not in the first are 3 and 13. Find L.C.M.

$$30 \times 7 \times 11 \times 3 \times 13 = 90090.$$

37. What is the smallest number by which 732 must be multiplied to produce a number which shall be exactly divisible by H.C.F. of 1980 and 2628.

H.C.F. is 36 and $732 \div 36$ leaves a remainder of 12; $36 \div 12 = 3$, that is, 732 must be multiplied by 3 to make it exactly divisible by 36. ANS. 3.

38. On a rock are three revolving lights; one revolves in 132 seconds; another in 187 seconds, and the third in 204 seconds. All were seen at midnight, when next will they be seen? ANS. in 37 min. 24 sec.

VI. VULGAR FRACTIONS.

1. A man spent $\$75\frac{3}{10}$ and then had left $\$69\frac{3}{4}$ more than he spent. How much had he at first? ANS. $\$220\frac{7}{20}$.

2. By selling a quantity of wheat for $\$256\frac{16}{25}$ I lost $\$22\frac{4}{5}$. For how much should it have been sold to gain $\$17\frac{1}{2}$? ANS. $\$296\frac{47}{50}$.

3. A farmer exchanged $155\frac{5}{8}$ ac. of land for 135 horses valued at \$83 each. What is the land worth per acre? ANS. \$72.

4. A farmer raised $31\frac{1}{2}$ t. of hay on $16\frac{4}{5}$ ac. How many acres should yield $63\frac{3}{4}$ t.? ANS. 34 ac.

5. Reduce 2 da. 11 hr. 44 min. to the fraction of a week. ANS. $\frac{16}{45}$.

6. A man owning $\frac{7}{8}$ of a boat sold $\frac{3}{4}$ of his share for \$1785. Find the value of the boat and the value of the part he had left. ANS. \$2720; \$595.

7. A drover sold 504 pigs at $\$7\frac{3}{5}$ each and invested $\frac{9}{16}$ of the proceeds in sheep at $\$11\frac{4}{10}$ each. How many sheep did he buy? ANS. 189.

8. The fencing of a field whose area is $2032\frac{3}{4}$ sq. rd. cost $\$149\frac{9}{25}$. If the field is in the form of a rectangle $34\frac{3}{5}$ rd. wide, find the cost per rd.

Length is $58\frac{3}{4}$ rd.; perimeter is $186\frac{7}{10}$ rd.; cost per rd.$=$ $\$(149\frac{9}{25}\div186\frac{7}{10})=\$\frac{4}{5}$.

9. Two men, A and B, start to walk towards each other from London and Montreal, respectively, at the same time, one going $9\frac{7}{10}$ mi. and the other $8\frac{3}{4}$ mi. per hr. If they meet in $24\frac{8}{9}$ hr., how far is London from Montreal? ANS. $459\frac{1}{5}$ mi.

10. What part of 3 cwt. 25 lb. 2 oz. is 1 cwt. 80 lb. 10 oz? ANS. $\frac{5}{9}$.

11. A merchant paid $\$247\frac{1}{2}$ for apples at $\$2\frac{16}{25}$ a bbl. and sold $\frac{8}{15}$ of them at a profit of $\$\frac{9}{25}$ per bbl. How much did he receive for what he sold? ANS. \$150.

12. If one edge of a cubical block of marble is 4 ft. 6 in. long, what will it cost to polish 4 of its faces at $\$1\frac{7}{25}$ per sq. ft.? ANS. $\$103\frac{17}{25}$.

13. A man divided $\frac{5}{8}$ of his farm between his two sons giving to the younger $\frac{7}{9}$ as much as to the elder. If the elder received 25 acres more than the younger, how many acres are there in the farm?

In every 16 ac. divided the difference is 2 ac.; hence, 200 ac. were divided, or $\frac{5}{8}$ farm. ANS. 320 ac.

14. A travels $562\frac{1}{2}$ mi. in $16\frac{7}{8}$ da.; B $618\frac{3}{4}$ mi. in $19\frac{4}{5}$ da. How far will they be apart in 12 da., if they travel in the same direction, starting from the same place at the same time.

Each day A goes $562\frac{1}{2}$ mi.$\div16\frac{7}{8}$, or $33\frac{1}{3}$ mi.; each day B goes $618\frac{3}{4}$ mi.$\div19\frac{4}{5}$, or $31\frac{1}{4}$ mi.; difference in 1 day is $2\frac{1}{12}$ mi.; in 12 da. it is 25 mi.

15. Divide the difference between $\dfrac{2\frac{5}{6}}{3\frac{7}{9}}$ and $\dfrac{3\frac{3}{5}}{12}$ by the product. ANS. 2.

15. I have \$2354. If I expend $\frac{5}{8}$ of it for wheat at $87\frac{1}{2}$c. a bu.; $\frac{3}{10}$ for barley at $56\frac{1}{4}$c. a bu. and the rest in corn at $31\frac{1}{4}$c. per bu. How many bushels of each do I receive?

Wheat, $1681\frac{3}{7}$ bu.; barley, $1255\frac{7}{15}$ bu.; corn, $564\frac{24}{25}$ bu.

16. Write the following fractions in the order of magnitude: $\frac{12}{2847}$, $\frac{9}{2812}$, $\frac{16}{2881}$, $\frac{18}{2911}$.

Reduce to common Numerator, then the fraction with smallest denominator will be the greatest, etc. The order is $\frac{18}{2911}$, $\frac{16}{2881}$, $\frac{12}{2847}$, $\frac{9}{2812}$.

17. Find the least number which added to the sum of $\frac{3}{4}$, $\frac{7}{8}$ and $\frac{17}{12}$, will make the result a whole number. ANS. $\frac{23}{24}$.

18. Reduce $\frac{7}{35200}$ ac. to the fraction of a sq. ft. ANS. $\frac{693}{80}$ sq. ft.

19. Reduce $\frac{88}{125}$ c. ft. to the fraction of a cord. ANS. $\frac{11}{2000}$ cord.

20. What part is 15 lb. 7 oz. 12 dwt. of 27 lb. 4 oz. 6 dwt? ANS. $\frac{4}{7}$.

21. 13 ac. 80 sq. rd. 24 sq. yd. is the unit. Find the measure of 11 ac. 40 sq. rd. 20 sq. yd. ANS. $\frac{5}{6}$.

22. Simplify $\dfrac{\text{£1 2s. 11d.}}{\text{£1 0s. 10d.}}+\dfrac{\text{1 t. 17 cwt. 50 lb.}}{\text{2 t. 10 cwt.}}\div\dfrac{\text{1 oz. 15 dwt.}}{\text{4 oz. 7 dwt. 12 gr.}}$
ANS. $1\frac{9}{20}$.

23. If $(\frac{2}{3}+\frac{1}{21})$ of an estate is worth £6023 2s. 6d., find the value of $(4\frac{1}{2}\div11\frac{1}{4})$ of it. ANS. £3372 19s.

24. Find the value of $\frac{2}{3}$ of 10s. 6d.$+\frac{7}{8}$ of 27s.$-\frac{8}{11}$ of 3s. 8d. ANS. £1 8s. $7\frac{1}{2}$d.

25. Simplify $\dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}$ of $\dfrac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}+\frac{1}{5}}$ of $\dfrac{\frac{1}{6}-\frac{1}{7}}{\frac{1}{6}+\frac{1}{7}}$ of 2340. ANS. 4

26. Find the value of $\frac{2}{3}$ of 5s.$+\frac{3}{4}$ of $8\frac{1}{2}$s.$+1\frac{7}{17}$ of £$2\frac{1}{8}$; and reduce the result to the fraction of $\frac{1}{2}$ of £13 18s. 10d. ANS. $\frac{1}{2}$.

27. Find the G.C.M. of $19\frac{1}{8}$, $19\frac{5}{6}$, $21\frac{1}{4}$. ANS. $\frac{17}{24}$.

28. Find the L.C.M. of $2\frac{2}{3}$, $2\frac{1}{4}$, $2\frac{2}{5}$, $2\frac{4}{7}$, $2\frac{10}{13}$. ANS. 72.

29. Find the G.C.M. and L.C.M. of $2\frac{1}{4}$, $\frac{1}{3}$, $7\frac{1}{5}$, $\frac{3}{4}$, $\frac{1}{5}$. ANS. G.C.M.$=\frac{1}{60}$; L.C.M.$=36$.

30. Divide the L.C.M. of $1\frac{11}{24}$, $1\frac{2}{3}$, $1\frac{1}{6}$, $2\frac{7}{9}$ by the G.C.M. of $\frac{1}{2}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{5}{21}$. ANS. $\frac{175}{3}\div\frac{1}{210}$, or 12250.

31. Three girls step at the same moment, and then step at intervals of $\frac{3}{5}$, $\frac{3}{4}$ and $\frac{4}{5}$ seconds respectively. How long will it be before they again step together? ANS. 12 sec.

32. Find the smallest fraction which when divided by either $\frac{3}{14}$, $\frac{6}{35}$ or $\frac{4}{63}$ will give a whole number for quotient. ANS. $\frac{12}{7}$.

33. What is the largest sum of money that can be paid an exact number of times from either £$25\frac{21}{80}$, or £$186\frac{209}{240}$? ANS. 43d.

VII. DECIMALS.

1. At the rate of \$.015 each, how many pencils can I get for \$36.72? ANS. 2448 pencils.

2. A boy had earned \$129.75; he spent .87 of it for a bicycle. What had he left?

Sum left is $.13 \times \$129.75$, or \$$16.86\frac{3}{4}$.

3. A man travelled 420.75 mi. in 6 da. On each of the first 5 da. he went an equal distance and on the sixth he went 93.25 mi. How far did he go the first da.? ANS. 65.5 mi.

4. A man receives \$9.12 for 4.75 days' work of 8 hr. each. What does he get per hr.? ANS. 24c.

5. A man had 608 bu. of wheat; he sold .25 of it at \$.80 per bu. and .5 of the rest at \$.72 per bu. How much did he receive altogether and how many bu. had he left? ANS. \$285.76; 238 bu.

6. If .65 of a piece of cloth is worth \$57.20, what is the rest of it worth at .875 as much per yd. as the first lot? ANS. \$26.95.

7. A merchant paid \$$116\frac{2}{3}$ for 87.5 yd. of cloth. He sold .4 of it at a gain of \$$.16\frac{2}{3}$ per yd. and the rest at a gain of \$.125 per yd. Find the average selling price per yd. ANS. \$1.475.

8. Find the sum of .8 of a gal. + .8 qt. + .8 pt. + .8 gill. ANS. 1 gal. 1 pt.

NOTE.—4 gills = 1 pint.

9. A's money is .8 of B's and B's is .75 of C's. If C has \$120 more than A, how much has each? ANS. A, \$180; B, \$225; C, \$300.

10. The fore wheel of a buggy is 11.25 ft. round and the hind wheel 13.5 ft. How many more revolutions will one make than the other in going 2.25 mi.?

$$\frac{2.25 \times 5280}{11.25} - \frac{2.25 \times 5280}{13.5} = 176.$$

11. A grain merchant paid \$1600 for corn at \$.625 per bu. and sold .5625 of it for \$1260 and the rest of it for \$1050. Find the gain per bu. on each lot. ANS. \$.25 and \$.3125.

12. A lot is 36.8 rd. long and 27.5 rd. wide. It produces potatoes at the rate of 240 bu. per acre. Find the value of the crop at \$.65 per bu. ANS. \$986.70.

13. Simplify $\frac{.045}{.72} \times \frac{3.125}{87.5} \div \frac{28.125}{18.9}$. ANS. .0015.

14. The fencing of a field 42.15 rd. long in the form of a rectangle and containing 1277.145 sq. rd. cost \$217.35. Find the cost per rd. ANS. \$1.50.

15. A man owning .68 of a tract of land sold .45 of his share for \$3901.50. What is the value of the whole tract? ANS. \$12750.

16. My watch at 9.15 a.m. on June 30th, 1900, was 12.95 min. slow, but it gains .0025 hr. every 3.75 hr. How much too slow will it be in a week? ANS. 6.23 min.

17. Find the value of $2.\dot{3}\dot{5}$ of a mile. ANS. 2 mi. 2 fur. 33 per. 2 ft. 2 in.

18. Find the value of 14.1275 of an acre. ANS. 14 ac. 20 per. 12 yd. 129.6 in.

19. What decimal of a sovereign is .215 of £4.16. ANS. 1.032.

20. Reduce 3 pk. 1 gal. 2 qt. 1 pt. to the decimal of 8 bu. ANS. .119140625.

21. From $5.\dot{1}4285\dot{7}$ take 5.142857. ANS. $.00000\dot{1}4285\dot{7}$.

22. Find the decimal which will become 2 when multiplied by $2\frac{3}{5} \times \frac{2}{9}$. ANS. $3.\dot{4}6153\dot{8}$.

23. If 1 lb. of metal consisting of copper and zinc in the proportion of .84 to .16 be mixed with 2 lb. of the same metals mixed in the proportion of .75 to .25, find how much copper and zinc are in the mixture.

Copper in first mixture= .84 lb.;
" " second " =1.50 lb.;
Total copper=2.34 lb.=2 lb. 5.44 oz.

Similarly,
Zinc in first mixture=.16 lb.;
" " second " =.5 lb.;
Total zinc=.66 lb.=10.56 oz.

VIII. TRADE DISCOUNT.

1. The retail price of an invoice of groceries was \$1350, but the purchaser received a discount of $\frac{1}{5}$ and 12% off. What was the cost?

$$\text{Cost}=\tfrac{88}{100} \text{ of } \tfrac{4}{5} \text{ of } \$1350=\$950.40$$

2. I bought 36 kegs of nails, each containing 100 lb. at 6c. a lb., but received a discount of $12\frac{1}{2}\%$ and 10% off. What was the net price?

Net price $= \frac{9}{10}$ of $\frac{7}{8}$ of \$216 = \$170.10.

3. A merchant sold goods at 20%, 15% and 10% off, and the discounts amounted to \$485. Find the list price.

20%, 15% and 10% off = 38.8% off;
∴ 38.8% of list price = \$485;
List price = \$1250.

4. Sold goods at 25%, 20% and $16\frac{2}{3}\%$ off, and the discounts amounted to \$187.50. Find the list price. Ans. \$375.

5. The net price of an invoice of goods was \$1060.29, the purchaser having been allowed 16%, 15% and 10% off. What was the list price?

16%, 15% and 10% off = 35.74% off;
Hence, 64.26% of list price = \$1060.29. Ans. \$1650.

6. What is the difference between 25% off and 15% and 10% off? Ans. $1\frac{1}{2}\%$.

7. What is the difference between 15% and 10% off and 10% and 15% off. Ans. No difference.

8. I bought 300 yd. of cloth for cash, the list price being 80c. a yard, at 15% and 10% off and an additional 5% being allowed for cash. Find the cost of the cloth. Ans. \$174.42.

9. I bought apples at discounts of 20% and $12\frac{1}{2}\%$ off. The list price was \$1.25 per bbl. How many barrels did I buy if the discount amounted to \$112.50?

Single discount = 30%; 30% of cost = \$112.50; ∴ cost = \$375 00; No. of bbl. = 375 ÷ 1.25 = 300.

IX. PROFIT AND LOSS.

1. A merchant buys 400 yd. of silk for \$320; he sells 300 yd. at \$1.10 per yd. and the rest at 40c. per yd. Find the gain or loss %.

300 yd. at \$1.10 + 100 yd. at 40c. = \$370; \$370 − \$320 = \$50. Ans. gain $15\frac{5}{8}\%$.

2. If I buy two lots of tea, one at \$1.00 per lb. and the other at \$75 per cwt. How must I mix them to sell at 99c. a lb. and gain 10%?

Selling price without gain = 90c. a lb.; loss on 1st, 10c. a lb.; gain on 2nd 15c. a lb.; proportion in mixture 3 of 1st to 2 of 2nd.

3. I paid \$70 for apples at 40c. a bushel, part being damaged were lost, and the remainder sold at 50% profit for \$90. How many were lost?

No. of bushels bought=175;
∴ " " " sold $=\frac{9000}{60}=150$;
Loss=(175−150) bu.=25 bu.

4. My retail price for cloth is \$3.75 per yd., and my gain $33\frac{1}{3}$%. I sell to a customer and give him 20% discount from the retail price. Find my gain %.

$\frac{4}{3}$ of the cost is \$3.75; cost is \$2.8125; price to customer \$3; gain on \$2.8125 is \$.1875, or $6\frac{2}{3}$%.

5. If a wholesaler sells at 10% profit and a retailer at 50% profit. What proportion of the price paid by a customer is profit?

Wholesaler's selling price is $\frac{110}{100}$ of cost; retailer's selling price is $\frac{150}{100}$ of $\frac{110}{100}$ of cost; profit is $\frac{65}{100}$ of cost; profit is $\frac{65}{165}$ or $\frac{13}{33}$ of price paid.

6. A merchant selling goods at a certain price loses 10% but if he sold them for \$54 more he would have gained 8%. Find the cost of the goods.

Difference between the two prices=(10+8)% of cost=\$54;
∴ cost=\$300.

7. By selling sugar at \$4.50 per cwt. a merchant gains $12\frac{1}{2}$% on his outlay. How much greater would have been his gain % on his original outlay had he sold at 5c. a lb.

$\frac{112\frac{1}{2}}{100}$ of cost=\$4.50; ∴ cost=\$4.00; 5c. a lb.=\$5.00 per cwt.; gain on \$4 cost=\$1, or 25%; $(25-12\frac{1}{2})$%$=12\frac{1}{2}$%.

8. At what price must goods which cost \$16.80 be marked so that when sold at 25% discount the merchant may still realize 25% on his outlay?

\$1 cost realizes \$1.25; \$16.80 realizes 16.80×\$1.25, or \$21.00, but \$21.00 is 75% of marked price. Ans. \$28.00.

9. If goods are listed at \$12 a dozen and I buy at 40% and $33\frac{1}{3}$% off, what discount must I make from the list prices in addition to 40% to still realize 25% on the net cost?

List price=\$12; net cost=\$4.80; ∴ net selling price=$\frac{125}{100}$ of \$4.80=\$6.00. But the selling price at 40% off list price=\$7.20; ∴ additional discount=\$(7.20−6.00)=\$1.20, or $16\frac{2}{3}$%.

10. An agent sold wheat for \$460 on a commission of $3\frac{1}{2}$%. After deducting his commission for buying at 5%, he invested the

proceeds in cotton at 15c. a yd. If he bought at a discount of 10% and 6% off, how many yd. did he buy?

Amount to be invested is \$422.7619; cost per yd. after discounts are allowed=12.69c. ANS. 3331.45 yd.

X. COMMISSION.

1. An agent sold a house for \$4900 at $1\frac{1}{2}$% commission. What did he send to his employer. ANS. \$4826.50.

2. An agent's commission for selling \$13860 worth of goods was \$415.80. Find the rate of commission. ANS. 3%.

3. A commission merchant sold rice on a commission of $1\frac{1}{2}$%, and sent his employer \$9357.50. What was the amount of commission. ANS. \$142.50.

4. The net proceeds of sale were \$3494.45 after paying \$56.75 expenses and $3\frac{1}{2}$% commission. What was the amount of the sale?

$100\%-3\frac{1}{2}\%=96\frac{1}{2}\%$; $96\frac{1}{2}\%$ of sales=\$3494.45+56.75; ∴ sales= \$3680.

5. I received \$3825 to invest in wheat at 2% commission. After reserving my commission, how many bushels can I buy at 75c. per bushel? ANS. 5000 bu.

6. A merchant sold lumber on 2% commission for \$5096, and invested the proceeds in goods on 4% commission. What sum did he invest?

2% commission on \$5096=\$101.92; ∴ \$(5096−101.92)=\$4994.08; sum invested=$\frac{100}{104}$ of \$4994.08=\$4802.

7. A commission merchant sold 1800 lb. of turkeys at 25c. a lb. and retained for his services \$22.50. What rate of commission did he charge?

Amount received for turkeys=\$450; \$450 gives \$22.50 commission, or 5%.

8 A commission merchant pays \$2.26 freight, deducts commission, and sends \$22.22 to his employer as the net proceeds of a firkin of butter. Find his commission at 4%.

Freight and remittance=\$24.48; commission merchant has 4% commission, hence, 96% of sales=\$24.48; ∴ his commission=$\frac{1}{24}$ of \$24.48=\$1.02.

9. An agent sold 255 bbl. sugar, 300 lb. each, at $4\frac{4}{5}$c. a lb. Find his commission at $\frac{1}{4}$c. a lb. ANS. \$191.25.

10. A commission merchant sold 250 bbl. sugar of 300 lb. each at $4\frac{1}{2}$c. a lb., and 120 bbl. syrup of 40 gal. each at $39\frac{1}{2}$c. a gal. Find his whole commission at 2%.

Price of sugar=\$3375
" " syrup= 1896
Total price=\$5271
Commission at 2%=\$105.42.

XI. INSURANCE.

1. A house is worth \$3000, and furniture \$2000. The house is insured for \$2500 at 3% and the furniture for \$1500 at 2%. Find the cost of insurance. ANS. \$105.

2. I have a house worth \$5520 insured for $\frac{5}{8}$ of its value. The premium is \$92. Find the rate. ANS. $2\frac{2}{3}$%.

3. Insured a store for $\frac{7}{8}$ of its value at $2\frac{1}{2}$%. The premium was \$40.25. Find the value of the store.

$2\frac{1}{2}$% of the amount of insurance=\$40.25; ∴ amount of insurance=\$1610; ∴ $\frac{7}{8}$ of the value of the store=\$1610; ∴ value=\$1840.

4. I paid \$81 for insuring a dwelling worth \$5625 at $1\frac{4}{5}$%. What fraction of its value was insured?

$1\frac{4}{5}$% of policy=\$81.00; policy=\$4500=$\frac{4}{5}$ of its value.

5. A store is worth \$5280; contents, \$6240. The store is insured at $\frac{2}{3}$ of its value at $\frac{5}{8}$%, and the contents at $\frac{3}{4}$ of their value at $\frac{4}{5}$%. The policy cost \$2.25. How much was paid for the insurance?

$\frac{2}{3}$ of \$5280=\$3520; $\frac{5}{8}$% of \$3520=\$22.00; $\frac{3}{4}$ of \$6240=\$4680; $\frac{4}{5}$% of \$4680=\$37.44; total cost=\$(22.00+37.44+2.25)=\$61.69.

6. A merchant insured his stock for \$30000; $\frac{1}{3}$ in the Western at $1\frac{1}{2}$%; $\frac{1}{4}$ in the Equity at 2%, and the balance in the London Mutual at $1\frac{1}{5}$%. What was the total premium?

$1\frac{1}{2}$% of \$10000=\$150; 2% of \$7500=\$150; $1\frac{1}{5}$% of \$12500=\$150. Total premium=\$450.

7. A house was insured at $4\frac{1}{4}$% for $\frac{2}{3}$ of its value. The company received \$180, including \$1.50 for the policy. Find the value of the house.

Premium=\$(180—1.50)=\$178.50; $4\frac{1}{4}$% of policy=\$178.50; ∴ policy=\$4200; ∴ $\frac{2}{3}$ of value of house=\$4200; ∴ value=\$6300.

8. James Smith owns $\frac{5}{8}$ of a business which is insured for $\frac{2}{3}$ of its value at $1\frac{1}{2}$%. His total premium is \$480. Find how much the business is woıth.

$1\frac{1}{2}\%$ of Smith's policy=\$480; ∴ Smith's policy=\$32000; ∴ $\frac{2}{3}$ of Smith's share=\$32000; ∴ Smith's share=\$48000; and whole business=\$72000.

9. The contents of a canning factory were insured at $1\frac{1}{2}\%$. The goods were damaged by fire and water to the extent of \$18750, which was $\frac{2}{3}$ of the amount insured. If the amount of the insurance was $\frac{3}{4}$ of the total value of the contents, find their value.

$\frac{2}{3}$ of insurance=\$18750; ∴ insurance=\$28125; $\frac{3}{4}$ of the value of contents=\$28125; ∴ value of contents=\$37500.

10. Mr. Smith paid \$100 for insuring $\frac{4}{5}$ of the value of his house at $2\frac{1}{2}\%$. Mr. Jones paid \$110 for insuring $\frac{5}{6}$ of the value of his house at $2\frac{3}{4}\%$. Find the difference in value of the two houses.

$2\frac{1}{2}\%$ of Smith's policy=\$100; ∴ Smith's policy=\$4000; $\frac{4}{5}$ of value of Smith's house=\$4000; ∴ value of Smith's house=\$5000; $2\frac{3}{4}\%$ of Jones' policy=\$110; ∴ Jones' policy=\$4000; $\frac{5}{6}$ of value of Jones' house=\$4000; ∴ value of Jones' house=\$4800. ANS. \$200.

XII. TAXES.

1. The cost of building a school house to be paid by a tax of 23 mills on the dollar was \$3795; what was the assessment? ANS. \$165000.

2. If a man's property is assessed at \$2560 and he pays \$67.20 taxes, what is the total amount received in taxes if the assessment is \$907200?

Taxes on \$2560 =\$67.20;

∴ " " \$907200=$\left(\$907200\times\frac{6720}{256000}\right)$=\$23814.

3. The tax is \$6352.50; what is the assessment if A pays \$49.50 on \$2250?

Assessment to give \$49.50=\$2250; ∴ assessment to give \$6352.50=\$288750.

4. The city of London requires \$12000 for building and repairs, \$7500 for salaries, \$2500 for other expenses. The government grant is \$1500. The rest is to be raised by tax on property valued at \$4100000. Find the rate.

Amount required=\$(12000+7500+2500−1500)=\$20500; \$4100000 gives \$20500; ∴ rate, 5 mills.

5. A farm worth \$10000 is rated by the assessor at $\frac{3}{4}$ of its value. The rate of taxation is 3 mills on the dollar. Find the amount of taxes paid.

$\frac{3}{1000}$ of $\frac{3}{4}$ of \$10000=\$22.50.

6. The assessed value of A's property is \$30000. He pays $3\frac{1}{2}$ mills city tax, 13 mills school tax, $\frac{1}{2}$ mill poor tax, and \$1.50 poll tax. What is the amount of his taxes.

\$30000 at 17 mills in dollar=\$510; \$$(510+1\frac{1}{2})$=\$511.50.

7. I own farm property worth \$6800, personal property worth \$3150 and mortgages worth \$4360. I pay $4\frac{3}{4}\%$ on $\frac{1}{4}$ of the value of the farm; 5% on $\frac{1}{3}$ of the value of personal property, and 3 mills in the dollar on the mortgages. Find the amount of taxes. Ans. \$146.33.

8. In a town of 840 voters, \$3680 is to be raised. Each voter pays \$1.25 poll tax and the rest is raised by uniform tax of 3 mills on the dollar. Find the assessment.

Poll tax=840×\$1.25=\$1050; assessment=$\frac{1000}{3}$ of \$(3680−1050) =\$876666$\frac{2}{3}$.

9. A county requires \$5820 to build a bridge, allowing 3% for collecting and 5% commission on cost of bridge for inspection. What must be the assessment if taxes are levied at 4 mills on the dollar to raise the amount required?

Cost of bridge and inspection=\$(5820+291);
Sum to be raised=$\frac{100}{97}$ of \$(5820+291);
Assessment=$\frac{1000}{4}$ of $\frac{100}{97}$ of \$6111
=\$1575000.

10. A town-hall is to be built for \$5986. The property is valued at \$665870. What will be the tax on \$1000? Ans. \$8.99.

XIII. INTEREST.

1. Find the interest on a note drawn Jan. 1st, 1899, and due August 8th, 1900, for \$3860 at $4\frac{1}{2}\%$. Ans. \$277.92.

2. What principal will amount to \$1416 at 5% from March 9th, 1897, to Oct. 14th, 1900?

Principal=$\frac{100}{118}$ of \$1416=\$1200.

3. Principal \$250; interest \$33.75; time 1 yr. 6 mo. Find the rate %. Ans. 9%.

4. Principal \$468; interest \$98.28; rate 6%. Find the time.

Interest in 1 yr. on \$468 at 6%=\$28.08; \$98.28÷\$28.08=$3\frac{1}{2}$; time 3 yr. 6 mo.

5. A certain sum of money invested at 6% simple interest for 11 yr. amounted to \$1162. In what time would it amount to \$1266.00. Ans. 13 yr. 6 mo.

6. A friend loaned me $463.50 at 6%, which I kept till it amounted to $472.77. How long did I keep it. ANS. 4 mo.

7. $1800 was borrowed Jan. 1st, 1899, at 5% per annum. What sum will repay the debt on October 20th? ANS. $1872.

8. At what rate per cent. will a given sum double itself in 30 years? ANS. $3\frac{1}{3}$%.

9. A man borrowed a certain sum May 11th, 1898, at 6%, and on Feb. 27th, 1900, paid $637.10, which was the amount due. How much did he borrow?

Time $1\frac{4}{5}$ years; rate 6%; amount of $100 for given time and rate=$110.80; $\therefore$ amount borrowed$=\frac{100}{110.8}$ of $637.10=$575.

10. I loaned Mr. Jones a sum of money July 7th, 1897, at 8% simple interest, and on Nov. 30th, 1900, he paid the interest due, which was $183.60. How much did I loan him?

Interest on $100 for $3\frac{2}{5}$ yr. @ 8%=$27.20. ANS. $675.

11. Find the compound interest on $2080 in 3 yr. at 4% per annum. ANS. $259.71.

12. What principal will amount to $4250 in 2 years at 5% compound interest? ANS. $3854.875.

13. What is the difference between the simple and compound interest on $1500 for 3 yr. at 6%? ANS. $16.524.

14. What is the amount of $2000 for $1\frac{1}{2}$ yr. at 6% per annum, payable half yearly?

There will be 3 payments at 3%; Amount=$\$[2000\ (1.03)^3]=$ $2185.45.

15. Find the compound interest on $3000 for 2 years at 8%, payable half yearly. ANS. $509.58.

16. Find the amount of $1700 for $2\frac{3}{4}$ yr. at 4% per annum compound interest.

$$\text{Amount}=\$(1700\times\tfrac{104}{100}\times\tfrac{104}{100}\times\tfrac{103}{100})=\$1893.88.$$

17. Find the amount of $6282 in 2 yr. at $3\frac{1}{2}$% per annum compound interest.

$$\text{Amount}=\$[6282\times(1.035)^2]=\$6729.4355.$$

18. What principal will amount to $8279 in $2\frac{1}{2}$ yr. at 3% compound interest?

$$\text{Principal}=\$\left[8279\times\tfrac{100}{103}\times\tfrac{100}{103}\times\frac{100}{101\frac{1}{2}}\right]$$
$$=\$[8279\times(\tfrac{100}{103})^2\times\tfrac{200}{203}]=\$7688.432.$$

19. What is the difference between the simple and compound interest on \$1250 from June 30th, 1898, to December 31st, 1900, at 8%?

$$\text{Amount}=\$[1250\ (\tfrac{108}{100})\ (\tfrac{108}{100})\ (\tfrac{104}{100})=\$1516.32;$$

∴ compound interest=\$(1516.32−1250)=\$266.32;
Simple " =\$250;
∴ difference=\$16.32.

20. Find the amount of \$1080 for 3½ yr. at 3⅓% compound interest.

$$\text{Amount}=\$\left[1080\left(\frac{103\frac{1}{3}}{100}\right)^3\times\frac{101\frac{2}{3}}{100}\right]=\$[1080\ (\tfrac{31}{30})^3\times\tfrac{61}{60}]=\$1211.50.$$

XIV. BANK DISCOUNT.

1. A note of \$1350 was dated Oct. 19th, 1899, payable in 5 mo. and discounted Dec. 17th, 1899, at 4%. Find the proceeds.

Time, 95 days; rate, 4%; interest on \$1350 for given time and rate=\$14.05; proceeds \$1335.95.

2. \$750. LONDON, June 1st, 1900.

Eleven months after date, for value received, I promise to pay J. M. Northcott, or order, seven hundred and fifty dollars with interest at 8%. REGINALD HOWARD.

Discounted Jan. 15th, 1901, at 5%. Find the proceeds. ANS. \$792.98.

3. A note for \$1314, dated Jan. 1st, 1899, is payable in 3 mo. and is discounted Feb. 12th, at 2½%. Find the discount.

The note is legally due April 4th; number of days from Feb. 12th to April 4th=51; interest on \$1314 for 51 da. at 2½%=\$4.59.

4. A note for \$600 dated Oct. 12th, for 60 da. at 6% interest was discounted Oct. 15th at 6%. Find the proceeds.

The note is due Dec. 14th; time of discount, 60 days; amount of note when due at 6%=\$606.214; interest on \$606.214 for 60 days=\$6.06; net proceeds \$600.15.

5. For what sum must a 70 day note be drawn without interest so that when discounted at the bank at 6% the proceeds will be \$573.90?

Discount on \$100 for 73 da. @ 6%=\$1.20; note to produce \$98.80=\$100; note to produce \$573.90=\$580.87.

6. The discount on a note for \$1825, legally due on 21st August and discounted June 24th, was \$40.60. At what rate was it discounted?

Time 58 days; interest \$40.60.

Interest on \$100 for 365 da.=$\$\frac{100\times365\times40.60}{58\times1825}$=\$14. ANS. 14%.

7. The face of a note is \$960; date Feb. 8th, 1900; time 90 days; interest 5%; discounted Feb. 20th, 1900, at 6%. Find the proceeds.

Amount of note when due in 93 days=\$972.23;
Discount off \$972.23 at 6% for 81 da.=\$12.945;
Proceeds=\$(972.23—12.945)=\$959.285.

8. I wish to use \$3224.20 for 2 mo. For what sum must I give my note that when discounted at the bank at 12% per annum I shall receive that amount. Days of grace not taken.

Face of note=$\frac{100}{98}$ of \$3224.20=\$3290.

9. TORONTO, Dec. 23rd, 1900.

Three months after date, for value received, I promise to pay Frank Stover, or order, sixteen hundred dollars with interest at 6%. Discounted Jan. 9th, at 6%; no days of grace. Find proceeds.

Amount of note at maturity=\$1624; due March 23rd; term of discount 73 da.; interest on \$1624=\$19.48$\frac{4}{5}$. ANS. \$1604.51$\frac{1}{5}$.

10. Find the face of a 70 da. note that will yield \$1482 when discounted at 6%.

Face of note—$\frac{73}{365}$ of $\frac{6}{100}$ of face of note=\$1482. ANS. \$1500.

XV. STOCKS.

1. Find the cost of eighty \$100 shares of Bank of Ontario stock at 121; brokerage, $\frac{1}{4}$%.

Cost of 1 share=\$(121+$\frac{1}{4}$)=\$121$\frac{1}{4}$: ∴ cost of 80 shares= 80×\$121$\frac{1}{4}$=\$9700.

2. What sum must I invest in 5% stock at 103 to yield an income of \$720?

Investment to yield \$5=\$103; ∴ investment to yield \$720= \$14832.

3. What income will I derive from \$6240 invested in 5% stock at 104?

Income from investing \$104=\$5; ∴ income from investing \$6240=\$300.

4. A man bought bank stock at 3$\frac{1}{4}$% discount, and sold at 4$\frac{3}{4}$% premium, and gained \$960 by the transaction. Find the par value of the stock.

\$8 is gained from \$100 stock; ∴ \$960 is gained from \$12000 stock.

5. Which is the more profitable investment—$46800 in 7% stock at 90, or the same sum in 10% stock at 130, and how much?

Income from 1st=$3640
" " 2nd=$3600
The 1st is better by $40

6. A man invests 40% of his capital in 4% stock at 90 and the remainder in 7% stock at 135; which yields him an income of $2200 a year. Find his capital. Ans. $45000.

7. What rate per cent. do I receive on my money by investing in stock at $95\frac{7}{8}$, brokerage $\frac{1}{8}$, and paying a yearly dividend of 8%.

$100 stock costs $\$(95\frac{7}{8}+\frac{1}{8})$, or $96; ∴ $96 yields $8 interest and rate=$8\frac{1}{3}$%.

8. If I buy stock at 115 and thereby realize 6% on the investment, at what rate should I have bought it to realize $7\frac{1}{2}$%?

$$\text{Rate}=\frac{6}{7\frac{1}{2}} \text{ of } \$115=\$92.$$

9. I bought $9500 stock at 75, at what price must I sell to gain $570?

$9500 stock at 75 cost $7125; ∴ s. p.=$(7125+570)=$7695; ∴ selling price of stock=81.

10. Sold stock at $17\frac{1}{2}$% discount and realized 10% on my money. Find the rate of discount at which I bought.

$100 stock sells for $\$82\frac{1}{2}$; $\frac{110}{100}$ of cost price=$\$82\frac{1}{2}$; ∴ cost=$75; discount=25%.

XVI. EQUATION OF PAYMENTS.

1. A debt of $600 is to be paid as follows:—$150 at once, $150 in 6 mo., $150 in 12 mo. and $150 in 18 mo. When should it all be paid together? Ans. 9 mo.

2. A man lends $10000 in four sums. He gets 3% for $2000; $3\frac{1}{2}$% for $4000 and 5% for $2500. What per cent. must he realize on the remainder, if his average rate is $3\frac{17}{20}$%?

Interest from $(2000+4000+2500)=$325; total interest at $3\frac{17}{20}$% =$385; ∴ $(385−325)=$60 to be realized on remainder $1500; ∴ rate=4%.

3. A merchant purchased goods on April 1st, 1900, as follows: —$700 on 2 mo.; $1000 on 3 mo., and $1400 on 6 mo. credit. What is the equated time of payment? Ans. $4\frac{4}{31}$ mo., or Aug. 5.

4. I borrowed $720 for 10 mo. and $1080 for 8 mo. I paid $1200 at end of 7 mo. How long after the equated time for the payment of the whole may I keep the balance?

Equated time for payment of all, $8\frac{4}{5}$ mo.; \$1200 is paid $1\frac{4}{5}$ mo. before it is due and hence \$600 may be kept $3\frac{3}{5}$ mo. after it is due.

5. If A lends B \$1350 for 6 mo., how long ought B to lend A \$900 to repay the favor? Ans. 9 mo.

6. E owes F \$4800 payable in 8 mo. without interest. At the end of 2 mo. he pays \$1200; in 3 mo. \$1600, and in 4 mo. \$400. How long should E be allowed to keep the remainder after maturity?

The interest on the money paid before it is due is equal to the interest on \$16800 for 1 mo.; hence, \$1600, the remainder may be held $\frac{16800}{1600}$ mo. after it is due. Ans. $10\frac{1}{2}$ mo.

XVII. PARTNERSHIP.

1. A, B and C sell grain for \$2550. How much should each receive if for every 4 bu. A sells, B sells 5 bu.; and for every $2\frac{1}{2}$ bu. B sells, C sells 3 bu.? The grain is all equal in quality and price per bu. Ans. \$680, \$850, \$1020, respectively.

2. A and B enter into partnership with \$4500 and \$6750 respectively. In 8 mo. A puts in \$3750. 15 mo. from the beginning their profits were \$3120. How should they be divided? Ans. A=\$1500; B=\$1620.

3. A, B and C agree to pay a hotel bill in the proportions of 4, 5 and 6, respectively. A pays the first day's bill, \$12; B pays the second day's bill, \$14.50: and C the third, \$15.50. How must they settle accounts?

Amount of bill=\$42; A should pay $\frac{4}{15}$, or \$11.20; B $\frac{5}{15}$, or \$14; C $\frac{6}{15}$, or \$16.80; ∴ C must pay A 80c. and B 50c.

4. A began business with \$4500 and is joined by B with \$13500. They divide the profits equally at the end of the year. When did B join?

B's capital for 4 mo.=A's for 12 mo.; ∴ B joined 8 mo. after A began business.

5. A and B rent a pasture for 10 mo. at \$6 per mo. A has 10 horses for 9 mo. How many can B put in for the remaining mo. if he pays \$12 less than A?

A pays \$36; B, \$24. \$36 pays for 90 horses for 1 mo.; ∴ \$24 pays for 60 horses for 1 mo. Ans. 60 horses.

6. A, B and C join in partnership. The capital is contributed in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ respectively. In 4 mo. A takes out

one-half of his, and at the end of the year the profits amount to $5280. What should A receive?

Capital of each in the proportion of 6, 4, 3; A's=48 for 1 mo., B's 48 and C's 36; ∴ A gets $\frac{48}{132}=\frac{4}{11}$ of gain—$\frac{4}{11}$ of $5280=$1920.

7. Three brothers, John, James and Henry engage in business with a capital of $20000. John puts in $2000 more than James, and James $1500 more than Henry. The profits amount to $1360. What amount should John receive?

John's capital=$8500; James', $6500; Henry's, $5000; ∴ John receives $\frac{85}{200}$ or $\frac{17}{40}$ of $1360, or $578.

8. A and B enter into partnership with a capital of $6390, and the gain is $3550, of which A takes $250 more than B. How much capital had each?

A=$3420; B=$2970.

9. A, B and C are partners; A's money is in 9 mo., B's 8 mo., C's 7 mo., and they gain $900, $1000 and $1050 respectively; if A's capital is $4500, what is B's and C's?

A's monthly gain=$100; B's $125; C=$150; ∴ if $4500 gives $100 a mo. for A; then B's capital=$5625; C=$6750.

10. A man left his property to his children in the following proportion $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. His property amounted to $14976. What should each receive? ANS. $6912, $4608, $3456.

XVIII. APPLICATION OF INVOLUTION AND EVOLUTION.

1. Find by factoring the sq. root of (*a*) $12\times35\times105$. (*b*) $8\times14\times21\times27$; (*c*) $21\times77\times132\times6\times24$. ANS. (*a*) 210; (*b*) 252; (*c*) 5544.

2. A sq. field contains 18 ac. 36 sq. rd. Find the cost of fencing it at $68\frac{3}{4}$c. a rd.

Perimeter=216 rd.; ∴ $216\times68\frac{3}{4}$c.=$148.50.

3. A man has a field three times as long as it is wide, containing 37 ac. 155 sq. rd. What will fencing it cost at $1.75 per rd.?

Number of sq. rd.=6075. Divide field into three equal parts by lines parallel to the end and each part will be a square and contain 2025 sq. yd. Width is 45 rd. and length 135 rd.; perimeter 360 rd. Cost=360×$1.75=$630.

4. A boat in crossing a river 736 yd. wide landed on the opposite side 273 yd. farther down the stream. How far are the starting and landing points distant from each other in a straight line?

($\sqrt{736^2+273^2}$) yd.=785 yd.

5. What distance would be saved by walking along the diagonal instead of the two sides of a rectangular field 63 rd. wide and 84 rd. long? ANS. 42 rd.

6. How much more fencing would be required for a field 160 rd. long and 90 rd. wide than a square field of equal area? ANS. 20 rd.

7. The diagonal of a square field is 84 rd.; find the diagonal of another square field 4 times as large. ANS. 168 rd.

8. A pile of wood in the form of a rectangular solid is 48 ft. wide, 36 ft. high and 216 ft. long. If it were in the shape of a cube what would be the length of one edge? ANS. 72 ft.

9. A cubical block of stone contains 2406104 c. in.; what is the area of its faces? ANS. 748 sq. ft. 24 sq. in.

10. A river is 880 mi. long 2640 ft. wide and 12 ft. deep; find the depth of a cubical reservoir that would hold all the water in the river.

Depth of reservoir $=(\sqrt[3]{880\times5280\times2640\times12})$ ft. $=5280$ ft.

11. Find the area of a triangle whose sides are 52 ft. 56 ft. and 60 ft. ANS. 1344 sq. ft.

12. Find the value of $4\sqrt{5625}\times8\sqrt[3]{19683}\div4^2$. ANS. 4050.

13. Find the value of $8\sqrt{4096}\div4\sqrt[3]{32768}$. ANS. 4.

14. The surface of a cube is 3750 sq. ft.; find its solid content.
$\frac{3750}{6}$ sq. ft. $=625$ sq. ft. $=$ surface of one face; side $=25$ ft. ANS. 15625 c. ft.

15. Find the area of the largest circle that can be drawn in a square, 56 in. on each side.

Area of circle $=(28^2\times\frac{22}{7})$ sq. in. $=2464$ sq. in.

16. If a horse is tied to graze on an acre, what is the length of the rope?

Radius $=(\sqrt{4840\div\frac{22}{7}})$ yd. $=39.24$ yd.

17. The volume of a cubical block is 60698.457 c. ft. Find its surface.

$\sqrt[3]{60698.457}=$ edge $=39.3$ ft.; surface $=[6\times(39.3)^2]$ sq. ft. $=$ 9266.94 sq. ft.

18. A square field contains 5 ac. 46 rd. $8\frac{1}{2}$ yd. How many acres are there in a rectangular field whose adjacent sides are respectively 82 yd. and 72 yd. longer than the former?

Side of square field $=(\sqrt{25600})$ yd. $=160$ yd.; area of rectangular field $=(242\times232)$ sq. yd. $=11$ ac. 96 sq. rd.

19. A rectangular vessel is 143 in. wide, 1573 in. long and 13 in. deep. Find the side of a cubical vessel of equal capacity.

Side of vessel $=[\sqrt[3]{(143\times13\times1573)}]$ in. $=143$ in.

20. Find the contents of a cubical cistern that contains 17150 gal. of water.

$$\text{Side of cistern}=\left[\sqrt[3]{\left(17150\times\frac{10}{62\frac{1}{2}}\right)}\right]\text{ ft.}=14\text{ ft.}$$

21. How long will it take to ride around a square field containing 52 ac. 324 sq. yd. at $3\frac{1}{2}$ mi. an hr.?

Perimeter=2008 yd. ANS. $19\frac{4}{7}\frac{3}{7}$ hr.

22. A square field contains 3 ac. 121 sq. yd.; another field in the form of a rectangle contains 4 ac. 160 sq. yd. more, and is 56 yd. wider. Find its length.

Length of side of 1st field=121 yd.; area of 2nd=7 ac. 281 sq. yd.=34161 sq. yd.; width of 2nd=(121+56) yd.=177 yd.; ∴ length=(34161÷177) yd.=193 yd.

23. A square plot contains 824464 sq. yd. What will be the area of a rectangular field whose length is three times and breadth twice as much?

Area=$(3\times2\times824464)$ sq. yd.=4946784 sq. yd.

24. The area of 6 faces of a cube is $37\frac{1}{2}$ sq. ft., find its weight assuming that a cu. ft. weighs 1648 oz.

Side of cube=2.5 ft.; ∴ cu. content 15.625 cu. ft.; (1648×15.625) oz.=25750 oz.

25. By what number must the following be multiplied to make them perfect squares?

775; 539; 42525, and 972. Factors of $775=(5\times5)\times31$: ∴ other factor 31; similarly the other numbers require 11; 21; and 3 respectively.

26. What is the area of an isosceles triangle whose base is 12 ft. and each of the equal sides 10 ft.? ANS. 48 sq. ft.

27. Find the area of a triangle whose sides are 409 ft., 169 ft. and 510 ft. long. ANS. 30600 sq. ft.

28. By what number must the square of 126 be multiplied to become a perfect cube?

$126^2=2\times3\times3\times7\times2\times3\times3\times7=2^2\times3^3\times3\times7^2$; hence, the multiplier is $2\times3^2\times7$, or 126.

29. Find the largest perfect square contained in 2016.

The factors are $2^5\times3^2\times7$; ∴ highest sq. No.=144.

30. What is the least number by which the following may be multiplied to make them complete squares:—48, 252, 3000?

$48=4^2\times3$; $\because$ 3 is the multiplier; similarly 7 and 30.

31. Find the smallest perfect cube that contains 288 without a remainder.

$288=2^3\times2^2\times3^2$; number required$=2^3\times2^3\times3^3=1728$.

32. Find the smallest square and cube numbers which contain both 12 and 27 as factors.

L.C.M. of 27 and $12=108$; the factors of 108 are $2^2\times3^2\times3$; $\therefore$ square number$=2^2\times3^2\times3^2=324$; cube number$=2^3\times3^3=216$.

33. What perfect square factors are contained in 33075. ANS. 9, 25, 49, 225, 441, 1225, 11025.

34. A farmer owns a farm in the form of a rectangle 58619 ft. long, and 18491 ft. wide. How many more rods of fencing will it take to enclose it than a square farm of equal area?

Side of square field$=(\sqrt{58619\times18491})$ ft.$=32923$ ft.; perimeter of rectangle$=154220$ ft.; perimeter of square$=131692$ ft.: difference$=22528$ ft.$=1365\frac{1}{3}$ rd.

35. What is the 5th power of the square root of .0004?

$\sqrt{.0004}=.02$; $(.02)^5=.0000000032$.

36. Find the area of a circular ring of which the inner and outer diameters are 73.6 ft. and 106.4 ft. respectively. ANS. $4638\frac{6}{7}$ sq. ft.

37. Find the volume of the moon whose radius is 1080 miles. ANS. $5278793142\frac{6}{7}$ c. mi.

38. Find the quantity of iron in a spherical shell whose inner and outer radii are $10\frac{1}{2}$ in. and 14 in. respectively.

Quantity of iron$=(\frac{4}{3}\times\frac{22}{7})\ (14^3-10\frac{1}{2}^3)$c. in.$=6647\frac{2}{3}$ c. in.

39. Find the weight of a 12 inch shell of iron, the iron being 1 in. thick, if a c. ft. of iron weighs 444 lb.

Quantity of iron$=(\frac{4}{3}\times\frac{22}{7})\ (6^3-5^3)$ c. in.$=\frac{1144}{3}$ c. in.; weight $=\left(\frac{1144}{3\times1728}\times444\right)$ lb.$=97\frac{53}{54}$ lb.

XIX. METRIC SYSTEM.

1. How many times will 84 m. go around a circular disk 7 cm. in circumference? ANS. 1200 times.

2. If the diameter of a cent is 25 mm., how many of them placed in a continuous straight line will reach a kilometre? ANS. 40000c.

3. A bicycle wheel being 2 m. 28 cm. round, what distance has been passed over when it has made 7930 revolutions? ANS. 18.0804 Km.

4. A draper bought 85.54 m. of cloth, and sold at one time 17.5 m., at another 6.5 m., at another 19.25 m., at another 23.84 m. How much had he left? ANS. 18.45 m.

5. A train goes 72 Km. per hour. How many metres does it go per second? ANS. 20 m.

6. A metre equals 39.3708 inches. How many millimetres are there in one inch? ANS. 25.3995.

7. How many litres remain after taking 3947 cl. from a cask containing 2.75 Hl. ANS. 235.53 l.

8. If 2.25 litres of milk cost one franc, how many hectolitres could be bought for 73 francs? ANS. 1.6425 Hl.

9. Find the difference in grams between 68.575 Kg. and 4982.5 Dg. ANS. 18750 gr.

10. How many square metres are there in a floor 5.64 m. long and 4.75 wide? ANS. 26.79 sq. m.

11. What is the number of square metres in a path 38.40 m. long and 85 cm. wide? ANS. 32.64 sq. m.

12. Find the number of hectares in a rectangular plot 1.225 Km. long and 556 m. wide. ANS. 68.11 ha.

13. A train leaves Lyons at 10.30 a.m. and runs 45.64 Km. per hour. At what rate must a train run which leaves 45 min. later and overtakes it at 15 min. past 2 p.m.? ANS. 57.05 Km.

14. If the interest on 568.25 fr. for 3 years is 127.86 fr. Find the rate of interest.

$$\text{Rate}=\left(\frac{100}{568.25} \text{ of } \tfrac{1}{3} \text{ of } 127.86\right)\%=7.5\% \text{ nearly.}$$

15. It costs 688.5 fr. to dig a ditch whose depth is .68 m. and width .48 m. If the cost of digging is 6.75 fr. per c. m., find the length in kilometres. ANS. .3125 Km.

16. The area of a field is 5.695 hectares. Find its size in acres, rods, etc.

1 hectare=11960 sq. yd.; $\therefore$ 5.695 hectares=5.695×11960= 68112.2 sq. yd.=14 ac. 11 sq. per. $19\frac{9}{20}$ sq. yd.

17. How many grams are there in 1 lb. avoir.? ANS. 451.61.

18. How many ares are there in 90 acres?

1 are=119.6034 sq. yd.; 90 ac.$=90\times4840$ sq. yd.=435600 sq. yd.; No. of ares$=435600\div119.6054=3641.976$.

19. How much is a pile of wood 20 ft. long 4 ft. wide and 5 ft. high worth at \$1.25 per stere?

1 stere=35.316 c. ft.;

$$\text{Value of wood}=\frac{20\times4\times5}{35.316}\times\$1.25=\$14.157.$$

20. A man divided 5 Hl. of wheat among 20 persons. How many litres did each receive?

Share of each$=\frac{5}{20}$ Hl.$=\frac{500}{20}$ l.$=25$ l.

21. If a person steps 80 cm. at each step, how many steps will he take in walking around a square field containing 18 hectares 625 square metres?

18 hectares 625 sq. m.=180625 sq. m.; ∴ side of field$=(\sqrt{180625})$ m.=425 m.; number of steps$=\frac{170000}{80}=2125$.

22. A cistern is 4 m. long 24 dm. wide and 80 cm. deep. Find its capacity in litres.

Capacity$=(400\times240\times80)$ c. cm.=7680 l.

23. At 62c. a litre, find the cost of 9 l., 8 dl. of molasses.

9.8 litres at 62c.=\$6.076.

24. How many letters, each weighing 3.5 g. will be required to weigh 1.75 kg.? Ans. 500.

XX. WORK PROBLEMS.

1. A can cut 3 ac. in $4\frac{1}{2}$ hr.; B, 3 ac. in 5 hr. They cut $10\frac{1}{2}$ ac. together for \$3.99; what share should each get?

In 1 hr. A cuts $\frac{2}{3}$ ac.; B, $\frac{3}{5}$ ac.; ∴ money is divided in proportion of $\frac{2}{3}$ to $\frac{3}{5}$, or of 10 to 9. A's share$=\frac{10}{19}$ of \$3.99=\$2.10; B's, \$1.89.

2. A can do as much work in 3 da. as B can in 4 da. If B has done $\frac{1}{2}$ of a work in 12 da., how long will it take both to finish it?

B can do the work in 24 da.; A, in $\frac{3}{4}$ of 24 da., or 18 da; ∴ A and B do $\frac{7}{72}$ in 1 da., or $\frac{1}{2}$ the work in $5\frac{1}{7}$ da.

3. A can cut a field of grain in 24 hr., B in 10 hr. and C in 27 hr. They work together for $4\frac{1}{2}$ hr. and then A finishes the work. How long did A work altogether?

A, B and C do $\frac{193}{240}$ of work in $4\frac{1}{2}$ hr.; time for A to do $\frac{47}{240}$ of work$=(\frac{47}{240}\div\frac{1}{24})$ hr.$=4\frac{7}{10}$ hr. Ans. $9\frac{1}{5}$ hr.

4. A and B do $\frac{3}{8}$ of a work in 2 da. and then B finishes it in 4 da. In what time could A do the work?

A and B do $\frac{6}{32}$ in 1 da.; B $\frac{5}{32}$ in 1 da. ANS. 32 da.

5. 12 men can do a work in $17\frac{2}{3}$ da. How long may 4 remain away and yet finish in the same time by bringing 9 men more with them?

4 men do $\frac{1}{3}$ of work in $17\frac{2}{3}$ da.; $\therefore$ 13 men do $\frac{1}{3}$ of work in $5\frac{17}{39}$ da.; $\therefore$ time 4 men may remain away$=(17\frac{2}{3}-5\frac{17}{39})$ da.$=12\frac{3}{13}$ da.

6. A certain number of men mow 4 ac. in 3 hr. and a certain number of others mow 8 ac. in 5 hr. How long will it take to cut 11 ac. all working together?

First lot mow $1\frac{1}{3}$ ac. in 1 hr.; second lot mow $1\frac{3}{5}$ ac. in 1 hr.; both mow $2\frac{14}{15}$ ac. in 1 hr. $\therefore$ time to mow 11 ac.$=\frac{11}{2\frac{14}{15}}=3\frac{3}{4}$ hr.

7. A and B agree to do a work for \$5.60. A could do it in 7 da., B in 8 da. Together, with a boy, they do it in 3 da. How much should the boy get?

A earns 80c. and B 70c. a da., and both in 3 da. earn \$4.50; $\therefore$ boy earns \$(5.60—4.50), or \$1.10.

8. A can do $\frac{1}{3}$ of a work in 8 da.; B $\frac{3}{5}$ of the remainder in 5 da., and C could finish it in 4 da. What time will it take for all working together to do the work?

A can do it in 24 da.; B in $12\frac{1}{2}$ da.; C in 15 da.; $\therefore$ altogether in $5\frac{35}{113}$ da.

XXI. CLOCK PROBLEMS.

1. Two clocks, one of which gains $5\frac{1}{2}$ min. a day and the other $3\frac{3}{4}$ min. per day, are set right on Tuesday at noon. In what time will one be half an hour ahead of the other?

Time for one clock to gain $1\frac{3}{4}$ min. on the other$=1$ da.;

$\therefore$ " " " " " " 30 " " " " $=\frac{30\times 1}{1\frac{3}{4}}$ da.

$=17\frac{1}{7}$ da.

2. At what time between 9 and 10 o'clock a.m. is the hour hand 31 minute-spaces in advance of the minute-hand? ANS. $17\frac{5}{11}$ min. past 9.

3. Two clocks gain and lose equally 25 sec. in 8 hr. They are together at noon. How long before they will differ in time by 1 hr.? ANS. 24 da.

A watch is right at noon but loses time at the rate of 4%. What is the true time when the watch shows 12 o'clock at night?

96 min. on the watch=100 min. true time:

$\therefore$ (12×60)" " " " $=\frac{12\times 60\times 100}{96}$ min. true time=$12\frac{1}{2}$ hr.

5. At what time is the number of minutes past 5 equal to $\frac{3}{5}$ of the number of minutes to 6?

No. of min. past 5=$\frac{3}{5}$ of (60—No. of min. past 5). ANS. $5.22\frac{1}{2}$.

6. One clock gains 4 min. in 6 da. and another loses 1 min. in 24 hr. They are together at noon; when will they differ by 15 min.?

Time to differ $1\frac{2}{3}$ min.=1 da.; $\therefore$ time to differ 15 min.=9 da.

7. A man walking, finds on looking at his watch that the hands are at right angles, and when he has gone 1920 yd. they are together for the first time. At what rate per hour is he travelling?

The hands are 15 min. apart and will be together in $16\frac{4}{11}$ min.; distance walked in 60 min.=$\frac{60\times 1920}{16\frac{4}{11}}$ yd.=4 mi.

8. I looked at my watch between 8 and 9 o'clock and also between 9 and 10, and found the hands had changed places. What time was it when I looked first at my watch?

When the hands have changed places, the sum of the distances moved must be an exact number of times around the face (in this case, once), or 60 min.-spaces. The hr.-hand goes $\frac{1}{13}$ and min.-hand $\frac{12}{13}$ of the distance. The hr.-hand goes $\frac{1}{13}$ of 60 spaces, or $4\frac{8}{13}$ spaces. This is the distance the min.-hand was ahead when I first looked; $\therefore$ it has gained $44\frac{8}{13}$ spaces since 8 o'clock. 11 spaces are gained in 12 min.; $44\frac{8}{13}$ spaces are gained in $48\frac{96}{143}$ min. Time, $48\frac{96}{143}$ min. past 8.

9. I looked at my watch between 11 and 12 and again between 2 and 3 and found the hands had changed places. At what time did I first look? ANS. $9\frac{3}{10}$ min. past 11.

XXII. PROBLEMS INVOLVING VELOCITIES.

1. A hare is 310 yd. ahead of a hound and takes 10 leaps of 7 ft. each in 6 seconds, the hound takes 12 leaps of 9 ft. $4\frac{1}{2}$ in. each 3 seconds. How long before the hound will catch the hare?

Hare goes 70 yd. in 18 seconds;
Hound " 225 " " 18 "
" gains 155 " " 18 "
" " 310 " " 36 "

2. A train 176 yd. long passes a telegraph post in 6 sec., and passes a man in another train going in the opposite direction in $3\frac{3}{7}$ sec. At what rate is each train running?

1st train goes 176 yd. in 6 sec.; ∴ 60 mi. per hr.
Distance travelled by 1st train in $3\frac{3}{7}$ sec. $= 100\frac{4}{7}$ yd.:
∴ " " " 2nd " " $3\frac{3}{7}$ sec. $= (176 - 100\frac{4}{7})$ yd.:
∴ " " " 2nd " " 1 hr. $= 45$ mi.

3. A can run 400 yd. in 42 sec.; B, 500 yd. in 56 sec. How much start must B have to come out even with A in a 200 yd. race?

A runs 400 yd. in 42 sec., or 200 yd. in 21 sec.; B runs $187\frac{1}{2}$ yd. in 21 sec.; ∴ start $= (200 - 187\frac{1}{2})$ yd. $= 12\frac{1}{2}$ yd.

4. In running 1760 yd. A beats B 32 yd. and C by 48 yd., by how much can B beat C in 1296 yd.?

A runs 1760 yd. while B runs 1728 yd., or C 1712 yd. ∴ B runs 1728 yd. while C runs 1712 yd., and 1296 yd. while C runs 1284 yd.; ∴ B beats C by 12 yd.

5. A man in a buggy goes $6\frac{1}{2}$ miles per hr. and starts at 8 a.m. A boy on a bicycle starts 1 hr. 50 min. later and overtakes the man at noon. At what rate did the boy ride?

The man goes $4 \times 6\frac{1}{2}$ miles $= 26$ miles; the boy goes 26 miles in $2\frac{1}{6}$ hr., or 12 mi. per hr.

6. A man standing at a station noticed a train pass him in $10\frac{1}{2}$ sec.; and that it passed completely over the distance between two posts 108 rd. apart in $37\frac{1}{2}$ sec. Find the length and rate of the train.

Distance the train goes in $(37\frac{1}{2} - 10\frac{1}{2})$ sec. $= 108$ rd.;
" " " " " 1 hr. $= 45$ mi.;
" " " " " $10\frac{1}{2}$ sec. $= 231$ yd.;
$=$ length of train.

7. The Imperial Limited travels from Montreal to Vancouver, 2700 miles, in 99 hours, but in the last half of the journey it goes 60 miles in the same time it took to go 50 miles in the 1st half. Find its rate per hour in the 2nd half of the journey.

Time to make the journey at 5 mi. for the first half and 6 mi. for the 2nd half $= (\frac{1350}{5} \times \frac{1350}{6})$ hr. $= 495$ hr.; this time is $\frac{495}{99}$ times as long as the actual time; ∴ the actual rate for 2nd half was $\frac{495}{99}$ as fast as 6 mi., or 30 mi.

8. A train running 25 mi. an hr. is 60 mi. ahead of another which runs 15 mi. in $\frac{1}{3}$ of the time it takes the first to run 25 mi. How long will it take the fast train to overtake the other?

Rate of fast train per hr. $= 45$ mi.; time to gain 60 mi. $= \frac{60}{45 - 25}$ hr. $= 3$ hr.

9. A train running 45 mi. an hr. crosses a bridge 240 yd. long in 36 sec. Find the length of the train.

Distance train goes in 36 sec. $=\frac{36\times45\times1760}{60\times60}$ yd. $=792$ yd.; length of train $=(792-240)$ yd. $=552$ yd.

10. A man can row 3 mi. an hr. in still water. How long will it take him to go 12 mi down stream and back again if the stream flows 1 mi. per hr.? ANS. 9 hr.

11. A train travels 25 mi. in $\frac{7}{10}$ hr.; find its rate in feet per min. ANS. $3142\frac{6}{7}$ ft.

12. A boat runs 24 mi. down stream in $1\frac{1}{2}$ hr. and 15 mi. up stream in the same time. After running down for 9 hr., how long will it be in returning? ANS. $14\frac{2}{5}$ hr.

13. A can run 8 mi. and B 7 mi. per hr. How much start can A give B in a mi. race and yet win by 44 yd.?

A runs 1760 yd., B runs (including start) 1716 yd. While A runs 1760 yd., B runs $\frac{7}{8}$ of it, or 1540 yd. $\therefore$ $(1716-1540)$ yd. $=176$ yd. start.

14. Two trains start from Toronto for the same place, one at 8 a.m. at 25 mi. an hr., the other at 10 a.m. at 30 mi. an hr. The faster arrives 1 hr. ahead of the other. Find the distance travelled.

Distance gained by faster train $=(2\times25+25)$ mi.; time to gain 75 mi. $=\frac{75}{5}$ hr. $=15$ hr.; distance travelled $=(15\times30)$ mi. $=450$ mi.

15. A train running 30 mi. per hr. overtakes a man going 5 mi. an hr. and passes him in 18 sec. Find the length of the train.

Distance gained by train in 1 hr. $=25$ mi.; distance gained by train in 18 sec. $=\frac{18\times25\times1760}{60\times60}$ yd. $=220$ yd.; length of train $=220$ yd.

16. A can walk 10 mi. in $2\frac{1}{4}$ hr.; B, 11 mi. in $2\frac{1}{2}$ hr. They start to walk a race of 55 mi, Which will win and by how much time?

A walks 10 mi. in $2\frac{1}{4}$ hr., or 55 mi. in $12\frac{3}{8}$ hr.; B walks 11 mi. in $2\frac{1}{2}$ hr., or 55 mi. in $12\frac{4}{8}$ hr.; $\therefore$ A wins by $(12\frac{4}{8}-12\frac{3}{8})$ hr. $=$ $7\frac{1}{2}$ min.

17. A train going 40 mi. an hr. takes 18 sec. to pass another standing on a switch. Find the length of the moving train if the other is 192 ft. long.

Distance travelled by train in 18 sec. $=\frac{18\times40\times5280}{60\times60}$ ft., or 1056 ft.; length of train $=(1056-192)$ ft. $=864$ ft.

18. A man can row down stream 16 miles per hr., and up 8 miles per hr. How far can he go down stream so as to be back at the starting point in 12 hours?

Time to go 1 mi. down stream and back $=(\frac{1}{16}+\frac{1}{8})$ hr.;
Distance gone down and back in $\frac{3}{16}$ hr. $=1$ mi.;
" " " " " " 12 hr. $=\dfrac{12\times 1}{\frac{3}{16}}$ mi. $=64$ mi.

19. A train 418 yd. long overtakes a man walking at 3 miles an hr., and passes him in 15 sec. Find the rate of the train in miles per hr.

Distance the man goes in 15 sec. $=22$ yd.;
" " train " " 1 hr. $=\dfrac{60\times 60\times(418+22)}{15\times 1760}$ mi. $=60$ mi.

XXIII. MISCELLANEOUS EXERCISES.

1. Simplify $\dfrac{5\frac{3}{4}-\frac{3}{7}\text{ of }15\frac{3}{4}+2\frac{2}{35}\div 2\frac{22}{25}\times 2}{\frac{3}{4}\text{ of }7\frac{3}{7}-5\frac{3}{5}\div 3\frac{4}{15}}.=$ ANS. $\frac{1}{9}$.

2. A train 264 ft. long takes 10 sec. to cross a bridge 132 ft. long. Find when it will be 25 miles farther away.

Train goes $(264+132)$ ft., or 396 ft. in 10 sec., or 2376 ft. per min.
No. of feet in 25 miles $=(25\times 5280)$ ft.;
$\therefore$ time $=\dfrac{25\times 5280}{2376}=55\frac{5}{9}$ min.

3. I buy three town lots whose cost is in the proportion of 4, 3, 2. I sell them at once and gain 10% on the first, 6% on the second and lose 4% on the third. My whole gain is $505. Find the value of each lot.

$4 investment gives 40c. gain; $3 investment gives 18c. gain; $2 investment gives 8c. loss; ∴ $9 investment gives 50c. gain; $9090 investment gives $505 gain; hence, the values are $4040; $3030; $2020.

4. A dealer buys pigs at $5 each and sells $\frac{1}{2}$ of them at 6\frac{1}{3}$, and the other half at 10% loss. The total gain was $25. How many pigs did he buy?

Gain on each of 1st half 1\frac{1}{3}$, or $$\frac{4}{3}$ on each; loss of 10% on 2nd half $=5\%$ on all $=25$c. a pig; ∴ net gain on each pig $=66\frac{2}{3}$c. -25c. $=41\frac{2}{3}$c., or $25 on 60 pigs.

5. Show that the product of 72 and 99 is the product of their L.C.M. and G.C.M.

6. A man has two fields; a square one containing $22\frac{1}{2}$ acres, and the other a rectangular field of $31\frac{1}{4}$ acres. A boy rides round the square at 12 miles an hr. Another round the rectangular field in

$1\frac{13}{20}$ min. longer time. Find the rate per hour of the 2nd boy, if the shorter side of the rectangle is the same as the side of the square.

Side of square=60 rd.; ∴ sides of rectangle=(60 and 84) rd.; time riding round square=$3\frac{3}{4}$ min.: time riding round rectangle=$(3\frac{3}{4}+1\frac{13}{20})$ min.=$5\frac{2}{5}$ min.=10 mi. an hr.

7. 2, 3, 5, and 7, being known to be factors of a number, what other factors must it have? ANS. 1, 6, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210.

8. A note for a certain sum at 4% interest is due in $2\frac{1}{2}$ months. What fractions of the principal are the interest and true discount? ANS. $\frac{1}{120}$ and $\frac{1}{121}$.

9. The men in a factory work 10 hr. a day for $6.50 a week. How much per cent. will the wages be increased for doing the same work, working $9\frac{1}{2}$ hr. per day, and receiving $6.65 per week?

1st rate is $10\frac{5}{6}$c. per hr.; 2nd rate is $11\frac{2}{3}$c. per hr.; gain=$(11\frac{2}{3}-10\frac{5}{6})$c.=$\frac{5}{6}$c., or $7\frac{9}{13}$%.

10. Find the publisher's profit on the sale of 10400 copies of a book retailing at 70c. The publisher allows 25% discount to the retailer and also gives 13 copies for a dozen, and the cost of printing, binding, etc., is 25c. a copy.

The retailer pays $52\frac{1}{2}$c., but gets 13 for a dozen; net sum received for each copy published=$\frac{12}{13}$ of $52\frac{1}{2}$c.=$48\frac{6}{13}$c: net profit=$(48\frac{6}{13}-25)$c.=$23\frac{6}{13}$c. total profit=$(10400\times23\frac{6}{13})$c.=$2440.

11. A rectangular cistern $4\frac{1}{2}$ ft. long when full contains $28\frac{1}{8}$ c. ft. of water. If the breadth and depth are the same, what will be the cost of lining the inside of the cistern with lead (no lid) at 28c. per sq. ft.?

Breadth and depth, each=$\sqrt{\left(\frac{28\frac{1}{8}}{4\frac{1}{2}}\right)}$ ft.=$2\frac{1}{2}$ ft.; ∴ $46\frac{1}{4}$ sq. ft. to be lined. ANS. $12.95.

12. Show that the G.C.M. of the squares of two numbers is the square of the G.C.M.

13. A boy does in 4 days what a man does in 3 days, but a man gets $1.40 per day and a boy $1.00. How much will a man gain if he employs 7 boys for 12 days instead of men?

7 men will do the work in 9 da.; cost=$88.20; 7 boys will do the work in 12.; cost=$84.00; saving $4.20.

14. If I buy G.T.R. stock at $122\frac{1}{2}$, I get $7 per share of $100, but if I invest in mining stock at 105, I get $$5\frac{1}{4}$ per $100 share.

The former investment gives me \$185 more income than the latter. Find the amount invested.

In the 1st investment, every \$ gives $\$\frac{2}{35}$; in the 2nd investment, every \$ gives $\$\frac{1}{20}$; $\$(\frac{2}{35}-\frac{1}{20})=\$\frac{1}{140}$; $\therefore \frac{1}{140}$ of the investment is \$185, and the investment is \$25900.

15. A banker's discount on \$4920 for $2\frac{1}{2}$ years is \$615. What will be the true discount on \$18400 for 3 years at same rate, and simple interest?

The banker's rate is 5%; the true discount on \$18400 for 3 yr. at $5\% = \frac{15}{115}$ of \$18400=\$2400.

16. Find a number whose square is equal to the sum of the squares of 1392 and 580; also one whose square is equal to the difference of the squares of 1460 and 876, and subtract the one from the other.

1st=1508; 2nd=1168; difference=340.

17. A man meets 5 beggars and he divides a certain sum among four of them in the proportion of 1, 2, 3, 4, with the understanding that each must give the fifth $\frac{3}{16}$ of his share. The fifth gets \$5.25. Find the share of each of the others.

The fifth gets $\frac{30}{16}$ of the amount the first receives; the first receives $\frac{16}{30}$ of \$5.25, or \$2.80; etc.

18. A runs 800 yd. in 3 min.; B, 1100 yd. in 4 min. How much start must one give the other to reach the winning post together in a mile race?

A runs a mi. in $6\frac{3}{5}$ min.; B, in $6\frac{2}{5}$; $\therefore$ A must have $\frac{1}{5}$ min. start or $53\frac{1}{3}$ yd.

19. A man has an income of \$1660 from capital in 4% stock. He sells out at 119 and buys bonds which pay 5%; find the price of the bonds in order that his income may be \$1700.

He has \$41500 of 4% stock; he sells at 119 and realizes \$49385; bonds must=\$34000 to give \$1700 income; $\therefore$ price must be $145\frac{1}{4}$.

20. A note for \$24530 drawn March 14th at 9 mo. was discounted at a bank May 12th at $3\frac{1}{4}$%. Find the net proceeds.

Time of discount, 219 da.; bank discount is $\frac{3}{5}$ of $3\frac{1}{4}$% of \$24530, or \$478.335; hence net proceeds=\$(24530−478.335)=\$24051.665.

21. The rent of a square field at \$11 per ac. is \$110. Find the cost of fencing it at 15c. a yd. Ans. \$132.

22. A B C D is a four-sided field; A B is 48 rd. long; B C, 20 rd.; A C, 52 rd.; and the perpendicular from D on A C, 30 rd. Find the value of the field at \$80 an ac. Ans. \$630.

23. A man on a western cattle ranch increased the number of his cattle in four successive years by $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$ and $\frac{1}{9}$ of the number he had at first and at the end of 1st, 2nd and 3rd years respectively. He then had 4000 head. How many had he at first?

$\frac{10}{9}$ of $\frac{9}{8}$ of $\frac{8}{7}$ of $\frac{7}{6}$ of number at first=4000 head. ANS. 2400 head.

24. What is the weight of a circular brass plate 39 in. in diameter and $1\frac{1}{2}$ in. thick if 1 c. in. of brass weighs 5 oz.? ANS. 560 lb. $3\frac{1}{28}$ oz.

25. What is the highest cube number contained in 108086, and what is the least number as a multiplier that will make the given number a perfect cube.

Factors of $108086=17^3\times2\times11$: $\therefore$ $17^3=4913$, the highest cube number, and to make the given number a perfect cube, multiply by $2^2\times11^2$, or 484.

26. Two sums of money are divided among A, B and C; each gets $\frac{1}{3}$ of the first sum; and the 2nd sum is divided in the proportion of 3, 5, 8. A's share is \$53.80 and B's, \$67.80. Find the amount of each sum.

$\frac{1}{3}$ of 1st and $\frac{3}{16}$ of 2nd=\$53.80; $\frac{1}{3}$ of 1st and $\frac{5}{16}$ of 2nd=\$67.80; $\therefore$ 2nd=\$112; 1st=\$98.40.

27. The true discount on a sum of money due in 4 mo. at $4\frac{1}{2}\%$ is \$16. Find the interest on the sum for the same time at the same rate.

Interest on \$16 for 4 mo. at $4\frac{1}{2}\%$=24c.; $\therefore$ interest on the given sum, etc.,=\$16.24.

28. A man invests $\frac{2}{7}$ of his money at 6%; $\frac{2}{5}$ at $4\frac{1}{2}\%$ and the rest at $3\frac{1}{2}\%$. His income is \$1938. How much did he invest at each rate?

6% on $\frac{2}{7}=1\frac{5}{7}\%$ on all; $4\frac{1}{2}\%$ on $\frac{2}{5}=1\frac{4}{5}\%$ on all; $3\frac{1}{2}\%$ on $\frac{11}{35}=1\frac{1}{10}\%$ on all; $\therefore$ $4\frac{43}{70}\%$ of his money is \$1938; $\therefore$ his money=\$42000. 1st=\$12000; 2nd=\$16800; 3rd=\$13200.

29. How deep may a round cistern which is 4 ft. in diameter be made so as to have the same inside surface as a cubical one whose side is 4 ft., and compare their capacities? ANS. 5.3636 ft.; capacity, 1.0536:1.

30. A man invests \$28600, part in 3% stock at 88 and part in 5% railway bonds at $115\frac{1}{2}$. He wishes to get the same income from each investment; how much of each stock must he buy?

\$1 income is received from investing $\$29\frac{1}{3}$ in the first stock or $\$23\frac{1}{10}$ in the second. Hence, \$28600 must be divided in proportion to $29\frac{1}{3}$ and $23\frac{1}{10}$. ANS. $\$18181\frac{9}{11}$ of first and $\$10909\frac{1}{11}$ of second.

31. The weights of equal bulks of gold and silver being as 15:8 and a bar of gold 2 ft. long 2 in. wide and 3 in. thick weighs 100.56 lb.; find the weight of a bar of silver 6 in. long 3 in. wide and $1\frac{1}{2}$ in. thick.

Weight of 1 c. in. of gold is $(100.56 \div 144)$ lb.; weight of 27 c. in. of silver is $[\frac{8}{15}$ of $27 \times (100.56 \div 144)]$ lb., or 10.056 lb.

32. A man bought a farm and paid for it in 4 equal annual payments, including interest at 5%, of \$3241.35 each. What was the cash value of the farm? ANS. \$$11493\frac{2}{8}$.

33. A c. in. of iron weighs 100 dr. I have an iron rod $\frac{1}{4}$ in. in diameter, which weighs 119 dr. Find its length. ANS. $24\frac{64}{275}$ in.

34. The interest on a sum of money for 4 yr. is \$281.25, and the true discount for the same time and rate is \$225. Find the sum and rate %.

Interest=discount+interest on discount; $\therefore$ \$(281.25−225)= \$56.25=interest on discount for 4 yr.; $\therefore$ rate=$6\frac{1}{4}$%; sum \$1125.

35. A man has two chests of mixed green and black teas of 48 lb. and 42 lb. respectively. The proportion of black to green is 13:7 in the first and 18:17 in the second. The two chests are then mixed and 20 lb. green tea added. What will now be the proportions of black and green tea?

$\frac{13}{20}$ of 48 lb.$=31\frac{1}{5}$ black and remainder $16\frac{4}{5}$ green; $\frac{18}{35}$ of $42=21\frac{3}{5}$ black and remainder $20\frac{2}{5}$ green; now $(31\frac{1}{5}+21\frac{3}{5})$ lb.$=52\frac{4}{5}$ lb. black; $(16\frac{4}{5}+20\frac{2}{5}+20)$ lb.$=57\frac{1}{5}$ lb. green; hence, proportion 12:13.

36. If a note for \$11602.50 due in 12 mo. is discounted at 5% true discount, how much more will be paid than if 5% from the whole amount is deducted?

P. W. of 1st=\$11050; P. W. of 2nd=\$11022.375; difference= \$27.625.

37. The accounts of a company show that when 54% of the receipts are used for working expenses, 10% for reserve fund, and 6% preference dividend paid on \$1,000,000 of capital, \$120000 remains which is sufficient to pay $2\frac{1}{2}$% on the rest of the capital. Find capital and receipts.

Working expenses and reserve is 64% of receipts; 6% on \$1000000 is \$60000; hence, the remainder is 36% of the receipts −\$60000; the receipts are $\frac{100}{36}$ of \$(120000+60000), or \$500000; capital is $\frac{100}{2\frac{1}{2}}$ of \$120000+\$1000000, or \$5800000.

38. There are two wells, the area of the tops are 20 sq. ft. and 21 sq. ft. respectively. A hose that fills the first in 4 min. would

take 7 min. to fill the 2nd, and the water that would fill the smaller would fill the larger within 15 ft. of the top. Find the depth of each.

4 times larger=7 times smaller; 4 times smaller+4(15×21) c. ft.=7 times smaller; 3 times smaller=4(15×21) c. ft.; smaller =420 c. ft. and (420÷20) ft.=21 ft. the depth, 2nd is 35 ft. deep.

39. A person finds by investing money in G.T.R. bonds paying \$6 per \$100 share, when they are selling at \$132, that he will obtain \$54 more income than by loaning his money at $3\frac{7}{31}$ per cent. per annum. What amount has he for investment?

\$132 cash gives \$6 income; \$100 cash gives $\$4\frac{6}{11}$ income; a loan of \$100 gives $\$3\frac{7}{31}$ interest; hence, the gain on \$100 invested in bonds is $\$1\frac{109}{341}$; ∴ to gain \$54 he must invest \$4092.

40. The Lehigh train 264 ft. long, going 35 mi. per hr., meets the Atlantic Express, 264 ft. long and passes it at 12 o'clock, in 6 sec. Then $15\frac{1}{20}$ min. later the Lehigh meets the International Limited, 396 ft. long, and passes it also in 6 sec. How long before the International Limited will overtake the Atlantic Express? ANS. $15\frac{1}{20}$ min. past one.

41. A dealer mixes inferior brandy at \$2.88 per gal., with a better quality in the proportion of 3:1, and by selling the mixture at \$3.45 per gal. he realizes 15% gain. Find the price of the good brandy.

If sold at \$3.00 there will be neither gain nor loss; 4 gal. mixture at \$3.00=\$12.00; 3 gal. inferior=3×\$2.88=\$8.64; ∴ price of good brandy=\$3.36.

42. Two rectangular rooms of equal height were papered. The first was 16 ft. long and 14 ft. wide; the second 14 ft. long by 12 ft. wide. The paper was 21 inches wide and cost $5\frac{1}{2}$c. a yd., and the hanging $1\frac{1}{2}$c. a yd. The whole cost was \$20.16. Find the height of the rooms.

$5\frac{1}{4}$ sq. ft. cost 7c.; ∴ 2016c. would cover $\frac{2016}{7}\times5\frac{1}{4}$ sq. ft., or 1512 sq. ft.; 2(16+14)+2(14+12)=112 ft.=distance around both rooms; ∴ height=1512÷112=$13\frac{1}{2}$ ft.

43. A nugget weighing 62 oz. Avoir., cost $\$12\frac{1}{4}$ per oz. After separating the gold from the quartz, it was found that the gold was to the quartz as 21:4. The gold was sold at $\$19\frac{1}{3}$ per oz. Troy. Find the gain or loss.

Cost=62×$\$12\frac{1}{4}$=\$759.50; gold=52.08 oz. Avoir.=$47\frac{15}{32}$ oz. Troy; $47\frac{15}{32}$ oz. @ $\$19\frac{1}{3}$ per oz.=\$917.73; gain=\$(917.73−759.50)=\$158.23.

44. If I buy 50 shares of railway stock at \$82 per share, and 100 more at \$31 per share, and I receive 72c. a share half-yearly as a dividend, what % per annum do I receive?

50 shares at \$82=\$4100; 100 shares at \$31=\$3100; sum invested =\$7200; $\frac{1}{2}$ yearly dividend=150×72c.=\$108; ∴ yearly income= \$216, or 3%.

45. I wish to have an income of \$1840 per annum. I can invest in City Gas Co. stock at 87⅜ and paying 3% dividend, or in Bank stock which is selling at \$233 and paying a dividend of 7⅔% on each share of \$100. How much more would I require to invest in the latter than the former?

\$3 income is got from \$87⅜; \$1840 income is got from \$53590. Again, \$7⅔ income is got from \$233; \$1840 income is got from \$55920. Additional expenditure in latter is \$2330.

46. A body when weighed in one scale of a false balance weighs 81 oz. and in another 87⅑ oz. What would a merchant gain % by selling goods with such a scale?

Real weight=$\sqrt{87\frac{1}{9}\times 81}$=84 oz.; 84 oz. gain (87⅑−84) oz.= 3⅑ oz., or $3\frac{19}{27}$%.

47. A circular yard 240 ft. in diameter has a circular plot in the centre whose radius is 32 yd. A driveway is constructed around the outside of the plot and is 24 ft. wide. Find the cost of construction at 27c. per sq. yd.

Area of drive=$[(40^2-32^2)\times\frac{22}{7}]$ sq. yd.=1810$\frac{2}{7}$ sq. yd.; cost of 1810$\frac{2}{7}$ sq. yd. at 27c. is \$488.77$\frac{5}{7}$.

48. A room is 25 ft. long and 15 ft. wide and has a semicircular bow 20 ft. long thrown out on one side; find the area of the whole room.

Area of rectangular part=375 sq. ft.: area of semicircular part $=\frac{1}{2}(10^2\times\frac{22}{7})$ sq. ft.=157$\frac{1}{7}$ sq. ft.; whole area=(375+157$\frac{1}{7}$) sq. ft. =532$\frac{1}{7}$ sq. ft.

49. Three boys, A, B and C, start in a 3 mi. race; A goes 32 yd. while B goes 30 yd. and C 22 yd. How far from the winning post will B and C be when A reaches it? Ans. B, 330 yd.; and C, 1650 yd.

50. A invests a certain sum in 2½% stock at 90: B invests an equal amount at 89⅜ and his income is \$25 more; find the amount invested by each. Ans. \$128700.

51. An ornament of gold and silver contains 8½c. in. and weighs 64½ oz. A c. in. of gold weighs 10¼ oz. and of silver 5¾ oz. How much gold is there in the ornament?

If all was gold the weight would be $(8\frac{1}{2} \times 10\frac{1}{4})$ oz., or $87\frac{1}{8}$ oz.; the actual weight is $64\frac{1}{2}$ oz.; $(87\frac{1}{8} - 64\frac{1}{2})$ oz. $= 22\frac{5}{8}$ oz. Every oz. of silver reduces the weight $(10\frac{1}{4} - 5\frac{3}{4})$ oz., or $4\frac{1}{2}$ oz.; $\therefore 22\frac{5}{8} \div 4\frac{1}{2} = 5\frac{1}{36}$ = No. of c. in. of silver. Weight of gold is $[(8\frac{1}{2} - 5\frac{1}{36}) \times 10\frac{1}{4}]$ oz., or $35\frac{85}{144}$ oz.

52. A has sugar worth \$4.56 per 112 lb.; B exchanges flour with A for sugar, but gives him only $\frac{19}{21}$ of the weight to which he is entitled. A finding this out raises the sugar accordingly. What price per lb. does he charge B? ANS. $4\frac{1}{2}$c. a lb.

53. A has \$41030 deposited in the bank at 3%. He takes it out to invest it in G.T.R. $3\frac{1}{4}$% stock at $93\frac{1}{4}$, and thus increase his income, but before he has time to do so, the banker offers him $3\frac{1}{4}$%, and the stock rises to $93\frac{1}{2}$. How much more would his income be increased by the former than the latter arrangement?

I. Income from bank = \$1230.90;
Income from G.T.R. stock at $93\frac{1}{4}$ = \$1430.00;
Increase by change = \$(1430 − 1230.90) = \$199.10;
II. Income from stock at $93\frac{1}{2}$ = \$1426.17$\frac{1}{2}$ nearly;
Income from bank at $3\frac{1}{4}$% = \$1333.47$\frac{1}{2}$;
Increase by change to stock = \$92.70;
$\therefore$ gain by 1st over 2nd arrangement = \$(199.10 − 92.70) = \$106.40.

54. A room 44 ft. long, 32 ft. wide has a floor partly covered with carpet, and a margin 4 ft. wide round the outside painted. The carpet and painting together cost \$321.60. The carpet was \$$2\frac{1}{2}$ per sq. yd. Find cost of painting per sq. ft. ANS. 15c.

55. A man bought 126 pigs; he sold $\frac{4}{9}$ of them at 15% profit; $\frac{1}{7}$ at 50% profit, and the remainder at a loss of 25%. His gain altogether was \$19.25. Find the cost of the pigs.

$\frac{4}{9}$ at 15% gain $= 6\frac{2}{3}$% gain on all: $\frac{1}{7}$ at 50% gain $= 7\frac{1}{7}$% gain on all: $\frac{26}{63}$ at 25% loss $= 10\frac{20}{63}$% loss on all: $\therefore$ net gain % $= (6\frac{2}{3} + 7\frac{1}{7} - 10\frac{20}{63})$% $= 3\frac{31}{63}$%: $3\frac{31}{63}$% of cost = \$19.25; cost = \$551.25.

56. Divide \$13200 between John and James so that the simple interest on John's for 3 years at 4% will be equal to that on James' for $7\frac{1}{2}$ yr. at 5%.

John gets $\frac{25}{33}$ and James $\frac{8}{33}$ of the sum; John's, \$10000; James' \$3200.

57. At London the mean annual rainfall is estimated at 33 in.; find the side of a cubical reservoir that would hold all the water that falls on $\frac{3}{10}$ of an acre in one year. ANS. 33 ft.

58. What is the least number when divided by 98 leaves 97 as remainder, and by 35 leaves 34, and by 49 leaves 48.

The L.C.M. of 98, 35, 49 = 490; $\therefore$ 490 − 1 = 489 = No. divided by 98, 35 and 48, will leave the given remainders. ANS. 489.

59. A room is 3 times as long as it is wide, and costs \$39.69 to carpet it at \$1.08 per sq. yd.; the walls are papered at 18c. per sq. yd., and cost \$20.16. Find the dimensions. ANS. $10\frac{1}{2}$ yd. long, $3\frac{1}{2}$ wide, 4 yd. high.

60. A yard measure made of steel expands $\frac{1}{20}$ of an inch in warm weather and contracts $\frac{1}{50}$ in cold weather: what is the true length of a fence which measures $3\frac{1}{2}$ in. more in hot than in cold weather?

$\frac{1}{20}+\frac{1}{50}=\frac{7}{100}$; $\therefore$ $\frac{7}{100}$ of the measure$=3\frac{1}{2}$ in.; $\therefore$ the fence measures 50 yd.

61. A rectangular piece of lead 16 in. long $10\frac{1}{2}$ in. wide and $3\frac{3}{10}$ in. thick is melted. Find how many cubes $2\frac{1}{2}$ in. each way can be made from it and how many $\frac{1}{10}$ in. each way can be made from the remainder? ANS. 35 and 7525.

62. There are 150 bu. of grain in two bins, and in 1 bin there are 15 bu. less than $\frac{4}{7}$ as much as there are in the other. How many bu. are there in the smaller bin?

No. of bu. in larger bin$+\frac{4}{7}$ of No. of bu. in larger bin$=165$ bu. ANS. 45 bu.

63. If 30 c. in. of gunpowder weigh 1 lb., find the diameter of a hollow sphere that will hold 11 lb.

$$\tfrac{4}{3}\times\tfrac{22}{7}\times r^3=330;\ \therefore\ r=4.2863\ldots \quad \text{ANS. } 8.5726\ldots$$

64. A bought 16 cows and 120 sheep for \$465, the animals of the same kind costing a uniform price. He sold them for \$496.50, gaining $7\frac{1}{2}\%$ on the cows and 6% on the sheep. Find the cost of each per head.

Total gain is \$31.50; 6% gain on all is \$27.90. Hence, $1\frac{1}{2}\%$ of value of cows is \$(31.50—27.90), or \$3.60. ANS. \$15.00; \$1.87½.

65. There is a mixture of vinegar and water in the proportion of 93 parts of vinegar and 7 parts of water. How much water must be added so that in 25 qt. of the mixture there may be 2 qt. of water?

$\frac{93}{100}$ of original mixture is 23 qt. Hence, the original mixture was $\frac{2300}{93}$ qt. The new mixture is $\frac{2325}{93}$ qt. Hence, the first mixture has been increased by $\frac{1}{92}$ of itself.

66. I buy two horses for \$225, and sell one so as to lose 4% and the other so as to gain 5%. If on the whole, I neither gain nor lose, what did each horse cost?

4% of price of 1st=5% of price of second; $\therefore$ price of 1st$=\frac{5}{4}$ of price of second. ANS. \$125; \$100.

67. A laborer was engaged at \$1.25 and his board for each day he worked, but was charged 45c. for each day he was idle. At the end of 47 da. he received \$48.55. How many days did he work?

Sum lost by being idle=$(47\times 1.25-48.55)$=\$10.20; sum lost by being idle 1 da.=\$1.70. ANS. 41 da.

68. Divide \$7894.04 among 3 men, 4 women, and 6 children, so that each woman shall have twice as much as a child, and each man 5 times as much as a woman. What is the share of each? ANS. A man, \$1794.10; a woman, \$358.82; a child, \$179.41.

69. A certain number is divided by 9, and the quotient multiplied by 17. The product is then divided by 300, and 33 is added to the quotient. The result is next divided by 3, and to this quotient 19 is added, and the resulting sum divided by $12\frac{1}{2}$. Now $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{5}{8}$ of this last quotient is 3. Find the original number.

Begin with the last result, 3, and work towards the beginning. ANS. 81000.

70. Assuming that an express train runs 40 mi. an hour, and an ordinary train 30 mi. an hour, and that the express fare is $\frac{1}{4}$c. per mile more than the ordinary fare, find how much a man's time is worth, if it costs him the same to travel by one as by the other.

The *extra expense* by travelling 40 mi. by the express train is 10c. The *extra time* to travel 40 mi. by the ordinary train is $\frac{1}{3}$ hr. Hence, value of time per hr. is 30c.

71. At a game of skill A can give B 10 points out of 50, and B can give C 10 points out of 50. How many points can A give C out of 50, and yet be sure to win?

When A makes 50, B makes 40; when B makes 40, C makes 32. Hence, A can give (50—32) points, or 18.

72. The population of a certain town increases at a certain rate per cent. per annum. Now it is 194481. Four years ago it was 160000. What will be the population two years hence?

$160000\times(1+\text{rate})^4=194481$; rate=5%. ANS. 214415.

73. Gold weighs 19.3 times, and copper 8.89 times as much as water. How many times as heavy as water is a coin containing 11 parts of gold, and 1 part of copper.

No. of times=$(11\times 19.3+1\times 8.89)\div 12=18.4325$.

74. The true discount on a sum of money for one year @ 5% is \$1 greater than the sum of the true discounts of one-half of it @ 4%, and the other half @ 6%. Find the sum.

$\frac{1}{21}$ of sum $=\frac{1}{26}$ of half sum $+\frac{3}{53}$ of half sum $+\$1$; hence. $(\frac{1}{21}-\frac{1}{52}-\frac{3}{106})$ of sum $=\$1$; sum is \$11575.20.

75. My purse and the money in it are together worth \$66.50. If I spend $12\frac{1}{2}\%$ of my money, and sell the purse for $\frac{1}{2}$ its value, I shall then have \$57.25. Find the value of my purse.

$\frac{7}{8}$ money and $\frac{1}{2}$ value of purse are worth \$57.25; hence, $\frac{1}{8}$ money and $\frac{1}{2}$ value of purse are worth \$9.25; hence, money and 4 times value of purse are worth \$74; 3 times value of purse is worth \$7.50. ANS. \$2.50.

76. A man purchased a farm for \$3000, and agreed to pay principal and interest in 4 equal annual instalments. Interest being 6% per annum, what was the annual payment?

\$1 paid yearly amounts to $\$\frac{(1.06)^4-1}{1.06-1}$, or \$4.3746;

Debt amounts to $\$[3000\times(1.06)^4]$, or \$3787.4307; annual payment is $\$(3787.4307\div4.3746)$, or \$865.77.

77. A cube of lead, whose edge is 10 in., is wholly immersed in a cylindrical tub partly filled with water, whose internal diameter is 20 in. How much will it raise the water in the tub?

Area of bottom of tub $=(\frac{22}{7}\times100)$ sq. in.; hence, depth the water will rise is $[1000\div(\frac{22}{7}\times100)]$ in., or $3\frac{2}{11}$ in.

78. A, B and C form a company. A's stock is in trade 3 mo. and he claims $\frac{1}{12}$ of the gain; B's stock is 9 mo. in trade; and C advances \$3024 for 4 mo., and claims half the profit. How much do A and B contribute?

The profits are $\frac{1}{12}$, $\frac{5}{12}$ and $\frac{6}{12}$, respectively; $\therefore$ A's stock $=\$\frac{3024\times4}{6\times3}=\$672$; B's stock $=\$\frac{672\times3\times5}{9}=\$1120$.

79. A tea merchant mixes 40 lb. of tea @ 45c. per pound, with 50 lb. @ 27c. per pound, and sells the mixture @ 45c. per pound, What per cent. profit does he make? ANS. $28\frac{4}{7}$.

80. Divide \$2400 into two parts, such that the annual interest on the first part @ $3\frac{1}{2}\%$, added to that on the second @ $6\frac{1}{2}\%$ may equal that on the whole sum @ $5\frac{1}{4}\%$.

$5\frac{1}{4}\%-3\frac{1}{2}\%$ is $1\frac{3}{4}\%$; hence, $1\frac{3}{4}\%$ of all is 3% of second part. ANS. \$1000, \$1400.

81. A boat's crew can row 15 mi. down stream in the same time that they can row 7 mi. up it. If their rate of rowing is $5\frac{1}{4}$ mi. an hr. in still water, what is the velocity of the stream?

The rate in *still water* is to the rate of *the stream* as 11 to 4; hence, rate of stream is $\frac{4\times5\frac{1}{4}}{11}$ mi. per hr., or $1\frac{10}{11}$ mi.

82. The three per cents. are @ $91\frac{1}{2}$, and the three-and-one-half per cents. @ $96\frac{1}{2}$. A person has a sum to invest which will give him £100 more of the former stock than of the latter. Find the difference of income he could obtain by investing in the two stocks.

$$\frac{100\times\text{sum}}{91\frac{1}{2}}-\frac{100\times\text{sum}}{96\frac{1}{2}}=£100.\quad \text{Ans. } £6.15.$$

83. A conical tent, whose slant height is 12 ft., requires 66 sq. yd. of canvas to make it. How much ground does the floor of the tent cover? **Ans. $779\frac{5}{8}$ sq. ft.**

84. I buy two horses for \$260. If I sell the first at a gain of 5%, and the second at a loss of 5%, I would gain $\frac{5}{13}$%. What was the price of each horse?

My gain is $\frac{5}{1300}$ of \$260, or \$1. Hence, $\frac{5}{100}$ of cost of first$-\frac{5}{100}$ of (\$260—cost of first)=\$1. **Ans. \$140 and \$120.**

85. An agent sold 10 mowing machines @ \$80 each, and 15 @ \$100 each. He paid \$85 for transportation, and, after deducting his commission, remitted \$2100 to his employer. What was the rate of his commission? **Ans. 5%.**

86. A train, 352 ft. long, overtakes a man walking in the same direction, at the rate of 4 mi. per hr., and passes him completely in 15 sec. When going at the same rate, the train passes another man, walking in the opposite direction, in 9 sec. At what rate is the second man walking?

In 15 sec. the first man goes 88 ft.; hence, the train goes 440 ft. in 15 sec., or 264 ft. in 9 sec. In 9 sec. the second man must go (352—264) ft. **Ans. $6\frac{2}{3}$ mi. per hr.**

87. A lady wishing to give 25c. each to some poor women, found she had not money enough by 15c. She gave each of them 20c., and had 35c. left. How much money had she?

(25—20)c. less to each woman, was (15+35)c. less on the whole. Hence, there were $\frac{50}{5}$ women. Her money was $(10\times25-15)$c., or \$2.35.

88. From the first of two cannon, 36 shells are thrown before the second is ready for firing. Shells are thrown from both in the proportion of 8 from the first to 7 from the second. The second requires as much powder for 3 charges as the first does for 4. After how many shots will the quantity of powder consumed by the second equal that consumed by the first?

$1\frac{1}{3}$ charges of the first are gained every 7 shots of the second, and 36 charges of the first are gained every 189 shots of the second.

While the second is firing 189 shots the first fires 216. ANS. 36+216+189, or 441.

89. How many bricks, each 9 in. long, $4\frac{1}{2}$ in. broad and 4 in. thick, will be required for a wall 80 ft. long, 12 ft. high and $2\frac{1}{2}$ ft. thick, allowing $6\frac{1}{4}$% of the space for the mortar? ANS. 24000.

90. A farm cost $3\frac{3}{4}$ times as much as a house. By selling the house at a loss of 10%, and the farm at a gain of $7\frac{1}{7}$%, \$3993.30 is received. Find the cost of each.

$\frac{9}{10}$ of cost of house$+\frac{75}{70}$ of $\frac{15}{4}$ of cost of house$=$\$3993.30. ANS. House, \$812; farm, \$3045.

91. Calculate correctly to 7 places of decimals the value of $\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\frac{1}{1.2.3.4.5}=$&c.

$$\frac{1}{1.2}=.50000000$$
$$\frac{1}{1.2.3}=.16666666$$
$$\frac{1}{1.2.3.4}=.04166666$$
$$\frac{1}{1.2.3.4.5}=.00833333$$
$$\frac{1}{1.2.3.4.5.6}=.00138888$$
$$\frac{1}{1.2.3.4.5.6.7}=.00019841$$
$$\frac{1}{1.2.3.4.5.6.7.8}=.00002480$$
$$\frac{1}{1.2.3.4.5.6.7.8.9}=.00000275$$
$$\frac{1}{1.2.3.4.5.6.7.8.9.10}=.00000027$$
$$\frac{1}{1.2.3.4.5.6.7.8.9.10.11}=.00000002$$
$$\overline{.7182818}$$

92. In firing at a target A hits it twice out of three shots, B three times out of four, and C four times out of five. The target was hit 931 times. If each fired the same number of shots, find how many hits each made and the total number of shots fired.

Out of 60 shots by A there are 40 hits; by B, 45 hits; by C, 48 hits. There are 133 hits out of 180 shots. But there are 7 times 133 hits, hence there are 7 times 180 shots, or 1260, and A hits it 280 times, B 315 times, and C 336 times.

93. The money deposited in a savings bank during the year 1897 was 5% more than that deposited in 1896. In 1898 the deposits were $33\frac{1}{3}\%$ greater than in 1897, while the amount deposited in 1899, exceeded the average of the three previous years by 20%. The aggregate for the four years was \$150937.50. Find the amount deposited in each year.

Sum deposited in 1896$+\frac{21}{20}$ of this sum$+\frac{28}{20}$ of this sum$+\frac{6}{5}$ of $\frac{23}{20}$ of this sum$=$\$150937.50. Ans. \$31250; \$32812.50; \$43750; \$43125.

94. A compound of tin and lead weighs 10.43 times as much as an equal bulk of water, while tin weighs 7.44 times, and lead 11.35 times as much as equal bulks of water. Find the number of pounds of each metal in 765 lb. of the compound.

7.44 times the No. of lb. of tin$+$11.35 times (765$-$the No. of lb. of tin)$=$10.43 times 765. Ans. 180 lb. tin and 585 lb. lead.

95. A circular race-course 22 yd. wide covers 12 acres. Find the diameter of the inner circle. Ans. 818 yd.

96. On March 23rd a bank gave me \$845 for a note of \$860. Interest being charged at 8%, when was the note due?

The time for which \$15 is the interest on \$860 is 80 days. Ans. June 11th.

97. The map of a country is drawn on a scale of $\frac{1}{12}$ of an inch to a mile. What area on the map will represent 36000 ac.?

36000 a. is represented by $(\frac{36000}{640}\times\frac{1}{144})$ sq. in. Ans. $\frac{25}{64}$ sq. in.

98. A number of men can be formed either into a solid square, or into a hollow square 9 deep having 970 men in the front rank of each side. How many men are there in each side of the solid square?

$$\text{Number}=\sqrt{\{9\times[2\times970+2(970-18)]\}}=186.$$

99. There are three kinds of tea, valued respectively at 32c., 36c., and 42c., per lb. A mixture is to be made containing 6 lb. more of the second than of the first, and 8 lb. more of the third than of the first. What total quantity must be taken that the value of the mixture may be 38c. a lb.?

	38	
6	32	1+4=5
2	36	7+4=11
4	42	9+4=13

If there is 1 lb. at 32c., there must be 7 lb. at 36c. and 9 lb. at 42c. These give a mixture in which the gain is greater than the losses by 16c. The difference between the gains and losses by taking 1 lb. of each kind is 4c. Hence, 4 lb. must be taken to equalize this difference. Hence, there are 29 lb. in the mixture.

100. Two cogged wheels work together, there being 16 cogs on one and 27 on the other. The larger wheel makes 80 revolutions in $\frac{3}{4}$ of a minute. How often does the smaller turn in 8 seconds?

$$\text{No. of revolutions}=\tfrac{27}{16}\times\frac{8\times 80}{45}=24.$$

101. A box with a lid is made of material 1 inch thick. If the internal dimensions are 3 ft. by 2 ft. by 1 ft. 5 in. find the number of cubic inches of the material.

Cubic content of interior of box$=(36\times24\times17)$ c. in.$=14688$ c. in.; cubic content of entire box$=(38\times26\times19)$ c. in.$=18772$ c. in. ANS. 4084 c. in.

102. The French 20-franc piece weighs 6.45161 grams (a gram $=15.43235$ grains) and is $\frac{9}{10}$ pure gold. The sovereign is $\frac{11}{12}$ fine, weighs 123.274 grains, and is worth \$4.8665. How much is the 20-franc piece worth?

Value of ($\frac{11}{12}$ of 123.274) gr. of pure gold$=$\$4.8665; weight of pure gold in 20 fr.$=$($\frac{9}{10}$ of 6.45161×15.43235) gr. ANS. \$3.85 . . .

103. If 76 men and 59 boys can do as much work in 299 days as 40 men and 33 boys can do in 557 days, how many men will do as much work in a day as 15 boys?

The work of (299×76) men and (299×59) boys equals the work of (557×40) men and (557×33) boys. Hence, the work of 15 boys$=\frac{15\times444}{740}$ men$=9$ men.

104. Find the cost of polishing a marble pillar 2 ft. 4 in. in diameter, and 18 ft. long, at \$1.25 per sq. yd. ANS. \$18$\frac{1}{3}$.

105. A bought goods at \$5.70 on 4 months' credit, and sold them immediately at \$6.12 on such a term of credit as made his immediate gain $6\frac{2}{3}$%. Reckoning interest at 4% per annum, how long credit did A give?

If cash had been paid, the goods would have been sold for \$6.08; time for 4c. interest on \$6.08 at 4% is 60 days.

106. A bar of gold weighing 196 lb. 10 oz. 10 dwt. is cased in lead weighing 24 lb. 14 oz. Find the weight of the whole in avoirdupois.

196 lb. 10 oz. 10 dwt.$=$1134000 gr.$=$162 lb. ANS. 186.875 lb.

107. A person having a certain sum of money to invest finds that an investment in 5 per cent. stock at $117\frac{3}{8}$ will yield him \$29 more interest annually than an investment in 3 per cent. stock at $92\frac{1}{8}$. Brokerage being $\frac{1}{8}$%, how much money has he to invest?

$$\frac{5\times\text{sum}}{117\frac{1}{2}}-\frac{3\times\text{sum}}{92\frac{1}{4}}=\$29.\quad \text{ANS. } \$2890.50.$$

108. A bought 200 m. of cloth in France at $16\frac{1}{4}$ fr. a m.; he paid $12\frac{1}{2}$c. a yd. for duty and freight, and sold it in Toronto at $4.62\frac{1}{2}$ a yd. How much did he gain, a franc being $19\frac{1}{3}$c.?

$$\text{Cost of cloth} = (200 \times 16\tfrac{1}{4} \times 19\tfrac{1}{3})\text{ c.} \quad = \$628.33\tfrac{1}{3};$$

$$\text{Duty, etc.} = \left(\frac{200 \times 39.375}{36} \times 12\tfrac{1}{2}\right)\text{c.} \quad = \$27.34\tfrac{3}{8};$$

$$\text{Selling price} = \left(\frac{200 \times 39.375}{36} \times 462\tfrac{1}{2}\right)\text{c.} \quad = \$1011.71\tfrac{21}{24};$$

$$\text{Gain} = \$356.04\tfrac{1}{8}.$$

109. Each member of a pedestrian club walks as many miles as there are members in the club, and the expense of the trip is for each member as many cents per mile as there are members in the club. The total expense is $973.36. How many members are there in the club? ANS. 46.

110. A steel rod 1 ft. long and 1 in. square, weighs $3\frac{1}{2}$ lb. and will just support 50 t. What is the greatest length of steel wire, when hung up by one end, will just not break by its own weight?

$$\text{Length} = \frac{24 \times 50 \times 2000}{7}\text{ in.} = 5\tfrac{95}{231}\text{ mi.}$$

111. The sum of $540 is to be divided among 200 persons, consisting of men, women and children. The sums of the men's, women's and children's shares are proportional to the numbers 5, 4, and 3, but the individual shares of a man, woman, and child are proportional to the numbers 3, 2, 1. Find the number of men, women and children.

The men receive $225, the women, $180; and the children, $135. $\frac{225}{3} + \frac{180}{2} + \frac{135}{1} = 300$. The number of men is $\frac{2}{3}$ of 75; of women, $\frac{2}{3}$ of 90; of children, $\frac{2}{3}$ of 135.

112. One vessel contains 10 gal. of wine; another contains 10 gal. of water. 1 gal. is taken from each, and is then poured into the other. This is done 3 times. How much wine and how much water will the vessels then respectively contain?

At the end of the first pouring, the first contains $\frac{9}{10}$ of 10 gal. of wine; at the end of the second, $\frac{9}{10}$ of $\frac{9}{10}$ of 10 gal.; and the end of the third, $\frac{9}{10}$ of $\frac{9}{10}$ of $\frac{9}{10}$ of 10 gal., or 7.29. ANS. 7.29 gal. wine; 2.71 gal. water; and 7.29 gal. water and 2.71 gal. wine.

113. The sum of the ages of A and B is now 60 years, and 10 years ago their ages were as 5 to 3. Find their present ages.

Ten years ago the sum of their ages was 40 yr.; A's age was $\frac{5}{8}$ of 40, or 25; present age is 35.

114. A sum of money is put out at compound interest for 4 yr. During the first two years interest is paid at the rate of 4% per annum, but during the last two years at 5% per annum. The total amount at the end of that time was \$74529. What was the sum invested?

Sum $\times (1.04)^2 \times (1.05)^2 =$ \$74529. ANS. \$62500.

115. A bought $2\frac{3}{4}$% stock at 95. He sold it and with the proceeds bought $3\frac{1}{2}$% stock. He obtained \$900 less stock than before, but the same income. How much money did he originally invest?
$\frac{11}{400}$ of 1st stock $= \frac{14}{400}$ of second; 1st stock $= \frac{14}{11}$ of (1st stock $-$ \$900).
ANS. \$3990.

116. A rectangular solid is hammered until its length is increased 10 per cent., and its width 15 per cent. By how much per cent. has its thickness been diminished?

New thickness is $\frac{100}{110}$ of $\frac{100}{115}$ of old thickness; diminution is $\frac{53}{253}$ of old thickness, or 20.94 . . .%.

117. A founder is required to supply a ton of fusible metal consisting of 8 parts by weight of bismuth, 5 of lead, and 3 of tin. The only bismuth he has in stock is an alloy consisting of 9 parts bismuth, 4 lead, and 3 tin. How much alloy must he take and how much lead and tin must he add to fill the order?

The fusible is to have $\frac{1}{2}$ t. bismuth; $\frac{5}{16}$ t. lead and $\frac{3}{16}$ t. tin. It is evident $\frac{8}{9}$ t. will supply the weight of bismuth; but $\frac{8}{9}$ t. alloy supplies $\frac{2}{9}$ t. lead and $\frac{3}{18}$ t. tin; hence, $(\frac{5}{16}-\frac{2}{9})$ t. lead, or $180\frac{5}{9}$ lb. lead and $(\frac{3}{16}-\frac{3}{18})$ t. tin, or $41\frac{2}{3}$ lb. must be taken.

118. If the manufacturer makes a profit of 10%, the wholesale dealer one of 15%, and the shopkeeper one of 20%, what was the cost to the manufacturer of an article bought at a shop for \$75.90?

$\frac{120}{100}$ of $\frac{115}{100}$ of $\frac{110}{100}$ of cost $=$ \$75.90. ANS. \$50.

119. A train 160 yd. long is overtaking another 150 yd. long on a parallel track. The first is going at the rate of 40 miles an hour and the second at the rate of 18. Find how long a person in the long train will see the other while passing it.

$$\text{Time} = \frac{150 \times 60 \times 60}{22 \times 1760} \text{ sec.} = 13\frac{229}{242} \text{ sec.}$$

120. A trader in London owes a debt of 1000 pistoles to one in Cadiz. Find what he gains by sending it to him through France, the exchanges being £1 = 25.4 francs; 19 francs = 1 Spanish pistole; 4 Spanish pistoles = £3.

By direct exchange 1000 pistoles=£750; by indirect exchange they=£748 0s. $7\frac{71}{127}$d. Ans. £1 19s. $4\frac{56}{127}$d.

121. If 60 c. in. of lead, together with 54 c. in. of cork are equal in weight to $1538\frac{8}{9}$ c. in. of fir, and the weights of equal bulks of lead and fir are represented by the numbers 11.324 and .45, what number represents the weight of an equal bulk of cork?

$$60\times 11.324+54\times \text{No.}=1538\tfrac{8}{9}\times .45.\quad \text{Ans. } .24.$$

122. A cistern containing 600 gal. measures externally 7 ft. in length, 2 ft. 11 in. in breadth and 5 ft. 6 in. in height. The sides being $1\frac{1}{2}$ in. thick, find the thickness of the bottom.

Space occupied by water is $\frac{6000\times 1728}{62\frac{1}{2}}$ c. in. and the depth of the water is $\frac{6000\times 1728}{81\times 32\times 62\frac{1}{2}}$ in., or 64 in. Ans. 2 in.

123. Two trains 92 ft. and 84 ft. long, respectively, when moving with uniform velocities on parallel rails in opposite directions pass each other in $1\frac{1}{2}$ sec., but when they move in the same direction the faster passes the other in 5 sec. Find the rates of speed in mi. per hr.

$$\text{Sum of rates per hr.}=\frac{60\times 60\times 176}{5280\times 1\frac{1}{2}}\text{ mi.}=80\text{ mi.};$$

$$\text{Difference `` `` `` ``}=\frac{60\times 60\times 176}{5280\times 5}\text{ mi.}=24\text{ mi.}$$

Ans. 52 mi. and 28 mi.

124. The capital of a railway company is \$18500000. It has borrowed \$11500000 at $4\frac{1}{4}$% per annum. Its receipts for the year are \$3175000. Its working expenses are 45% of the receipts. What rate of dividend will the shareholders receive for the year, \$55000 being retained as reserve fund?

Cash for dividend is \$(55% of 3175000—$4\frac{1}{4}$% of 11500000—55000). Ans. $6\frac{1}{2}$%.

125. A can reap a field in 15 days, and B in 12 days. The former works a certain time, and is joined by the latter, the whole work being completed in $7\frac{1}{2}$ days. How long did they work together?

A works all the time and does $\frac{1}{2}$ the work; B does the other half in 6 da. Ans. 6 da.

126. A travels 60 mi. in 3 hr., partly by stage and partly by rail. Had he gone all the way by rail he would have ended the journey an hour sooner and saved $\frac{2}{5}$ of the time he was on the stage. How far did he go by stage?

Rate by rail is 30 mi. per hr.; the time on the stage is $2\frac{1}{2}$ hr., and that on train $\frac{1}{2}$ hr.; distance travelled in train is 15 mi. ANS. 45 mi.

127. A vessel in the shape of a conic frustum is to hold $3\frac{1}{2}$ gal. If the diameters of its ends are 10 in. and 6 in. respectively, find the height of the frustum, the gal. being 277.274 c. in.

$\frac{h}{3}(\frac{22}{7}\times 25+\frac{22}{7}\times 9+\frac{22}{7}\times 15)=3\frac{1}{2}\times 277.274.$ ANS. 18.91 in. nearly.

128. The length, breadth and depth of a rectangular wooden box with a lid, are 16 in., 12 in. and 8 in. respectively, and the thickness of the wood is $\frac{1}{2}$ in. When the box is empty it weighs 3 lb., and when filled with sand it weighs 25 lb. Compare the weight of wood and sand.

Cubic content of box is 1536 c. in., and of sand is 1155 c. in.; hence, 381 c. in. of wood weigh 3 lb., and 1155 c. in. of sand 22 lb. ANS. 105 to 254.

129. A man running at the rate of 10 mi. per hr. towards a point where minute-guns are discharging, finds that 18 min. 45 sec. elapse between his hearing the 1st and 20th reports. Find the velocity of sound.

The man's rate per min. is 880 ft. In $18\frac{3}{4}$ min. he runs 16500 ft. This is the distance travelled by sound in 15 sec. ANS. 1100 ft.

130. A builder sells a house to B for \$2430 at a loss of 19%. B disposes of it to C at a price that would have given the builder 17% gain. Find B's gain.

The house cost the builder \$3000 and B's gain is \$(3510—2430). ANS. \$1080.

131. A lends B a certain sum. At the same time he insures B's life for \$2950.50, paying annual premiums of \$80. At the end of 3 yr., and, just before the fourth premium is to be paid, B dies, having never repaid anything. What sum must A have lent B in order that he may just have enough to recoup himself, together with 5% compound interest on the sum lent and on the premiums paid?

Sum $(1.05)^3+80[(1.05)^3+(1.05)^2+1.05]=2950.50$. ANS. \$2320.

132. Prove that the difference of any two numbers composed of the same digits is divisible by 9.

If a, b, c, d, etc., are digits of a number the number may be $a+10b+100c+1000d$, or $10a+100b+1000c+d$. The difference, $-9a-90b-900c+999d$ is evidently divisible by 9. The same may be shown for any other arrangement of the digits.

133. An agent sold a consignment of lumber for \$4056 and invested the proceeds, less his commission, in sugar. His total commission on the two transactions amounted to \$312. What rate did he charge, the rates on both sale and purchase being the same?

Commission on \$[4056+(4056−312)] is \$312. Ans. 4%.

134. A wholesale merchant allows three equal discounts on purchases. The three are equivalent to a single discount of 48.8%. Find the rate of discount allowed each time.

$$\left(1-\frac{\text{rate}}{100}\right)^3=\tfrac{512}{1000}. \quad \text{Rate}=20\%.$$

135. I bought through a broker in Toronto a bill of exchange on Glasgow for £600, paying the broker \$2915.27 for it. At what quotation was the bill purchased, allowing $\frac{1}{4}$% for brokerage?

Cost of bill of £601$\frac{1}{2}$ is \$2915.27. Ans. \$4.84$\frac{5}{8}$.

136. A person shooting at a target, distant 550 yards, hears the bullet strike the target four seconds after he fires. A spectator, equally distant from the target and the shooter, hears the shot strike the target two and a half seconds after he heard the report. Find the velocity of sound.

Time for ball to go to target and of sound to return to shooter is 4 sec. Time for ball to go to target and for sound to return half distance=time for sound to come from shooter and 2$\frac{1}{2}$ sec. Hence, time for ball to go to target is 2$\frac{1}{2}$ sec. Hence, sound travels the 550 yd. in 1$\frac{1}{2}$ sec. Ans. 1100 ft. per sec.

137. In building a house, A paid 2$\frac{1}{2}$ times as much for material as for labor. If the latter had cost 8% more, and the former 10% more, the whole cost would have been \$2872.50. Find the actual cost.

$\frac{108}{100}$ of cost of labor+$\frac{110}{100}$ of 2$\frac{1}{2}$ times cost of labor is \$2872.50. Ans. \$750; \$1875.

138. A starts to walk from P to Q at the rate of 4 mi. an hr., and 1 hr. afterwards B starts from P and overtakes A in 4 hr. Walking on B arrives at Q 2 hr. before A. Find the distance from P to Q.

B walks at the rate of 5 mi. per hr. In 4 hr. he goes 20 mi. To gain 2 hr. on A he must go 40 mi. Ans. 60 mi.

139. A is 12 yr. older than B. Should they both live 8 yr. longer, B's age will then be $\frac{7}{10}$ of A's. Find the present age of each.

A's age 8 yr. hence is to B's at that time as 10 is to 7. Hence, A's age:(A's−12)::10:7. A will then be 40 yr. Ans. 32 yr. and 20 yr.

140. An 8 gal. cask is full of brandy, and a 12 gal. cask is full of water. How much must be transferred from each cask to the other that the mixture may be of equal strength?

The contents of each cask must be $\frac{2}{5}$ brandy and $\frac{3}{5}$ water. Hence, $\frac{3}{5}$ of 8 gal. of brandy must be taken out. ANS. $4\frac{4}{5}$ gal.

141. A contractor undertakes to execute a certain work in a given time. He employs 55 men, who work 9 hr. daily. When $\frac{4}{5}$ of the time has expired, he finds that only $\frac{3}{7}$ of the work is done. How many men must he now employ 11 hr. a day to fulfil his contract?

The problem is this:—55 men working 9 hr. daily do $\frac{3}{7}$ of a work in $\frac{4}{5}$ of a time; how many men working 11 hr. daily can do $\frac{4}{7}$ of the work in $\frac{1}{5}$ of the time? ANS. 180.

142. Three men form a partnership, with a capital of \$3200. A's stock was in trade 6 mo., B's 12 mo., and C's 15 mo., and at the settling of accounts, A receives \$240 of the gain, B \$800 and C \$400. Find each person's stock.

A's stock earns \$40 per mo.; B's, \$66$\frac{2}{3}$, and C's, \$26$\frac{2}{3}$. Hence, A has $\frac{40}{133\frac{1}{3}}$ of the capital; etc. ANS. A, \$960; B, \$1600; C, \$640.

143. In a pond the top of a plant was 9 in. above the water but forced by the wind the plant leaned over and was submerged at a distance of 3 ft. from where it stood erect. Find the depth of the water.

$(\text{Measure of depth}+9)^2-36^2=(\text{Measure of depth})^2$. ANS. $67\frac{1}{2}$ in.

144. A merchant bought 500 lb. of sugar and 80 lb. of tea. The cost of the sugar per lb. was $16\frac{2}{3}\%$ that of the tea. He sold the tea at a loss of $8\frac{1}{3}\%$ and the sugar at a gain of 25% and gained \$4.25 on the whole transaction. Find the selling prices per lb.

80 lb. of tea=480 lb. of sugar; $91\frac{2}{3}\%$ of 480 times cost of 1 lb. sugar+125% of 500 times cost of 1 lb. sugar=980 times cost of 1 lb. sugar+\$4.25. ANS. $6\frac{1}{4}$c.; $27\frac{1}{2}$c.

145. A railroad train travels $\frac{1}{3}$ of the distance at the rate of 30 mi. per hr., the next $\frac{1}{3}$ of the distance at the rate of 35 mi. per hr., and the remaining distance at the rate of 40 mi. per hr. What is the average rate in mi. per hr.?

If the distance is 3 mi. and each section 1 mi., it will require $(\frac{1}{30}+\frac{1}{35}+\frac{1}{40})$ of an hr. to run 3 mi.; $3\div(\frac{1}{30}+\frac{1}{35}+\frac{1}{40})=34\frac{38}{73}$. ANS. $34\frac{38}{73}$ mi.

146. How may 7961 pounds be weighed with the weights, 1 lb., 3 lb., 9 lb., 27 lb., etc., using only one of each kind?

3)7961
3)2654—1
3)885—1
3)295 0
3)98 1
3)33—1
3)11 0
3)4—1
1 1

Put 1 lb., one 3 lb. wt., one 3^4 lb. wt. and one 3^6 lb. wt. along with the weight and one 3^3 lb. wt., one 3^7 lb. wt., and one 3^8 lb. wt. to balance these.

147. A number of two digits is multiplied by 3, and the product placed to the left of the original number. Show that the number so formed is always exactly divisible by 7.

If $10a+b$ represent the number, then $3000a+300b+10a+b$ will represent the new number. It is evident that this is exactly divisible by 7.

148. A number of men and women earned $93 a day, each man getting $2.25, and each woman $1.50. Had there been 6 more men and 7 more women the whole number of women would have earned the same as the whole number of men. Find the actual number of each.

With the additional number of men and women the total sum earned would be $117. The men earn half of this. Hence, number of men is $58\frac{1}{2}-2\frac{1}{4}$, or 26, and number of women is $58\frac{1}{2}\div1\frac{1}{2}$, or 39. ANS. 20 and 32.

149. A ditch has to be made 360 ft. long, 10 ft. wide at the top and 3 ft. wide at the bottom, the angle of the slope of each side being 45°. Find the number of c. yd. to be excavated.

By drawing a diagram of a cross section, it will be seen that the depth will be $3\frac{1}{2}$ ft.; No. of c. yd$=\left(360\times\frac{10+3}{2}\times3\frac{1}{2}\right)\div27=303\frac{1}{3}$.

150. A dealer bought 100 animals for $100; calves @ $2½, lambs @ $1½, and turkeys @ $½. How many animals were there of each kind?

Diffs.	1		
1½	2½	2:0	2
½	1½	0:1	1
½	½	6:1	7

2 calves and 6 turkeys average $1 each; 1 lamb and 1 turkey average $1 each. Hence, 2 calves, 1 lamb and 7 turkeys average $1 each. Hence, as 100 animals are required, 20 calves, 10 lambs and 70 turkeys will give this average.

Other solutions as 17 calves, 16 lambs and 67 turkeys may be found.

PART V. A TABLE OF SQUARES, CUBES, AND ROOTS.

No.	Square	Cube.	Square Root.	Cube Root.	No.	Square.	Cube.	Square Root.	Cube Root.
1	1	1	1·0000000	1·000000	64	4096	262144	8·0000000	4·000000
2	4	8	1·4142136	1·259921	65	4225	274625	8·0622577	4·020726
3	9	27	1·7320508	1·442250	66	4356	287496	8·1240384	4·041240
4	16	64	2·0000000	1·587401	67	4489	300763	8·1853528	4·061548
5	25	125	2·2360680	1·709976	68	4624	314432	8·2462113	4·081656
6	36	216	2·4494897	1·817121	69	4761	328509	8·3066239	4·101566
7	49	343	2·6457513	1·912931	70	4900	343000	8·3666003	4·121285
8	64	512	2·8284271	2·000000	71	5041	357911	8·4261498	4·140818
9	81	729	3·0000000	2·080084	72	5184	373248	8·4852814	4·160168
10	100	1000	3·1622777	2·154435	73	5329	389017	8·5440037	4·179339
11	121	1331	3·3166248	2·223980	74	5476	405224	8·6023253	4·198336
12	144	1728	3·4641016	2·289428	75	5625	421875	8·6602540	4·217163
13	169	2197	3·6055513	2·351335	76	5776	438976	8·7177979	4·235824
14	196	2744	3·7416574	2·410142	77	5929	456533	8·7749644	4·254321
15	225	3375	3·8729833	2·466212	78	6084	474552	8·8317609	4·272659
16	256	4096	4·0000000	2·519842	79	6241	493039	8·8881944	4·290841
17	289	4913	4·1231056	2·571282	80	6400	512000	8·9442719	4·308870
18	324	5832	4·2426407	2·620741	81	6561	531441	9·0000000	4·326749
19	361	6859	4·3588989	2·668402	82	6724	551368	9·0553851	4·344481
20	400	8000	4·4721360	2·714418	83	6889	571787	9·1104336	4·362071
21	441	9261	4·5825757	2·758924	84	7056	592704	9·1651514	4·379519
22	484	10648	4·6904158	2·802039	85	7225	614125	9·2195445	4·396830
23	529	12167	4·7958315	2·843867	86	7396	636056	9·2736185	4·414005
24	576	13824	4·8989795	2·884499	87	7569	658503	9·3273791	4·431047
25	625	15625	5·0000000	2·924018	88	7744	681472	9·3808315	4·447960
26	676	17576	5·0990195	2·962496	89	7921	704969	9·4339811	4·464745
27	729	19683	5·1961524	3·000000	90	8100	729000	9·4868330	4·481405
28	784	21952	5·2915026	3·036589	91	8281	753571	9·5393920	4·497941
29	841	24389	5·3851648	3·072317	92	8464	778688	9·5916630	4·514357
30	900	27000	5·4772256	3·107232	93	8649	804357	9·6436508	4·530655
31	961	29791	5·5677644	3·141381	94	8836	830584	9·6953597	4·546836
32	1024	32768	5·6568542	3·174802	95	9025	857375	9·7467943	4·562903
33	1089	35937	5·7445626	3·207534	96	9216	884736	9·7979590	4·578857
34	1156	39304	5·8309519	3·239612	97	9409	912673	9·8488578	4·594701
35	1225	42875	5·9160798	3·271066	98	9604	941192	9·8994949	4·610436
36	1296	46656	6·0000000	3·301927	99	9801	970299	9·9498744	4·626065
37	1369	50653	6·0827625	3·332222	100	10000	1000000	10·0000000	4·641589
38	1444	54872	6·1644140	3·361975	101	10201	1030301	10·0498756	4·657010
39	1521	59319	6·2449980	3·391211	102	10404	1061208	10·0995049	4·672329
40	1600	64000	6·3245553	3·419952	103	10609	1092727	10·1488916	4·687548
41	1681	68921	6·4031242	3·448217	104	10816	1124864	10·1980390	4·702669
42	1764	74088	6·4807407	3·476027	105	11025	1157625	10·2469508	4·717694
43	1849	79507	6·5574385	3·503398	106	11236	1191016	10·2956301	4·732624
44	1936	85184	6·6332496	3·530348	107	11449	1225043	10·3440804	4·747459
45	2025	91125	6·7082039	3·556893	108	11664	1259712	10·3923048	4·762203
46	2116	97336	6·7823300	3·583048	109	11881	1295029	10·4403065	4·776856
47	2209	103823	6·8556546	3·608826	110	12100	1331000	10·4880885	4·791420
48	2304	110592	6·9282032	3·634241	111	12321	1367631	10·5356538	4·805896
49	2401	117649	7·0000000	3·659306	112	12544	1404928	10·5830052	4·820284
50	2500	125000	7·0710678	3·684031	113	12769	1442897	10·6301458	4·834588
51	2601	132651	7·1414284	3·708430	114	12996	1481544	10·6770783	4·848808
52	2704	140608	7·2111026	3·732511	115	13225	1520875	10·7238053	4·862944
53	2809	148877	7·2801099	3·756286	116	13456	1560896	10·7703296	4·876999
54	2916	157464	7·3484692	3·779763	117	13689	1601613	10·8166538	4·890973
55	3025	166375	7·4161985	3·802953	118	13924	1643032	10·8627805	4·904868
56	3136	175616	7·4833148	3·825862	119	14161	1685159	10·9087121	4·918685
57	3249	185193	7·5498344	3·848501	120	14400	1728000	10·9544512	4·932424
58	3364	195112	7·6157731	3·870877	121	14641	1771561	11·0000000	4·946088
59	3481	205379	7·6811457	3·892996	122	14884	1815848	11·0453610	4·959675
60	3600	216000	7·7459667	3·914867	123	15129	1860867	11·0905365	4·973190
61	3721	226981	7·8102497	3·936497	124	15376	1906624	11·1355287	4·986631
62	3844	238328	7·8740079	3·957892	125	15625	1953125	11·1803399	5·000000
63	3969	250047	7·9372539	3·979057	126	15876	2000376	11·2249722	5·013298

No.	Square	Cube.	Square Root.	Cube Root.	No.	Square.	Cube.	Square Root.	Cube Root
127	16129	2048383	11·2694277	5·026526	190	36100	6859000	13·7840488	5·748897
128	16384	2097152	11·3137085	5·039684	191	36481	6967871	13·8202750	5·758965
129	16641	2146689	11·3578167	5·052774	192	36864	7077888	13·8564065	5·768998
130	16900	2197000	11·4017543	5·065797	193	37249	7189057	13·8924440	5·778996
131	17161	2248091	11·4455231	5·078753	194	37636	7301384	13·9283883	5·788960
132	17424	2299968	11·4891253	5·091643	195	38025	7414875	13·9642400	5·798890
133	17689	2352637	11·5325626	5·104469	196	38416	7529536	14·0000000	5·808786
134	17956	2406104	11·5758369	5·117230	197	38809	7645373	14·0356688	5·818272
135	18225	2460375	11·6189500	5·129928	198	39204	7762392	14·0712473	5·828648
136	18496	2515456	11·6619038	5·142563	199	39601	7880599	14·1067360	5·838476
137	18769	2571353	11·7046999	5·155137	200	40000	8000000	14·1421356	5·848035
138	19044	2628072	11·7473444	5·167649	201	40401	8120601	14·1774469	5·857766
139	19321	2685619	11·7898261	5·180101	202	40804	8242408	14·2126704	5·867464
140	19600	2744000	11·8321596	5·192494	203	41209	8365427	14·2478068	5·877130
141	19881	2803221	11·8743421	5·204828	204	41616	8489664	14·2828569	5·886765
142	20164	2863288	11·9163753	5·217103	205	42025	8615125	14·3178211	5·896368
143	20449	2924207	11·9582607	5·229321	206	42436	8741816	14·3527001	5·905941
144	20736	2985984	12·0000000	5·241483	207	42849	8869743	14·3874946	5·915483
145	21025	3048625	12·0415946	5·253588	208	43264	8998912	14·4222051	5·924993
146	21316	3112136	12·0830460	5·265637	209	43681	9123329	14·4568323	5·934473
147	21609	3176523	12·1243557	5·277632	210	44100	9261000	14·4913767	5·943921
148	21904	3241792	12·1655251	5·289572	211	44521	9393931	14·5258390	5·953341
149	22201	3307949	12·2065556	5·301459	212	44944	9528128	14·5602198	5·962731
150	22500	3375000	12·2474487	5·313293	213	45369	9663597	14·5945195	5·972091
151	22801	3442951	12·2882056	5·325074	214	45796	9800344	14·6287388	5·981426
152	23104	3511808	12·3288280	5·336803	215	46225	9938375	14·6628783	5·990727
153	23409	3581577	12·3693169	5·348481	216	46656	10077696	14·6969385	6·000000
154	23716	3652264	12·4096736	5·360108	217	47089	10218313	14·7309199	6·009244
155	24025	3723875	12·4498996	5·371685	218	47524	10360232	14·7648231	6·018463
156	24336	3796416	12·4899960	5·383213	219	47961	10503459	14·7986486	6·027650
157	24649	3869893	12·5299641	5·394691	220	48400	10648000	14·8323970	6·036811
158	24964	3944312	12·5698051	5·406120	221	48841	10793861	14·8660687	6·045943
159	25281	4019679	12·6095202	5·417501	222	49284	10941048	14·8996644	6·055048
160	25600	4096000	12·6491106	5·428835	223	49729	11089567	14·9331845	6·064126
161	25921	4173281	12·6885775	5·440122	224	50176	11239424	14·9666295	6·073178
162	26244	4251528	12·7279221	5·451362	225	50625	11390625	15·0000000	6·082201
163	26569	4330747	12·7671453	5·462556	226	51076	11543176	15·0332964	6·091199
164	26896	4410944	12·8062485	5·473704	227	51529	11697083	15·0665192	6·100170
165	27225	4492125	12·8452326	5·484806	228	51984	11852352	15·0996689	6·109115
166	27556	4574296	12·8840987	5·495865	229	52441	12008989	15·1327460	6·118033
167	27889	4657463	12·9228480	5·506879	230	52900	12167000	15·1657509	6·126925
168	28224	4741632	12·9614814	5·517848	231	53361	12326391	15·1986842	6·135792
169	28561	4826809	13·0000000	5·528775	232	53824	12487168	15·2315462	6·144634
170	28900	4913000	13·0384048	5·539658	233	54289	12649337	15·2643375	6.153449
171	29241	5000211	13·0766968	5·550499	234	54756	12812904	15.2970585	6·162239
172	29584	5088448	13·1148770	5·561298	235	55225	12977875	15·3297097	6·171005
173	29929	5177717	13·1529464	5·572055	236	55696	13144256	15·3622915	6·179747
174	30276	5268024	13·1909060	5·582770	237	56169	13312053	15.3948043	6·188463
175	30625	5359375	13·2287566	5·593445	238	56644	13481272	15·4272486	6·197154
176	30976	5451776	13·2664992	5·604079	239	57121	13651919	15·4596248	6·205821
177	31329	5545233	13·3041347	5·614673	240	57600	13824000	15·4919334	6·214464
178	31684	5639752	13·3416641	5·625226	241	58081	13997521	15·5241747	6.223084
179	32041	5735339	13·3790882	5·635741	242	58564	14172488	15·5563492	6·231679
180	32400	5832000	13·4164079	5·646216	243	59049	14348907	15·5884573	6·240251
181	32761	5929741	13·4536240	5·656651	244	59536	14526789	15·6204994	6·248800
182	33124	6028568	13·4907376	5·667051	245	60025	14706125	15·6524758	6·257324
183	33489	6128487	13·5277493	5·677411	246	60516	14886936	15·6843871	6·265826
184	33856	6229504	13·5646600	5·687734	247	61009	15069223	15·7162336	6·274305
185	34225	6331625	13·6014705	5·698019	248	61504	15252992	15·7480157	6·282760
186	34596	6434856	13·6381817	5·708267	249	62001	15438249	15·7797338	6·291194
187	34969	6539203	13·6747943	5·718479	250	62500	15625000	15·8113883	6·299604
188	35344	6644672	13·7113092	5·728654	251	63001	15813251	15·8429795	6·307993
189	35721	6751269	13·7477271	5·738794	252	63504	16003008	15·8745079	6·316359

No.	Square	Cube.	Square Root.	Cube Root.	No.	Square.	Cube.	Square Root.	Cube Root
253	64009	16194277	15·9059737	6·324704	316	99856	31554496	17·7763888	6·811284
254	64516	16387064	15·9373775	6·333026	317	100489	31855013	17·8044938	6·818462
255	65025	16581375	15·9687194	6·341326	318	101124	32157432	17·8325545	6·825624
256	65536	16777216	16·0000000	6·349604	319	101761	32461759	17·8605711	6·832771
257	66049	16974593	16·0312195	6·357861	320	102400	32768000	17·8885438	6·839904
258	66564	17173512	16·0623784	6·366095	321	103041	33076161	17·9164729	6.847021
259	67081	17373979	16.0934769	6·374311	322	103684	33386248	17·9443584	6·854124
260	67600	17576000	16·1245155	6·382504	323	104329	33698267	17.9722008	6·861212
261	68121	17779581	16·1554944	6·390676	324	104976	34012224	18·0000000	6.868285
262	68644	17984728	16·1864141	6·398828	325	105625	34328125	18·0277564	6·875344
263	69169	18191447	16·2172747	6·406958	326	106276	34645976	18·0554701	6·882388
264	69696	18399744	16·2480768	6·415068	327	106929	34965783	18·0831413	6·889419
265	70225	18609625	16·2788206	6·423158	328	107584	35287552	18·1107703	6·896435
266	70756	18821096	16·3095064	6·431228	329	108241	35611289	18·1383571	6·903436
267	71289	19034163	16·3401346	6·439277	330	108900	35937000	18·1659021	6·910423
268	71824	19248832	16·3707055	6·447305	331	109561	36264691	18·1934054	6·917396
269	72361	19465109	16·4012195	6·455315	332	110224	36594368	18·2208672	6·924355
270	72900	19683000	16·4316767	6·463304	333	110889	36926037	18·2482876	6·931301
271	73441	19902511	16·4620776	6·471274	334	111556	37259704	18·2756669	6·938232
272	73984	20123648	16·4924225	6·479224	335	112225	37595375	18·3030052	6·945149
273	74529	20346417	16·5227116	6·487154	336	112896	37933056	18·3303028	6·952053
274	75076	20570824	16·5529454	6·495065	337	113569	38272753	18·3575598	6·958943
275	75625	20796875	16·5831240	6·502956	338	114244	38614472	18·3847763	6·965819
276	76176	21024576	16·6132477	6·510830	339	114921	38958219	18·4119526	6·972683
277	76729	21253933	16·6433170	6·518684	340	115600	39304000	18·4390889	6·979532
278	77284	21484952	16·6733320	6·526519	341	116281	39651821	18·4661853	6·986368
279	77841	21717639	16·7032931	6·534335	342	116964	40001688	18·4932420	6·993191
280	78400	21952000	16·7332005	6·542133	343	117649	40353607	18·5202592	7·000000
281	78961	22188041	16·7630546	6·549912	344	118336	40707584	18·5472370	7·006796
282	79524	22425768	16·7928556	6·557672	345	119025	41063625	18·5741756	7·013579
283	80089	22665187	16·8226038	6·565415	346	119716	41421736	18·6010752	7·020349
284	80656	22906304	16·8522995	6·573139	347	120409	41781923	18·6279360	7·027106
285	81225	23149125	16·8819430	6·580844	348	121104	42144192	18·6547581	7·033850
286	81796	23393656	16·9115345	6·588532	349	121801	42508549	18·6815417	7·040581
287	82369	23639903	16·9410743	6·596202	350	122500	42875000	18·7082869	7·047298
288	82944	23887872	16·9705627	6·603854	351	123201	43243551	18·7349940	7·054004
289	83521	24137569	17·0000000	6·611489	352	123904	43614208	18·7616630	7·060696
290	84100	24389000	17·0293864	6·619106	353	124609	43986977	18·7882942	7·067376
291	84681	24642171	17·0587221	6·626705	354	125316	44361864	18·8148877	7·074044
292	85264	24897088	17·0880075	6·634287	355	126025	44738875	18·8414437	7.080699
293	85849	25153757	17·1172428	6·641852	356	126736	45118016	18·8679623	7·087341
294	86436	25412184	17·1464282	6·649399	357	127449	45499293	18·8944436	7·093971
295	87025	25672375	17·1755640	6·656930	358	128164	45882712	18·9208879	7·100588
296	87616	25934336	17·2046505	6·664444	359	128881	46268279	18·9472953	7·107194
297	88209	26198073	17·2336879	6·671940	360	129600	46656000	18·9736660	7·113786
298	88804	26463592	17·2626762	6·679420	361	130321	47045881	19·0000000	7·120367
299	89401	26730899	17·2916165	6·686882	362	131044	47437928	19·0262976	7·126936
300	90000	27000000	17·3205081	6·694329	363	131769	47832147	19·0525589	7·133492
301	90601	27270901	17·3493516	6·701759	364	132496	48228544	19·0787840	7·140037
302	91204	27543608	17·3781472	6·709173	365	133225	48627125	19·1049732	7·146569
303	91809	27818127	17·4068952	6·716570	366	133956	49027896	19·1311265	7·153090
304	92416	28094464	17·4355958	6·723951	367	134689	49430863	19·1572441	7·159599
305	93025	28372625	17·4642492	6·731316	368	135424	49836032	19·1833261	7·166096
306	93636	28652616	17·4928557	6·738665	369	136161	50243409	19·2093727	7·172580
307	94249	28934443	17·5214155	6·745997	370	136900	50653000	19·2353841	7·179054
308	94864	29218112	17·5499288	6.753313	371	137641	51064811	19·2613603	7·185516
309	95481	29503629	17·5783958	6·760614	372	138384	51478848	19·2873015	7·191966
310	96100	29791000	17·6068169	6·767899	373	139129	51895117	19·3132079	7·198405
311	96721	30080231	17·6351921	6·775169	374	139876	52313624	19·3390796	7·204832
312	97344	30371328	17·6635217	6·782423	375	140625	52734375	19·3649167	7·211248
313	97969	30664297	17·6918060	6·789661	376	141376	53157376	19·3907194	7·217652
314	98596	30959144	17·7200451	6·796884	377	142129	53582633	19·4164878	7·224045
315	99225	31255875	17·7482393	6·804092	378	142884	54010152	19·4422221	7·230427

No.	Square	Cube.	Square Root.	Cube Root.	No.	Square.	Cube.	Square Root.	Cube Root.
379	143641	54439939	19·4679223	7·236797	442	195364	86350888	21·0237960	7·617412
380	144400	54872000	19·4935887	7·243156	443	196249	86938307	21·0475652	7·623152
381	145161	55306341	19·5192213	7·249504	444	197136	87528384	21·0713075	7·628884
382	145924	55742968	19·5448203	7·255841	445	198025	88121125	21·0950231	7·634607
383	146689	56181887	19·5703858	7·262167	446	198916	88716536	21·1187121	7·640321
384	147456	56623104	19·5959179	7·268482	447	199809	89314623	21·1423745	7·646027
385	148225	57066625	19·6214169	7·274786	448	200704	89915392	21·1660105	7·651725
386	148996	57512456	19·6468827	7·281079	449	201601	90518849	21·1896201	7·657414
387	149769	57960603	19·6723156	7·287362	450	202500	91125000	21·2132034	7·663094
388	150544	58411072	19·6977156	7·293633	451	203401	91733851	21·2367606	7.668766
389	151321	58863869	19·7230829	7·299894	452	204304	92345408	21·2602916	7·674430
390	152100	59319000	19·7484177	7·306143	453	205209	92959677	21·2837967	7·680086
391	152881	59776471	19·7737199	7·312383	454	206116	93576664	21·3072758	7·685733
392	153664	60236288	19·7989899	7·318611	455	207025	94196375	21·3307290	7·691372
393	154449	60698457	19·8242276	7·324829	456	207936	94818816	21·3541565	7·697002
394	155236	61162984	19·8494332	7·331037	457	208849	95443993	21·3775583	7·702625
395	156025	61629875	19·8746069	7·337234	458	209764	96071912	21·4009346	7·708239
396	156816	62099136	19·8997487	7·343420	459	210681	96702579	21·4242853	7·713845
397	157609	62570773	19·9248588	7·349597	460	211600	97336000	21·4476106	7·719442
398	158404	63044792	19·9499373	7·355762	461	212521	97972181	21·4709106	7·725032
399	159201	63521199	19·9749844	7·361918	462	213444	98611128	21·4941853	7·730614
400	160000	64000000	20·0000000	7·368063	463	214369	99252847	21·5174348	7·736188
401	160801	64481201	20·0249844	7·374198	464	215296	99897344	21·5406592	7·741753
402	161604	64964808	20·0499377	7·380322	465	216225	100544625	21·5638587	7·747311
403	162409	65450827	20·0748599	7·386437	466	217156	101194696	21·5870331	7·752861
404	163216	65939264	20·0997512	7·392542	467	218089	101847563	21·6101828	7·758402
405	164025	66430125	20·1246118	7·398636	468	219024	102503232	21·6333077	7·763936
406	164836	66923416	20·1494417	7·404720	469	219961	103161709	21·6564078	7·769462
407	165649	67419143	20·1742410	7·410795	470	220900	103823000	21·6794834	7·774980
408	166464	67917312	20·1990099	7·416859	471	221841	104487111	21·7025344	7·780490
409	167281	68417929	20·2237484	7·422914	472	222784	105154048	21·7255610	7·785993
410	168100	68921000	20·2484567	7·428959	473	223729	105823817	21·7485632	7·791487
411	168921	69426531	20·2731349	7·434994	474	224676	106496424	21·7715411	7·796974
412	169744	69934528	20·2977831	7·441019	475	225625	107171875	21·7944947	7·802454
413	170569	70444997	20·3224014	7·447034	476	226576	107850176	21·8174242	7·807925
414	171396	70957944	20·3469899	7·453040	477	227529	108531333	21·8403297	7·813389
415	172225	71473375	20·3715488	7·459036	478	228484	109215352	21·8632111	7·818846
416	173056	71991296	20·3960781	7·465022	479	229441	109902239	21·8860686	7·824294
417	173889	72511713	20·4205779	7·470999	480	230400	110592000	21·9089023	7·829735
418	174724	73034632	20·4450483	7·476966	481	231361	111284641	21·9317122	7·835169
419	175561	73560059	20·4694895	7·482924	482	232324	111980168	21·9544984	7·840595
420	176400	74088000	20·4939015	7·488872	483	233289	112678587	21·9772610	7·846013
421	177241	74618461	20·5182845	7·494811	484	234256	113379904	22.0000000	7·851424
422	178084	75151448	20·5426386	7·500741	485	235225	114084125	22·0227155	7·856828
423	178929	75686967	20·5669638	7·506661	486	236196	114791256	22·0454077	7·862224
424	179776	76225024	20·5912603	7·512571	487	237169	115501303	22·0680765	7·867613
425	180625	76765625	20·6155281	7·518473	488	238144	116214272	22·0907220	7·872994
426	181476	77308776	20·6397674	7·524365	489	239121	116930169	22·1133444	7·878368
427	182329	77854483	20·6639783	7·530248	490	240100	117649000	22·1359436	7.883735
428	183184	78402752	20·6881609	7·536121	491	241081	118370771	22·1585198	7·889095
429	184041	78953589	20·7123152	7·541986	492	242064	119095488	22·1810730	7·894447
430	184900	79507000	20·7364414	7·547842	493	243049	119823157	22·2036033	7·899792
431	185761	80062991	20·7605395	7·553688	494	244036	120553784	22·2261108	7·905129
432	186624	80621568	20·7846097	7·559526	495	245025	121287375	22·2485955	7·910460
433	187489	81182737	20·8086520	7·565355	496	246016	122023936	22·2710575	7·915783
434	188356	81746504	20·8326667	7·571174	497	247009	122763473	22·2934968	7·921100
435	189225	82312875	20·8566536	7·576985	498	248004	123505992	22·3159136	7·926408
436	190096	82881856	20·8806130	7·582786	499	249001	124251499	22·3383079	7·931710
437	190969	83453453	20·9045450	7·588579	500	250000	125000000	22·3606798	7·937005
438	191844	84027672	20·9284495	7·594363	501	251001	125751501	22·3830293	7·942293
439	192721	84604519	20·9523268	7·600138	502	252004	126506008	22·4053565	7·947574
440	193600	85184000	20·9761770	7·605905	503	253009	127263527	22·4276615	7·952848
441	194481	85766121	21·0000000	7·611662	504	254016	128024064	22·4499443	7·958114

No.	Square	Cube.	Square Root.	Cube Root.	No.	Square.	Cube.	Square Root.	Cube Root.
505	255025	128787625	22·4722051	7·963374	568	322624	183250432	23·8327506	8·218635
506	256036	129554216	22·4944438	7·968627	569	323761	184220009	23·8537209	8·286493
507	257049	130323843	22·5166605	7·973873	570	324900	185193000	23·8746728	8·291344
508	258064	131096512	22·5388553	7·979112	571	326041	186169411	23·8956063	8·296190
509	259081	131872229	22·5610283	7·984344	572	327184	187149248	23·9165215	8·301030
510	260100	132651000	22·5831796	7·989570	573	328329	188132517	23·9374184	8·305865
511	261121	133432831	22·6053091	7·994788	574	329476	189119224	23·9582971	8·310694
512	262144	134217728	22·6274170	8·000000	575	330625	190109375	23·9791576	8·315517
513	263169	135005697	22·6495033	8·005205	576	331776	191102976	24·0000000	8·320335
514	264196	135796744	22·6715681	8·010403	577	332929	192100033	24·0208243	8·325147
515	265225	136590875	22·6936114	8·015595	578	334084	193100552	24·0416306	8·329954
516	266256	137388096	22·7156334	8·020779	579	335241	194104539	24·0624188	8·334755
517	267289	138188413	22·7376340	8·025957	580	336400	195112000	24·0831892	8·339551
518	268324	138991832	22·7596134	8·031129	581	337561	196122941	24·1039416	8·344341
519	269361	139798359	22·7815715	8·036293	582	338724	197137368	24·1246762	8·349126
520	270400	140608000	22·8035085	8·041451	583	339889	198155287	24·1453929	8·353905
521	271441	141420761	22·8254244	8·046603	584	341056	199176704	24·1660919	8·358678
522	272484	142236648	22·8473193	8·051748	585	342225	200201625	24·1867732	8·363446
523	273529	143055667	22·8691933	8·056886	586	343396	201230056	24·2074369	8·368209
524	274576	143877824	22·8910463	8·062018	587	344569	202262003	24·2280829	8·372967
525	275625	144703125	22·9128785	8·067143	588	345744	203297472	24·2487113	8·377719
526	276676	145531576	22·9346899	8·072262	589	346921	204336469	24·2693222	8·382465
527	277729	146363183	22·9564806	8·077374	590	348100	205379000	24·2899156	8·387206
528	278784	147197952	22·9782500	8·082480	591	349281	206425071	24·3104916	8·391942
529	279841	148035889	23·0000000	8·087579	592	350464	207474688	24·3310501	8·396673
530	280900	148877000	23·0217289	8·092672	593	351649	208527857	24·3515913	8·401398
531	281961	149721291	23·0434372	8·097759	594	352836	209584584	24·3721152	8·406118
532	283024	150568768	23·0651252	8·102839	595	354025	210644875	24·3926218	8·410833
533	284089	151419437	23·0867928	8·107913	596	355216	211708736	24·4131112	8·415542
534	285156	152273304	23·1084400	8·112980	597	356409	212776173	24·4335834	8·420246
535	286225	153130375	23·1300670	8·118041	598	357604	213847192	24·4540385	8·424945
536	287296	153990656	23·1516738	8·123096	599	358801	214921799	24·4744765	8·429638
537	288369	154854153	23·1732605	8·128145	600	360000	216000000	24·4948974	8·434327
538	289444	155720872	23·1948270	8·133187	601	361201	217081801	24·5153013	8·439010
539	290521	156590819	23·2163735	8·138223	602	362404	218167208	24·5356883	8·443688
540	291600	157464000	23·2379001	8·143253	603	363609	219256227	24·5560583	8·448360
541	292681	158340421	23·2594067	8·148276	604	364816	220348864	24·5764115	8·453028
542	293764	159220088	23·2808935	8·153294	605	366025	221445125	24·5967478	8·457691
543	294849	160103007	23·3023604	8·158305	606	367236	222545016	24·6170673	8·462348
544	295936	160989184	23·3238076	8·163310	607	368449	223648543	24·6373700	8·467000
545	297025	161878625	23·3452351	8·168309	608	369664	224755712	24·6576560	8·471647
546	298116	162771336	23·3666429	8·173302	609	370881	225866529	24·6779254	8·476289
547	299209	163667323	23·3880311	8·178289	610	372100	226981000	24·6981781	8·480926
548	300304	164566592	23·4093998	8·183269	611	373321	228099131	24·7184142	8·485558
549	301401	165469149	23·4307490	8·188244	612	374544	229220928	24·7386338	8·490185
550	302500	166375000	23·4520788	8·193213	613	375769	230346397	24·7588368	8·494806
551	303601	167284151	23·4733892	8·198175	614	376996	231475544	24·7790234	8·499423
552	304704	168196608	23·4946802	8·203132	615	378225	232608375	24·7991935	8·504035
553	305809	169112377	23·5159520	8·208082	616	379456	233744896	24·8193473	8·508642
554	306916	170031464	23·5372046	8·213027	617	380689	234885113	24·8394847	8·513243
555	308025	170953875	23·5584380	8·217966	618	381924	236029032	24·8596058	8·517840
556	309136	171879616	23·5796522	8·222898	619	383161	237176659	24·8797106	8·522432
557	310249	172808693	23·6008474	8·227825	620	384400	238328000	24·8997992	8·527019
558	311364	173741112	23·6220236	8·232746	621	385641	239483061	24·9198716	8·531601
559	312481	174676879	23·6431808	8·237661	622	386884	240641848	24·9399278	8·536178
560	313600	175616000	23·6643191	8·242571	623	388129	241804367	24·9599679	8·540750
561	314721	176558481	23·6854386	8·247474	624	389376	242970624	24·9799920	8·545317
562	315844	177504328	23·7065392	8·252371	625	390625	244140625	25·0000000	8·549879
563	316969	178453547	23·7276210	8·257263	626	391876	245314376	25·0199920	8·554437
564	318096	179406144	23·7486842	8·262149	627	393129	246491883	25·0399681	8·558990
565	319225	180362125	23·7697286	8·267029	628	394384	247673152	25·0599282	8·563538
566	320356	181321496	23·7907545	8·271904	629	395641	248858189	25·0798724	8·568081
567	321489	182284263	23·8117618	8·286773	630	396900	250047000	25·0998008	8·572619

No.	Square	Cube.	Square Root.	Cube Root.	No.	Square.	Cube.	Square Root.	Cube Root.
631	398161	251239591	25·1197134	8·577152	694	481636	334255384	26·3438797	8·853598
632	399424	252435968	25·1396102	8·581681	695	483025	335702375	26·3628527	8·857849
633	400689	253636137	25·1534913	8·586205	696	484416	337153536	26·3818119	8·862095
634	401956	254840104	25·1793566	8·590724	697	485809	338608873	26·4007576	8·866337
635	403225	256047875	25·1992063	8·595238	698	487204	340068392	26·4196896	8·870576
636	404496	257259456	25·2190404	8·599747	699	488601	341532099	26·4386081	8·874810
637	405769	258474853	25·2388585	8·604252	700	490000	343000000	26·4575131	8·879040
638	407044	259694072	25·2586619	8·608753	701	491401	344472101	26·4764046	8·883266
639	408321	260917119	25·2784493	8·613248	702	492804	345948408	26·4952826	8·887488
640	409600	262144000	25·2982213	8·617739	703	494209	347428927	26·5141472	8·891706
641	410881	263374721	25·3179778	8·622225	704	495616	348913664	26·5329983	8·895920
642	412164	264609288	25·3377189	8·626706	705	497025	350402625	26·5518361	8·900130
643	413449	265847707	25·3574447	8·631183	706	498436	351895816	26·5706605	8·904336
644	414736	267089984	25·3771551	8·635655	707	499849	353393243	26·5894716	8·908538
645	416025	268336125	25·3968502	8·640123	708	501264	354894912	26·6082694	8·912737
646	417316	269586136	25·4165301	8·644585	709	502681	356400829	26·6270539	8·916931
647	418609	270840023	25·4361947	8·649044	710	504100	357911000	26·6458252	8·921121
648	419904	272097792	25·4558441	8·653497	711	505521	359425431	26·6645833	8·925308
649	421201	273359449	25·4754784	8·657946	712	506944	360944128	26·6833281	8·929490
650	422500	274625000	25·4950976	8·662391	713	508369	362467097	26·7080598	8·933668
651	423801	275894451	25·5147016	8·666831	714	509796	363994344	26·7207784	8·937843
652	425104	277167808	25·5342907	8·671266	715	511225	365525875	26·7394839	8·942014
653	426409	278445077	25·5538647	8·675697	716	512656	367061696	26·7581763	8·946181
654	427716	279726264	25·5734237	8·680124	717	514089	368601813	26·7768557	8·950344
655	429025	281011375	25·5929678	8·684546	718	515524	370146232	26·7955220	8·954503
656	430336	282300416	25·6124969	8·688963	719	516961	371694959	26·8141754	8·958658
657	431649	283593393	25·6320112	8·693376	720	518400	373248000	26·8328157	8·962809
658	432964	284890312	25·6515107	8·697784	721	519841	374805361	26·8514432	8·966957
659	434281	286191179	25·6709953	8·702188	722	521284	376367048	26·8700577	8·971101
660	435600	287496000	25·6904652	8·706587	723	522729	377933067	26·8886593	8·975240
661	436921	288804781	25·7099203	8·710983	724	524176	379503424	26·9072481	8·979376
662	438244	290117528	25·7203607	8·715373	725	525625	381078125	26·9258240	8·983509
663	439569	291434247	25·7487864	8·719759	726	527076	382657176	26·9443872	8·987637
664	440896	292754994	25·7681975	8·724141	727	528529	384240583	26·9629375	8·991762
665	442225	294079625	25·7875939	8·728518	728	529984	385828352	26·9814751	8·995883
666	443556	295408296	25·8069758	8·732892	729	531441	387420489	27·0000000	9·000000
667	444889	296740963	25·8263431	8·737260	730	532900	389017000	27·0185122	9·004113
668	446224	298077632	25·8456960	8·741624	731	534361	390617891	27·0370117	9·008223
669	447561	299418309	25·8650343	8·745985	732	535824	392223168	27·0554985	9·012329
670	448900	300763000	25·8843582	8·750340	733	537289	393832837	27·0739727	9·016431
671	450241	302111711	25·9036677	8·754691	734	538756	395446904	27·0924344	9·020529
672	451584	303464448	25·9229628	8·759038	735	540225	397065375	27·1108834	9·024624
673	452929	304821217	25·9422435	8·763381	736	541696	398688256	27·1293199	9·028715
674	454276	306182024	25·9615100	8·767719	737	543169	400315553	27·1477439	9·032802
675	455625	307546875	25·9807621	8·772053	738	544644	401947272	27·1661554	9·036886
676	456976	308915776	26·0000000	8·776383	739	546121	403583419	27·1845544	9·040965
677	458329	310288733	26·0192237	8·780708	740	547600	405224000	27·2029410	9·045041
678	459684	311665752	26·0384331	8·785029	741	549081	406869021	27·2213152	9·049114
679	461041	313046839	26·0576284	8·789346	742	550564	408518488	27·2396769	9·053183
680	462400	314432000	26·0768096	8·793659	743	552049	410172407	27·2580263	9·057248
681	463761	315821241	26·0959767	8·797968	744	553536	411830784	27·2763634	9·061310
682	465124	317214568	26·1151297	8·802272	745	555025	413493625	27·2946881	9·065367
683	466489	318611987	26·1342687	8·806572	746	556516	415160936	27·3130006	9·069422
684	467856	320013504	26·1533937	8·810868	747	558009	416832723	27·3313007	9·073473
685	469225	321419125	26·1725047	8·815160	748	559504	418508992	27·3495887	9·077520
686	470596	322828856	26·1916017	8·819447	749	561001	420189749	27·3678644	9·081563
687	471969	324242703	26·2106848	8·823731	750	562500	421875000	27·3861279	9·085603
688	473344	325660672	26·2297541	8·828009	751	564001	423564751	27·4043792	9·089639
689	474721	327082769	26·2488095	8·832285	752	565504	425259008	27·4226184	9·093672
690	476100	328509000	26·2678511	8·836556	753	567009	426957777	27·4408455	9·097701
691	477481	329939371	26·2868789	8·840823	754	568516	428661064	27·4590604	9·101726
692	478864	331373888	26·3058929	8·845085	755	570025	430368875	27·4772633	9·105748
693	480249	332812557	26·3248932	8·849344	756	571536	432081216	27·4954542	9·109766

No.	Square	Cube.	Square Root.	Cube Root.	No.	Square	Cube.	Square Root.	Cube Root.
757	573049	433798093	27·5136330	9·113781	820	672400	551368000	28·6356421	9·359902
758	574564	435519512	27·5317998	9·117793	821	674041	553387661	28·6530976	9·363705
759	576081	437245479	27·5499546	9·121801	822	675684	555412248	28·6705424	9·367505
760	577600	438976000	27·5680975	9·125805	823	677329	557441767	28·6879766	9·371302
761	579121	440711081	27·5862284	9·129806	824	678976	559476224	28·7054002	9·375096
762	580644	442450728	27·6043475	9·133803	825	680625	561515625	28·7228132	9·378887
763	582169	444194947	27·6224546	9·137797	826	682276	563559976	28·7402157	9·382675
764	583696	445943744	27·6405499	9·141788	827	683929	565609283	28·7576077	9·386460
765	585225	447697125	27·6586334	9·145774	828	685584	567663552	28·7749891	9·390242
766	586756	449455096	27·6767050	9·149757	829	687241	569722789	28·7923601	9·394020
767	588289	451217663	27·6947648	9·153737	830	688900	571787000	28·8097206	9·397796
768	589824	452984832	27·7128129	9·157714	831	690561	573856191	28·8270706	9·401569
769	591361	454756609	27·7308492	9·161686	832	692224	575930368	28·8444102	9·405339
770	592900	456533000	27·7488739	9·165656	833	693889	578009537	28·8617394	9·409105
771	594441	458314011	27·7668868	9·169622	834	695556	580093704	28·8790582	9·412869
772	595984	460099648	27·7848880	9·173585	835	697225	582182875	28·8963666	9·416630
773	597529	461889917	27·8028775	9·177544	836	698896	584277056	28·9236646	9·420387
774	599076	463684824	27·8208555	9·181500	837	700569	586376253	28·9309523	9·424142
775	600625	465484375	27·8388218	9·185443	838	702244	588480472	28·9482297	9·427894
776	602176	467288576	27·8567766	9·189402	839	703921	590589719	28·9654967	9·431642
777	603729	469097433	27·8747197	9·193347	840	705600	592704000	28·9827535	9·435388
778	605284	470910952	27·8926514	9·197289	841	707281	594823321	29·0000000	9·439131
779	606841	472729139	27·9105715	9·201229	842	708964	596947688	29·0172363	9·442870
780	608400	474552000	27·9284801	9·205164	843	710649	599077107	29·0344623	9·446607
781	609961	476379541	27·9463772	9·209096	844	712336	601211584	29·0516781	9·450341
782	611524	478211768	27·9642629	9·213025	845	714025	603351125	29·0688837	9·454072
783	613089	480048687	27·9821372	9·216950	846	715716	605495736	29·0860791	9·457800
784	614656	481890304	28·0000000	9·220873	847	717409	607645423	29·1032644	9·461525
785	616225	483736625	28·0178515	9·224791	848	719104	609800192	29·1204396	9·465247
786	617496	485587656	28·0356915	9·228707	849	720801	611960049	29·1376046	9·468966
787	619369	487443403	28·0535203	9·232619	850	722500	614125000	29·1547595	9·472682
788	620944	489303872	28·0713377	9·237528	851	724201	616295051	29·1719043	9·476395
789	622521	491169069	28·0891438	9·240433	852	725904	618470208	29·1890390	9·480106
790	624100	493039000	28·1069386	9·244335	853	727609	620650477	29·2061637	9·483813
791	625681	494913671	28·1247222	9·248234	854	729316	622835864	29·2232784	9·487518
792	627264	496793088	28·1424946	9·252130	855	731025	625026375	29·2403830	9·491220
793	628849	498677257	28·1602557	9·256022	856	732736	627222016	29·2574777	9·494919
794	630436	500566184	28·1780056	9·259911	857	734449	629422793	29·2745623	9·498615
795	632025	502459875	28·1957444	9·263797	858	736164	631628712	29·2916370	9·502308
796	633616	504358336	28·2134720	9·267680	859	737881	633839779	29·3087018	9·505998
797	635209	506261573	28·2311884	9·271559	860	739600	636056000	29·3257566	9·509685
798	636804	508169592	28·2488938	9·275435	861	741321	638277381	29·3428015	9·513370
799	638401	510082399	28·2665881	9·279308	862	743044	640503928	29·3598365	9·517051
800	640000	512000000	28·2842712	9·283178	863	744769	642735647	29·3768616	9·520730
801	641601	513922401	28·3019434	9·287044	864	746496	644972544	29·3938769	9·524406
802	643204	515849608	28·3196045	9·290907	865	748225	647214625	29·4108823	9·528079
803	644809	517781627	28·3372546	9·294767	866	749956	649461896	29 4278779	9·531749
804	646416	519718464	28·3548938	9·298624	867	751689	651714363	29·4448637	9·535417
805	648025	521660125	28·3725219	9·302477	868	753424	653972032	29·4618397	9·539082
806	649636	523606616	28·3901391	9·306328	869	755161	656234909	29·4788059	9·542744
807	651249	525557943	28·4077454	9·310175	870	756900	658503000	29·4957624	9·546403
808	652864	527514112	28·4253408	9·314019	871	758641	660776311	29·5127091	9·550059
809	654481	529475129	28·4429253	9·317860	872	760384	663054848	29·5296461	9·553712
810	656100	531441000	28·4604989	9·321697	873	762129	665338617	29·5465734	9·557363
811	657721	533411731	28·4780617	9·325532	874	763876	667627624	29·5634910	9·561011
812	659344	535387328	28·4956137	9·329363	875	765625	669921875	29·5803989	9.564656
813	660969	537367797	28·5131549	9·333192	876	767376	672221376	29·5972972	9·568298
814	662596	539353144	28·5306852	9·337017	877	769129	674526133	29·6141858	9·571938
815	664225	541343375	28·5482048	9·340838	878	770884	676836152	29·6310648	9·575574
816	665856	543338496	28·5657137	9·344657	879	772641	679151439	29·6479325	9·579208
817	667489	545338513	28·5832119	9·348473	880	774400	681472000	29·6647939	9·582840
818	669124	547343432	28·6006993	9·352286	881	776161	683797841	29·6816442	9·586468
819	670761	549353259	28·6181760	9·356095	882	777924	686128968	29·6984848	9·590094

No.	Square	Cube.	Square Root.	Cube Root.
883	779689	688465387	29·7153159	9·593716
884	781456	690807104	29·7321375	9·597337
885	783225	693154125	29·7489496	9·600955
886	784996	695506456	29·7657521	9·604570
887	786769	697864103	29·7825452	9·608182
888	788544	700227072	29·7993289	9·611791
889	790321	702595369	29·8161030	9·615398
890	792100	704969000	29·8328678	9·619002
891	793881	707347971	29·8496231	9·622603
892	795664	709732288	29·8663690	9·626201
893	797449	712121957	29·8831056	9·629797
894	799236	714516984	29·8998328	9·633390
895	801025	716917375	29·9165506	9·636981
896	802816	719323136	29·9332591	9·640569
897	804609	721734273	29·9499583	9·644154
898	806404	724150792	29·9666481	9·647737
899	808201	726572699	29·9833287	9·651317
900	810000	729000000	30·0000000	9·654894
901	811801	731431701	30·0166620	9·658468
902	813604	733870808	30·0333148	9·662040
903	815409	736314327	30·0499584	9·665609
904	817216	738763264	30·0665928	9·669176
905	819025	741217625	30·0832179	9·672740
906	820836	743677416	30·0998339	9·676302
907	822649	746142643	30·1164407	9·679860
908	824464	748613312	30·1330383	9·683416
909	826281	751089429	30·1496269	9·686970
910	828100	753571000	30·1662063	9·690521
911	829921	756058031	30·1827765	9·694069
912	831744	758550528	30·1993377	9·697615
913	833569	761048497	30·2158899	9·701158
914	835396	763551944	30·2324329	9·704699
915	837225	766060875	30·2489669	9·708237
916	839056	768575296	30·2654919	9·711772
917	840889	771095213	30·2820079	9·715305
918	842724	773620632	30·2985148	9·718835
919	844561	776151559	30·3150128	9·722363
920	846400	778688000	30·3315018	9·725888
921	848241	781229961	30·3479818	9·729411
922	850084	783777448	30·3644529	9·732931
923	851929	786330467	30·3809151	9·736448
924	853776	788889024	30·3973683	9·739963
925	855625	791453125	30·4138127	9·743476
926	857476	794022776	30·4302481	9·746986
927	859329	796597983	30·4466747	9·750493
928	861184	799178752	30·4630924	9·753998
929	863041	801765089	30·4795013	9·757500
930	864900	804357000	30·4959014	9·761000
931	866761	806954491	30·5122926	9·764497
932	868624	809557568	30·5286750	9·767992
933	870489	812166237	30·5450487	9·771484
934	872356	814780504	30·5614136	9·774974
935	874225	817400375	30·5777697	9·778462
936	876096	820025856	30·5941171	9·782946
937	877969	822656953	30·6104557	9·785429
938	879844	825293672	30·6267857	9·788909
939	881721	827936019	30·6431069	9·792386
940	883600	830584000	30·6594194	9·795861
941	885481	833237621	30·6757233	9·799334
942	887364	835896888	30·6920185	9·802804
943	889249	838561807	30·7083051	9·806271
944	891136	841232384	30·7245830	9·809736
945	893025	843908625	30·7408523	9·813199
946	894916	846590536	30·7571130	9·816659
947	896809	849278123	30·7733651	9·820117
948	898704	851971392	30·7896086	9·823572
949	900601	854670349	30·8058436	9·827025
950	902500	857375000	30·8220700	9·830476
951	904401	860085351	30·8382879	9·833924
952	906304	862801408	30·8544972	9·837369
953	908209	865523177	30·8706981	9·840813
954	910116	868250664	30·8868904	9·844254
955	912025	870983875	30·9030743	9·847692
956	913936	873722816	30·9192497	9·851128
957	915849	876467493	30·9354166	9·854562
958	917764	879217912	30·9515751	9·857993
959	919681	881974079	30·9677251	9·861422
960	921600	884736000	30·9838668	9·864848
961	923521	887503681	31·0000000	9·868272
962	925444	890277128	31·0161248	9·871694
963	927369	893056347	31·0322413	9.875113
964	929296	895841344	31·0483494	9·878530
965	931225	898632125	31·0644491	9·881945
966	933156	901428696	31·0805405	9·885357
967	935089	904231063	31·0966236	9·888767
968	937024	907039232	31·1126984	9·892175
969	938961	909853209	31·1287648	9·895580
970	940900	912673000	31·1448230	9·898983
971	942841	915498611	31·1608729	9·902383
972	944784	918330048	31·1769145	9·905782
973	946729	921167317	31·1929479	9·909178
974	948676	924010424	31·2089731	9·912571
975	950625	926859375	31·2249900	9·915962
976	952576	929714176	31·2409987	9·919351
977	954529	932574833	31·2569992	9·922738
978	956484	935441352	31·2729915	9·926122
979	958441	938313739	31·2889757	9·929504
980	960400	941192000	31·3049517	9·932884
981	962361	944076141	31·3209195	9·936261
982	964324	946966168	31·3368792	9·939636
983	966289	949862087	31·3528308	9·943009
984	968256	952763904	31·3687743	9·946380
985	970225	955671625	31·3847097	9·949748
986	972196	958585256	31·4006369	9·953114
987	974169	961504803	31·4165561	9·956477
988	976144	964430272	31·4324673	9·959839
989	978121	967361669	31·4483704	9·963198
990	980100	970299000	31·4642654	9·966555
991	982081	973242271	31·4801525	9·969909
992	984064	976191488	31·4960315	9·973262
993	986049	979146657	31·5119025	9·976612
994	988036	982107784	31·5277655	9·979960
995	990025	985074875	31·5436206	9·983305
996	992016	988047936	31·5594677	9·986649
997	994009	991026973	31·5753068	9·989990
998	996004	994011992	31·5911380	9·993329
999	998001	997002999	31·6069613	9·996666
1000	1000000	1000000000	31·6227766	10·000000

5 2 29 | 998001 | 18414 21, 31, 201 &c | 38024 19 × 20 = 380

77 63 = 7 × 9
63 77 × 7 = 539
539 × 9 = 4851

13 13 + 4 = 17
14 3 × 4 = 12
182 170 + 12
teen by teen

81754
11
4, 4 + 5,
5 + 7 &c.

6 × 2 = 12
2 × 2 = 4 + 1 = 5
6 × 4 = 24 + 5 = 29
3 × 2 = 6 + 2 = 8
6 × 5 = 30
2 × 4 = 8 + 30 + 8 = 46
3 × 4 = 12 + 4 = 16
2 × 5 = 10 + 16 = 26
3 × 5 = 1872 = 17

any no. by any no. 326

26
42
92

(1)
6 × 2 = 12
2 × 2 = 4 + 1 = 5
6 × 4 = 24 + 5 = 29
2 × 4 = 8 + 2 = 10

(2) 542
176692

Why we teach division by factors.
reasons given by class 1909-10.

1. Sometimes quicker. 2. Less awkward.
3. Of use in reduction. 4. Assists in G.C.M. & L.C.M.
5. Pupils more apt to be correct in using small divisors.
6. Leads up to cancellation.
7. Gives opportunity for mental work.
8. Helps to a better understanding of division.
9. If Short Div. has been taught, larger divisors can be used.
10. More reasoning required, ∴ better training for p's.
11. Ch. get an idea of the composition of nos.

12. Helps to understand div. by such nos. as 60, 600, 6000 &c
13. May be used as a proof of division.